A Multiplex Transmission Scheme Based on Lattice Ruduction Decoding for MIMO System

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Abstract: - Lenstra-Lenstra-Lovász (LLL) is an effective lattice reduction algorithm in multi-input-multi-output (MIMO) system. The use of LLL lattice reduction can significantly improve the performance of zero-forcing and successive interference cancellation decoders in MIMO communications.But for conventional single channel transmission, when the data streams go through poor channel condition, it usually need higher transmission signal noise ratio(SNR) at transmit side to overcome the poor condition and guarantee better receiving performance at receive side.This may be a challenge for designing transmitter and receiver.In this paper, we introduce a multiplex transmission scheme based on LLL algorithm that transmits the same bit streams for multiple times.By means of appling LLL detection and hard decision majority logic decoding to the multi-channel streams of data, the final detection results will reach a lower bit error rate(BER) level at the same SNR. So it needs lower transmit power compared to original single channel transmission. Performance gain will be obtained by this multiplex transmission and majority logic decoding.
Key-Words: - LLL Algorithm, Multiplex Transmission, Majority Logical Decoding, MIMO System

1 Introduction

Multi-input multi-output (MIMO) systems have been widely adopted in several modern wireless 3G and 4G standards. Owing to its high spectral efficiency and large coverage, MIMO is still the key technology in future 5G standards to provide high data rate and throughput [1]-[2]. In MIMO communication systems, lattice reduction (LR) represents a main stream of decoding techniques for enhancing the performance of MIMO digital communication systems [3]-[4].When used in conjunction with traditional linear and nonlinear detectors, LR can substantially close the gap to fundamental performance limits with little additional system complexity [5].In 1982, A. K. Lenstra, H. Lenstra, Jr., J., and L. Lovász first introduced Lenstra-Lenstra-Lovász (LLL) algorithm which is regarded as the most practical and commonly used lattice reduction. LLL algorithm features polynomial complexity with respect to dimension n and several LLL-aided detectors can achieve the maximum receive diversity like maximum likelihood (ML) decoding [6]. By means of proximity factors analysis method, performance gap between LLL and ML decoding has been identified [7].

At first, these lattice reduction detectors are based on the traditional LLL reduction algorithm that was originally introduced for reducing real lattice basis .Recently complex Lenstra-Lenstra-Lovász algorithm (CLLL) straightforwardly performs the LR with a complex-valued matrix [8]. CLLL expands the definition of reducing basis to complex field and this can achieve a saving in complexity of nearly a half. Simulation results show CLLL requires less arithmetic operations than LLL algorithm and LR-aided linear equalizers collect the same diversity other as that exploited by the ML detector but with lower complexity [9]-[10].

On account of LLL algorithm pursues the optimal performance, so the whole complexity of LLL is extremely high. On the other hand, LLL algorithm features polynomial complexity with respect to the dimension n, this feature may not be enough strong for combination of quite a few equalizers [11].LLL algorithm consists of size reduction and column swap procedure. Only the algorithm satisfies Lovász condition, can the basis do column swap .Plenty of the recent research aims at changing Lovász condition for largely improving the speed of arithmetic operating and speeding the convergence of the algorithm. Capitalizing on the observation that the decision region of successive interference cancellation (SIC) is determined by Gram-Schmidt vectors rather than the basis itself, E-LLL is proposed for using in SIC decoding and sphere decoding. Effective LLL (E-LLL) [12] algorithm loosely imposes an ascending order on diagonal elements of channel matrix. So E-LLL will be a weaker version of LLL algorithm that it has a provable complexity bound $O\left(n^3 \log n\right)$ which is one order lower than LLL algorithm $O\left(n^4 \log n\right)$ [13].Furthermore, an even weaker criterion called diagonal reduction [14] only imposes one single constraint on diagonal elements .Diagonal reduction is weaker than E-LLL reduction, however like the E-LLL reduction , it has identical performance with LLL algorithm when applied for the sphere decoding and SIC decoding. It improves the efficiency of the E-LLL algorithm by significantly reducing the size reduction operations.

Meanwhile, except modifying conditions of size reduction and Lovász condition, redesigning process of algorithm is also the research focus. A feasible possible swap LLL algorithm (PSLLL) [15] for lattice reduction is modified by searching for the next column swap through the whole basis instead of the sequential procedure in the original LLL algorithm. Comparing to the LLL algorithm, the PSLLL algorithm enjoys fast termination property and lower computational complexity which can benefit practical hardware implementation. Other kinds of schemes combined with LLL algorithm, such as greedy column traverse strategy [16] and fast-Givens rotation scheme [17] also largely reduce the algorithm complexity. A slice of other details of MIMO detection technology and lattice reduction based on LLL algorithm can be seen in [18]-[20].

