On the Radio Capacity of a Spread Spectrum Cognitive Radio System

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Abstract: - In this paper, the problem of estimating the optimum radio capacity of a spread spectrum non-cellular Cognitive Radio (CR) system, operating in a Rayleigh fading environment, is examined. In contrast to previously published works, here, the radio capacity is expressed in terms of the number of simultaneously transmitting secondary users, considering only one primary user, which receives the spread spectrum simultaneously transmitted signals, through an optimum maximal-ratio combiner (MRC) RAKE receiver. Specifically, the problem to finding the number of secondary simultaneously transmitting users, which maximizes the normalized average channel capacity (expressed in bits/sec/Hz) available to primary user, is considered. Then, the theoretical analysis, avoiding the application of a complex and lengthy simulation process, leads to a simple but novel closed-form expression for the system's optimum radio capacity, which is related with the system's parameters and can be useful for the practical design of a CR system and for an initial quantitative analysis. Finally, numerical results and respective graphs are presented to illustrate the presented analysis.

Key-Words: Cognitive Radio systems, Radio capacity, Channel capacity, Rayleigh fading.

1 Introduction

The explosive growth of wireless applications along the increased demand for higher data rates imposes a challenge to develop more efficient spectrum utilization schemes. A future wireless system should be able to coexist with many others wireless technologies in a heterogeneous environment. The idea of CR developed firstly by Mitola, [1], is a promising way to solve this problem. Based on the spectrum sensing technology, the cognitive user can not only use the free spectrum, but also share the same channel with the primary user, if only the interference at the primary receivers caused by the cognitive users does not exceed a cer-tain threshold, i.e. the interference temperature, a concept proposed by the Federal Communications Commission (FCC), [2-6].

Recently, there have been many research works in the literature addressing different aspects of dynamic spectrum access (DSA) and information theory, [7-11]. In information theory, the channel capacity expression, (in the Shannon-Hartley sense), [12], establishes an upper bound limit for reliable information transmission over a band-limited additive white gaussian noise (AWGN) environment. When the channel side information (CSI) is not avail-able at the transmitter, the source data is transmitted at a constant rate. Since no CSI is available at the transmitter, data transmission takes place over all fading states including deep fades where the data is lost and hence the effective channel capacity is significantly reduced. In mobile radio, where signal fading is a considerable capacity degradation factor, channel capacity can be estimated in an average sense and used as a figure of merit for sys-tem operation, [13]. This average channel capacity formula would indeed provide the true channel capacity, if CSI were available at the receiver, [14].

Hence, in this paper, a novel closed-form expression for the optimum radio capacity, expressed here, in terms of the number of simultaneously transmitting secondary users of a spread spectrum non-cellular CR system, operating in a Rayleigh fading environment, is de-rived. Specifically, the radio capacity i.e. the number of simultaneously transmitting secondary users is estimated in the case where, the available average channel capacity of prima-ry user is maximized. The final expression is useful for the initial practical design of a spread spectrum CR system, specifically for the operation in a fading environment. The final simple but novel equation, theoretically derived, to the author's best knowledge, is the first time such expression has been exposed, thus avoiding complex algorithms or lengthy simulations. A comparison of the results derived here, with alternative results previously published in literature, [15,16], is not possible because these methodologies assume specific conditions for the system's operation and no one of these considers the case of the maximization of the average channel capacity available to the primary user. It must be noticed that, the analysis that follows does not solve the problem of the capacity region of primary and secondary users, but gives an optimistic upper bound, in an average sense. However, a simulation process, based on fact that, the channel capacity respects to error-free transmission and not to a specific bit error rate (BER) value, must be described analytically in future, in order to compare with the theoretical results of this paper, as also the case of multiple simultaneously transmitting primary and secondary users, in a multiple access CR system operating in a fading environment.

The remainder of this paper is organized as follows. Section 2 describes the system's model applied. In section 3, the operation of the considered CR system in a non-fading AWGN environment is examined. In section 4, the problem to estimate the optimum radio capacity of the considered spread spectrum CR non-cellular system, in a Rayleigh fading environment, is considered. The numerical results are presented in section 5, while the conclusions are outlined in the last section of the paper.