In original MIMO system, when we want the bit error rate (BER) reach a certain value, we sometimes have to improve signal noise ratio (SNR) at transmission side and choose a proper channel detection algorithm. For example, in our simulation when in an 8*8 MIMO system, that transmit antennas and receive antennas are both 8, in other to reach the level of BER at 10^{-5} , we should improve the transmission SNR to at least 22dB and the same time use LLL as signal detection scheme. LLL owns the best performance among the family of LLL algorithms so we choose it as detection strategy. When the transmission signal meets poor channel condition, in order to maintain the level of BER the transmission SNR have to continue to improving.

In this paper, we try to put forward a novel multiplex transmission scheme based on LLL algorithm. We choose three channel binary transmissions as an example. A binary imformation source has been generated and this time we transmit it for three times. Each bit streams will go through different channel condition and subsequently we use LLL algorithm as MIMO detection scheme. At the receiving terminal, we will get the final detection results of these three detection results. Theoretically, if there exists no bit error in all the three detection results, final detection results of the three channels will be the same. But usually the channel condition at different time and different environment changes a lot, so the final results may different from each other. At each bit location, if the transmission SNR is properly set, the probability of correct detection will be greater than the probability of error detection. So based on this criterion, we use multiplex transmission to acquire the correction copy of detection results and also we introduce the majority logic that by comparing the bit value at the same location to judge that 0 or 1 which is correct. If most of the detection results at the same location are 1, which means detection results of two channels or all the three channels is 1, so we have a large confidence that the detection results of this position is 1. So by means of majority logic among multiplex transmission channels, we can further modify the bit errors of the final detection results after detection.

Considering that at original transmission, if we want to acquire the final BER of 10^{-5}, we have to settle the transmission of SNR at least 22dB. But this time we introduce majority logic decoding among multiplex transmission channels, we can lower the SNR a little so that we can save large transmission power. As we know, with the higher of SNR, the higher of transmission power we should pay. Also a higher power amplifier is difficult to design and require higher expense. We should point out that all this theoretical analysis is based on that the transmission SNR is settled properly to maintain the final BER at an appropriate level. When the SNR is lower than a level, there exists no coding gain by using majority logic decoding scheme.

The rest of paper is organized as follows. Section Ⅱ presents the universally acknowledged MIMO system model and gives a brief introduction to lattice reduction algorithm. Details specification of LLL algorithm will be discussed in Section Ⅲ. In Section Ⅳ, we demonstrate ideas of the majority logic decoding scheme based on multiplex transmission .Our simulation results are shown in Section Ⅴ and at last a brief conclusion is given in Section Ⅵ.

Notation: In this article, $\Re(x)$ and $\Im(x)$ denote the real and imaginary part of x respectively .The inner product in the complex Euclidean space between vectors u and v is defined as $\langle u,v\rangle = u^H v$ and the Euclidean length is $\|u\| = \sqrt{\langle u,u\rangle}$ in $\Re^n$. Symbol $\lceil \bullet \rfloor$ represents an arbitrary integer closest to x and the transpose, Hermitian transpose, inverse of a matrix H defined by H^T, H^H, H^{-1} respectively. Expectation of a matrix H is represented by $E(H)$ and σ^2 is the variance. The big O notation $f(x) = O(g(x))$ means for sufficiently large x, $f(x)$ is bounded by a constant times $g(x)$ in absolute value.

2 MIMO System Model and LLL Algorithm

2.1 MIMO System Model

Here we use an $N \times M$ channel matrix to denote a MIMO system with M transmit antennas and N receive antennas. It is assumed that the transmitted signal at the m-th transmit antenna is x_m, and data received at the n-th receive antenna is y_m ($x_m \in \mathbb{Z} + j\mathbb{Z}$, $y_n \in \mathbb{Z} + j\mathbb{Z}$). A common signal alphabet $\mathbb{S}$ is used for all x_m. Over the MIMO channel, the received signal vector is represented as follows:

$$y = Hx + n \quad (1)$$

Matrix H consists of $M \times N$ independent and identically-distributed ($i.i.d$) complex Gaussian coefficients with zero mean and unit variance. Note that n is assumed to be an $i.i.d$ complex Gaussian vector with unit variance, $E[nn^H] = 2\sigma^2 I$.

2.2 Lattice Reduction

Definition 1 (Definition 1.9 [19]): Let $n \geq 1$ and let $x_1, x_2, x_3, ..., x_n$ be a basis of $\mathbb{R}^n$. The lattice with dimension n and basis $x_1, x_2, x_3, ..., x_n$ is the set L of all linear combinations of the basis vectors with integral coefficients:

$$L = \mathbb{Z}x_1 + \mathbb{Z}x_2 + ... + \mathbb{Z}x_n = \left\{ \sum_{i=1}^{n} a_i x_i \,\middle|\, a_1, a_2, ..., a_n \in \mathbb{Z} \right\} \quad (2)$$

The basis can be seen as span or generate of lattice and each lattice may be spanned by not only one basis. Different basis are linked by unimodular matrix whose determination ±1 and each of matrix element is integer.

Definition 2 (Lemma 1.10 [19]): $x_1, x_2, x_3, ..., x_n$ and $y_1, y_2, y_3, ..., y_n$ are two basis for the same lattice $L \in \mathbb{R}^n$. Let X, Y be the $n \times n$ matrix with x_i, y_i in row i for $j = 1,2,3,...,n$. The conclusion is that we can use a unimodular matrix to link the two basis x_i, y_i.

$$Y = C \times X \quad (3)$$

The Goal of lattice reduction is to find a lower complexity unimodular matrix to simplifying the basis. Once the unimodular matrix is obtained, the decoding procedure is listing below:

$$H = G \times U \quad (4)$$

$$y = G \times U \times s + n \quad (5)$$

$$y = G \times c + n \quad (6)$$

Where $c = U \times s$, U is a unimodular matrix and $s = U^{-1}c$.

2.3 LLL Algorithm

LLL algorithm is proposed to find a matrix whose column vectors are nearly orthogonal to generate the same lattice. Using LLL algorithm, the lattice reduction can be performed for the M-basis MIMO system with the channel matrix of size $N \times M$. In this paper, we only concentrate on real-valued matrix for lattice reduction in MIMO system.

Definition 3 (LLL Reduction [18]): A basis $H \in \mathbb{C}^{m \times n}$ is an LLL reduction process with $\delta(\frac{1}{4} < \delta < 1)$, and the upper triangular factor $R \triangleq [r_{i,j}]$ in its QR decomposition $H_r = Q_r \times R_r$ satisfies the following inequalities:

$$\left|[R_r]_{l,k}\right| \leq \frac{1}{2}\left|[R_r]_{l,l}\right| \quad ,1 \leq l < k \leq 2M \quad (7)$$

$$\delta [R_r]_{k-1,k-1}^2 \leq [R_r]_{k,k}^2 + [R_r]_{k-1,k}^2 \quad ,\ k = 2,...,2M \quad (8)$$

Where $[R_r]_{l,k}$ denotes the $(l,k)th$ entry of R_r. Ineq. (3) is the Lovász condition, and basis will do the column swap procedure only when Ineq. (3) is violated. δ is a real number arbitrarily chosen from $(\frac{1}{4},1)$ while $\delta=\frac{3}{4}$ is widely shared as meeting a good complexity-quality trade-off coefficient. At last, the LLL algorithm generates a LLL-reduced matrix from the real-valued channel matrix H_r.

3 A Novel Multiplex Transmission Scheme Based on Majority Logical Decoding

In this paper, we mainly consider the binary transmission. For original strategy of single channel transmission, if we want to receive a certain level of BER, we have to provide enough SNR at receiving terminal. Now we introduce a novel multiplex transmission scheme that can further ruduce the transmission power and improve the performance of whole system (in Fig.1).