2 System's Model

We assume that, the considered spread spectrum non-cellular CR system consists of m simultaneously transmitting secondary users, each of them transmitting in parallel, without loss of generality, the same spread spectrum of bandwidth *W*ss, after spreading the same baseband signal by the system's processing gain and where the suffice 'ss' indicates the spread spectrum transmission This spread spectrum CR system is applicable to the up-link of satellites, wireless local-area networks and single-cell indoor mobile radio systems. Although a dynamic user population is a reasonable assumption for a CR practical system, the theoretical results derived in the paper can be applied directly in a CR system, with a variable number of secondary users, considering that the number of secondary users m, represents the mean value of secondary users in a birth-death model describing the variable allocation of users, [17].

In addition, it is assumed that, in the considered CR system, exists only one primary user which receives the desired signal of bandwidth *W*ss with average transmitted power *S*p from one respective primary transmitting user (the new suffice 'p' indicates the primary transmitting user).

3 Operation in a non-fading AWGN Environment

Considering the primary user, the channel capacity (in bits/sec) required for error-less transmission of a signal of bandwidth W_{ss} will be given by the Shannon-Hartley theorem when arbitrarily complex coding and delay is applied, [12]:

$$\mathrm{C}_p = W_{ss} \cdot \log_2(1+\gamma_p) \tag{1}$$

where γ_p is the signal-to-interference plus noise ratio (SINR) received by the primary user, expressed as:

$$\gamma_p = \frac{S_p}{N_0 \cdot W_{ss} + \sum_{i=1}^{m} S_i} = \frac{S_p}{N_0 \cdot W_{ss} + m \cdot S_s} \tag{2}$$

assuming that, the total average interference power, resulting from the *m* simultaneously transmitting secondary users, is Gaussian distributed, [18], and where N_0 is the noise power spectral density of the AWGN channel and the suffice '*s*' indicates each one of the secondary transmitting users.. However, in eq.(2) a power control scheme is assumed, in order that the primary user receives equal average power S_i=S_s, 1≤*i*≤*m*, from all the secondary system's users. It must be noticed, that, this is not the real cognitive radio scenario, where all the secondary users control it's transmit power in order to maintain an interference limit on the primary user, but it is considered here, only for theoretical purposes and can be, in practice, the worst case scenario.

4 Estimation of Optimum Radio Capacity in a Rayleigh Fading Environment

The Rayleigh fading channel assigned to each system's transmitting user, is modeled as a tapped delay line, [19]. The radio channel is modeled as a slowly fading, time-invariant and discrete multipath channel and, thus, it appears to be frequency-selective to the transmitted spread spectrum signals. We assume that all the associated spread spectrum signals are received by a MRC RAKE receiver. In particular, the MRC RAKE receiver of the only one primary user, has M_{ss} taps corresponding to M_{ss} resolvable signal paths, on the condition that the transmitted spread spectrum signal bandwidth W_{ss} is much greater than the coherence bandwidth W_{coh} of the fading channel, [20], given by:

$$M_{ss}=[W_{ss}\cdot \Delta]+1 \tag{3}$$

where Δ is the total multipath spread of the Rayleigh fading channel (assumed known or measurable and much less than the bit interval in order to avoid inter-symbol interference (ISI)), and [.] returns the largest integer less than, or equal to, its argument. Although the number of resolvable paths M_{ss} may be a random number, it is approximated by eq.(3) in order to simplify the followed mathematical presentation. However, the performance of the MRC RAKE receiver depends on the number of the employed taps and the fading channel estimation. If the number of taps is less than the resolvable paths' number, the receiver performance will substantially be degraded because the power of the remaining "branches" will appear at the receiver output as self-noise power. Hence, in this paper, we consider the optimum operation of the MRC RAKE receivers where the number of taps employed is equal to the number M_{ss} of resolvable paths as given by eq.(3).