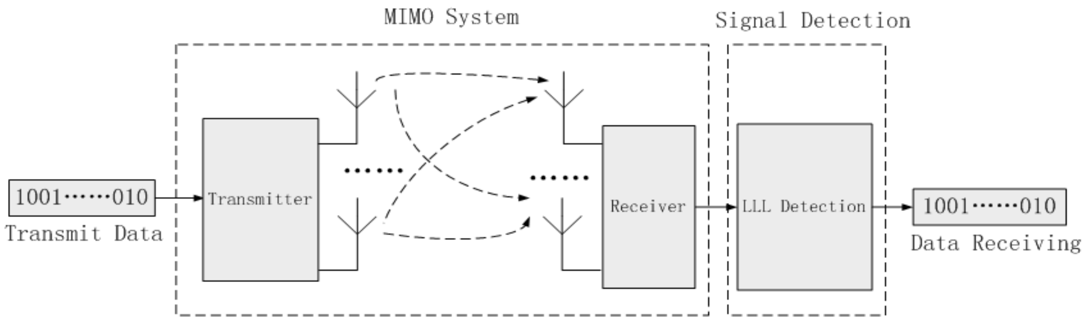

Fig 1 Original Binary Transmission and LLL algorithm for MIMO Detection

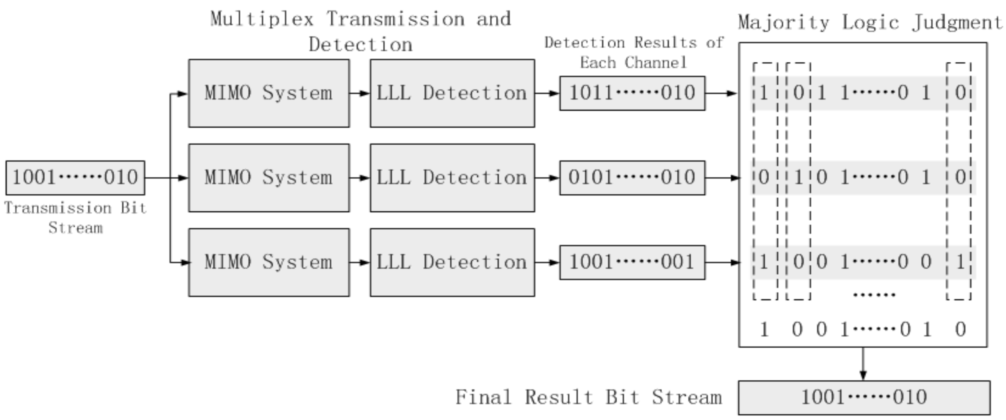

Fig 2 Multiplex Transmission Scheme Based on Hard Decision Majority Logic Decoding in Binary Transmission

We change the single transmission to multiplex transmission. We use three channels as an example to analysis. For the novel transmit strategy, the same data is transmitted for three times that the same three bit streams will go through different channel condition. By means of the MIMO detection algorithm and setting proper SNR at receiving terminal, if these entire three bit streams are corrected exactly, we certainly acquire the same detection results at three terminals respectively. But as mentioned above, non-error detection or higher order of BER usually needs higher SNR also means higher transmision power. It will enhance the difficulty for designing such signal receivers. Also the transmission power will usually be set extremely high to overcome the harmful channel condition. Different from single transmission, we have another two bit streams for further correcting the bit errors by doing majority logic decoding. At the same bit location, we do majority logic decoding that if there exist two or more same results such as 0, we have enough confidence that this bit at transmit terminal is 0(seen in Fig.2).Here in our analysis, we only consider uncoded consequence and majority logic decoding is based on hard decision.

Here we use mathematical formula to verify the theory. For example, at a certain SNR, if in each transmit bit stream, one bit has a probability x of error free transmission. Each channel is independent but for simply analysis, channel parameter is the same and we compute the probability of various cases, for a fixed bit location in all three bit streams:

All the three transmissions are right: x^3

There exist two error free transmissions: $C_3^2 x^2 (1-x)$

There exists only one error free transmission: $C_3^1 x (1-x)^2$

All the three transmissions are wrong: $(1-x)^3$

According to hard decision majority logic decoding criterion, that when there exists more than two error free transmissions that at a certain location the detection result is right. Details are listed in Table 2.

Table 1 Probability of Majority Logic Decoding

	Correct Decision		Wrong Decision	
Prob	x^3	$C_3^2 x^2 (1-x)$	$C_3^1 x (1-x)^2$	$(1-x)^3$

We will calculate the range of improving the probability of error free transmission:

$$x^3 + C_3^2 x^2 (1-x) > x \, , x \in \left(\frac{1}{2}, 1\right) \quad (9)$$

When probability of error free transmission belongs to the range of $\left(\frac{1}{2}, 1\right)$, multiplex transmission based on majority logic decoding will definitely improve the performance of system's reliability.