In general, the multipath-intensity profile (MIP) in a Rayleigh fading environment is exponential, but, in the present analysis, the MIP in a Rayleigh fading environment is assumed discrete and constant, so that the "resolvable" path model can be considered to have equal path strengths on the average. In a MRC RAKE receiver, the output's decision variable is identical to the decision variable which corresponds to the output of a M-branch space diversity MRC technique, with $M=M_{ss}$, [20]. Consequently, the MRC reception of spread signal, achieved by the considered RAKE receiver, is equivalent to a M_{ss}-branch space diversity MRC technique. Therefore, the probability density function (pdf) of the combined instantaneous SINR $\gamma_{p,c}$ of the spread signal over the bandwidth W_{ss}, with no correlation among the M_{ss} branches, will follow the distribution, [20],:

$$\mathrm{p}_{\gamma_{p,c}}\left(\gamma_{p,c}\right)=\frac{1}{(M_{ss}-1)!}\cdot\frac{\gamma_{p,c}^{\;M_{ss}-1}}{\left\langle\gamma_p\right\rangle^{\mathrm{M_{ss}}}}\cdot\exp\left(-\frac{\gamma_{p,c}}{\left\langle\gamma_p\right\rangle}\right) \tag{4}$$

where $\langle\gamma_p\rangle$ is the average received spread SINR value in the k-th, $k=[1,\ldots,M_{ss}]$, diversity branch (assumed equal for all the M_{ss} branches), M_{ss} is obtained from eq.(3), $\langle.\rangle$ indicates the average value and the suffice 'c' refers to the combined SINR. The statistics of each interfering signal, resulting for each secondary transmitting user, in eq.(4) need not considered separately since, either the total interference power at the MRC RAKE receiver output, even for a small number of secondary users, tends to be Gaussian, [18], and thus it can directly be incorporated in the Shannon-Hartley formula regardless the interference statistics.

We now estimate the average channel capacity $<C_p>$ available to the primary user, under the previously described MRC RAKE reception. Following eq.(4), the channel capacity C_p is averaged over the pdf of the combined SINR $\gamma_{p,c}$ at the MRC RAKE receiver output, so that:

$$\left\langle C_p\right\rangle = W_{SS}\cdot\int_0^\infty \log_2\left(1+\gamma_{p,c}\right)\cdot p_{\gamma_{p,c}}\left(\gamma_{p,c}\right)\cdot \mathrm{d}(\gamma_{p,c}) \tag{5}$$

and taking into account eq.(4):

$$\left\langle C_p\right\rangle=W_{ss}\cdot\int_0^\infty\log\left(1+\gamma_{p,c}\right)\cdot\frac{1}{(M_{ss}-1)!\left\langle\gamma_p\right\rangle^{M_{ss}}}\cdot\gamma_{p,c}^{\;M_{ss}-1}\cdot\exp\left(-\frac{\gamma_{p,c}}{\left\langle\gamma_p\right\rangle}\right)\mathrm{d}(\gamma_{p,c}) \tag{6}$$

Clearly, this channel capacity estimation indicates the average channel capacity value that appears at the MRC RAKE primary user's receiver output.

In the following, we consider the problem to finding the number m of secondary simultaneously transmitting users, which maximizes the normalized average channel capacity (expressed in bits/sec/Hz) available to primary user $\langle C_p\rangle/W_{ss}$, given an average transmit power constraint. Then, this problem of maximization of the normalized average channel capacity, can be stated as follows:

$$\operatorname{argmax}\left[\frac{\left\langle C_p\right\rangle}{W_{ss}}\right]_{m_{\mathrm{op}}} \tag{7}$$

where the new suffice 'op' refers to the estimated optimum value. Eq.(7), can be rewritten equivalently in the form:

$$\max\int_0^\infty\log_2\left(1+\gamma_{p,c}\right)\cdot\frac{\left(\gamma_{p,c}\right)^{\mathrm{M}-1}}{(M_{ss}-1)!\left(\left\langle\gamma_{\mathrm{p}}\right\rangle\right)^{M_{ss}}}\cdot\exp\left(-\frac{\gamma_{p,c}}{\left\langle\gamma_{\mathrm{p}}\right\rangle}\right)\cdot\mathrm{d}\gamma_{p,c} \tag{8}$$

The combined average spread SINR after diversity reception i.e. $M_{ss}\cdot\langle\gamma_p\rangle$, that maximizes eq.(8), equals to 6 dB, [13,21], i.e.,

$$M_{ss}\cdot\left\langle\gamma_{\mathrm{p}}\right\rangle=M_{ss}\cdot\frac{S_p}{N_0\cdot W_{ss}+m\cdot\cdot S_s}=10^{0.6} \tag{9}$$

where $\langle\gamma_p\rangle$ is the average SINR received by the primary user, in a Rayleigh fading channel, given, also in this case by eq.(2), since represents the final average received spread SINR in a Rayleigh fading channel and therefore is independent from the statistics of each separate Rayleigh fading interfering signal. Eq.(9) can be rewritten as:

$$M_{ss}\cdot\frac{\dfrac{S_p}{N_0\cdot W_{ss}}}{1+m\cdot\dfrac{S_s}{N_0\cdot W_{ss}}}=M_{ss}\cdot\frac{\gamma_{p,snr}}{1+m\cdot\gamma_{s,inr}}=10^{0.6} \quad (10)$$

where $\gamma_{p,snr}$ is the signal-to-noise ratio (SNR) received by the primary user over the bandwidth W_{ss} and $\gamma_{s,inr}$ is the interference-to-noise ratio (INR) received by the primary user over the bandwidth W_{ss}, due to secondary user transmission (the suffice '*inr*' refers to the INR value). Then, the optimum radio capacity m_{op}, can be found directly from eq.(10), as following i.e.:

$$m_{op}=\frac{M_{ss}}{10^{0.6}}\cdot\frac{\gamma_{p,snr}}{\gamma_{s,inr}}-\frac{1}{\gamma_{s,inr}}= \\ =\left(\frac{[W_{ss}\cdot\Delta]+1}{10^{0.6}}\right)\cdot\frac{\gamma_{p,snr}}{\gamma_{s,inr}}-\frac{1}{\gamma_{s,inr}} \quad (11)$$

5 Numerical Results

Eq.(11) serves as a general expression for estimating the optimum radio capacity m_{op} of a CR spread spectrum system, when Rayleigh fading is present. Then, the optimum radio capacity, m_{op}, is plotted as function of the SNR $\gamma_{s,inr}$ resulting from each one of the m_{op} simultaneously transmitting secondary users (expressed in dB) for a Rayleigh fading channel in Fig.1, for, W_{ss}=1.25MHz, total multipath delay spread of an urban Rayleigh fading channel: Δ=3μsec and $\gamma_{p,snr}$=20 dB. In addition, in Fig.2, the optimum radio capacity, m_{op}, is plotted as function of the number M_{ss} of resolvable signal paths assuming $\gamma_{p,snr}$=30 dB and $\gamma_{s,inr}$=20 dB. As it can be seen directly from Fig.1, the optimum radio capacity, m_{op}, is seriously decreased as the SNR $\gamma_{s,inr}$ is increased indicating that, the optimum radio capacity is seriously limited by the total interference power that appears in this case.

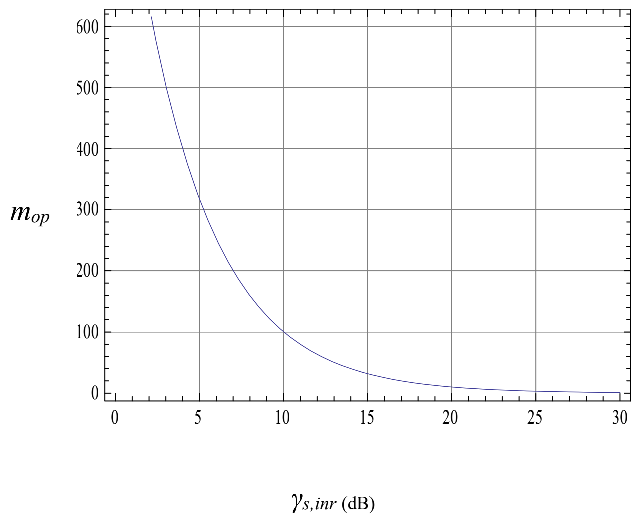

Fig.1 Optimum radio capacity m_{op} (in number of secondary users) of a CR spread spectrum system versus the SNR $\gamma_{s,inr}$ (expressed in dB) when operating in a Rayleigh fading environment for: $\gamma_{p,snr}$=20 dB, W_{ss}=1.25MHz and Δ=3μsec.

In addition, as it can be concluded from Fig.2, the optimum radio capacity, m_{op}, is increased when the number M_{ss} of resolvable signal paths is increased, indicating that, in this case, the "inherent" diversity potential provided by the spread spectrum transmission, can mitigate the total interference power that appears in a such CR spread spectrum system, leading directly to an increased system's radio capacity.

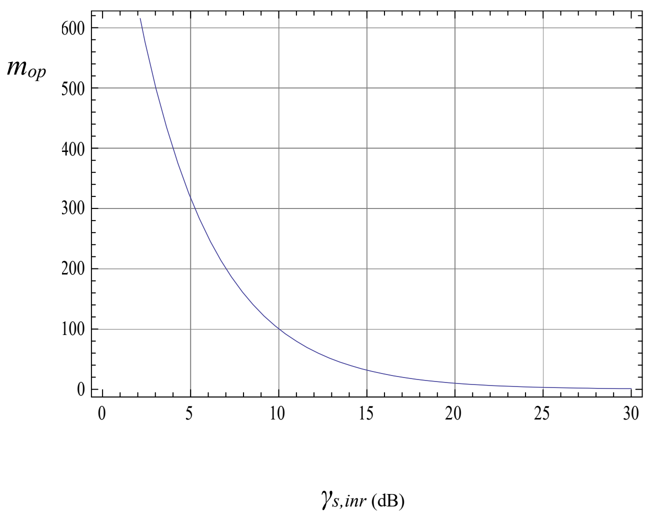

Fig.2 Optimum radio capacity m_{op} (in number of secondary users) of a CR spread spectrum system versus the number M_{ss} of resolvable signal paths when operating in a Rayleigh fading environment for: $\gamma_{p,snr}$=30 dB and $\gamma_{s,inr}$=20 dB.

Finally, in Fig.3, the optimum radio capacity, m_{op}, is plotted as function of the total multipath delay spread Δ of the Rayleigh fading channel. As it can be seen concluded from Fig.3, when the total multipath delay spread of the fading channel is increased, the optimum radio capacity can be also increased since, in this case, significant "inherent" diversity potential is achieved. In totally, you could say that overall all Fig.1, 2 and 3, relates the optimum radio capacity with all the critical system's parameters and provide a tool for a quantitative analysis of the considered CR system, under Rayleigh fading conditions.

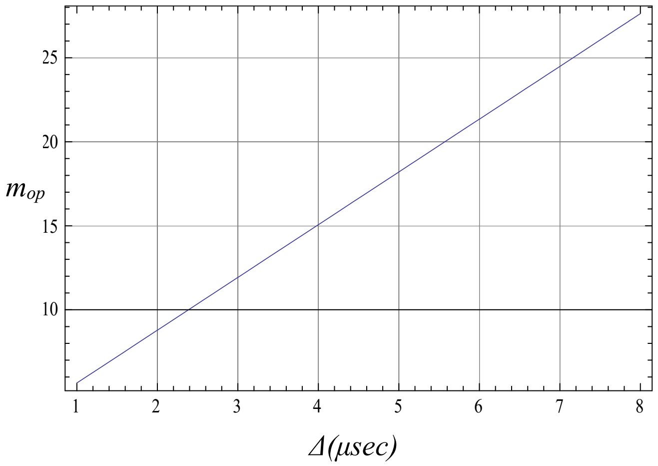

Fig.3 Optimum radio capacity m_{op} (in number of secondary users) of a CR spread spectrum system versus the total multipath delay spread Δ (expressed in μsec) of a Rayleigh fading channel for: W_{ss}=1.25MHz, $\gamma_{p,snr}$=30 dB and $\gamma_{s,inr}$=20 dB.

6 Conclusions

In this paper, a novel closed-form expression for the optimum radio capacity of a spread spectrum non-cellular CR system, when operating in a Rayleigh fading environment, has been derived. The optimum radio capacity estimation is based on the maximization of the average channel capacity (in the Shannon-Hartley sense) available to the primary user, under MRC RAKE reception. Although the final mathematical expression is simple, can be useful for an initial practical design of a CR non-cellular Rayleigh fading system, avoiding the use of a complex simulation process, and is applicable to the uplink of satellites, wireless local area networks or single-cell indoor mobile radio systems. It must be noticed that, the derived expression relates the optimal radio capacity with the critical system's parameters i.e., the average received SNR values, the totally allocated system's bandwidth, the signal's transmitted bandwidth and the total multipath delay spread of the fading channel. Finally, numerical results are presented to illustrate the analysis and show the effect of the total interference power on the system's optimum radio capacity. However, the case of multiple primary users and the description of a simulation process, are open research problems.

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