If we want the probability of error free transmission after hard decision majority logic decoding to exceed 0.999, which equals to $BER < 10^{-3}$:

$$x^3 + C_3^2 x^2 (1-x) > 0.999 \, , x \in (0.9816, 1) \quad (10)$$

Here we selcet three kinds of situations to analysis the performance of multiplex transmission based on hard decision majority logic decoding:

A. $BER = 10^{-1}$

For example, if the probability of error free decoding is $x = 0.9$, that means the BER equals to 10^{-1} . As a view of evaluation of a communication system, $BER = 10^{-1}$ usually indicates the performance of this system extremely poor. After applying the multiplex transmission and majority logic decoding criterion, the probability of error free decoding improves:

$$x^3 + C_3^2 x^2 (1-x) = 0.9^3 + C_3^2 0.9^2 * 0.1 = 0.972 \quad (11)$$

$0.972 > 0.9$, so the whole reliability of communication system is improved. But usually, the BER of a system may not too poor like the example.

B. $BER = 10^{-3}$

If the probability of error free decoding is $x = 0.999$, that means the BER is 10^{-3} and from the simulation results below, the receiving SNR usually is settled more than 20dB. After applying the multiplex transmission and majority logic decoding criterion, the probability of error free decoding improves:

$$x^3 + C_3^2 x^2 (1-x) = 0.999^3 + C_3^2 0.999^2 * 0.001 \approx 0.99999 \quad (12)$$

$0.99999 > 0.9$, that means system's BER equals to 1×10^{-5} .This improves nearly two orders that compared with original single channel transmission. So the multiplex transmission can largely improve the communication performance especially at a better channel condition.

If we can modify some bit error with the help of majority logic decoding, that we may not apply so much SNR at receiving terminal and transmit power. Although the three way transmission detection results don't arrive at a expect BER, by means of hard decision majority logic decoding can attribute non-error detection. So we may save a lot of transmission power and obtain coding gain.

Based on this multiplex transmission, if we continue adding more transmission path to transmit the same data, majority logic decoding will be more reliable. Details of simulation results will be discussed in section Ⅴ.

Also we should point out that if we insert error control coding technique and soft values decision, the whole multiplex transmission scheme will behave better.This need further research.

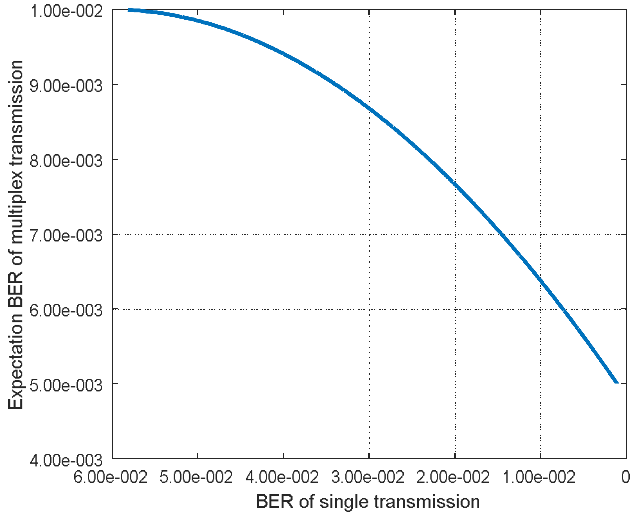

Fig 3 Expectation BER of multiplex transmission after applying majority logic decoding

Fig.3 shows the relationship between the error free probability of each channel (x-axis) and the final results of BER of hard decision majority logic decoding (y-axis).With the channel condition get better, the expectation BER of multiplex transmission is nearly more than one order higher than original singel channel transmission.

4 Simulation Results

In this section, we use computer simulations to verify theoretical claims on multiplex transmission schemes based on hard majority logic decoding. Channel matrix H and white Gaussian noise n are randomly generated. Each channel owns different channel parameters, but the same transmission channel at different receiving SNR will go through the same channel condition. Constellation mapping is settled 16QAM.Size of system we select is 4×4 (4 transmitting antennas and 4 receiving antennas) and 8×8.Total transmission bits are 100000 bits.SNR is defined as symbol energy per transmit antenna versus noise power spectral density. We use coding gain to measure that at a certain level of BER, the difference in SNR between two systems measure the performance among different algorithms at a fixed bit error rate (BER).LLL is chosen as the fixed MIMO detection algorithm in the all simulations.

Fig.3 and Fig.4 are respectively based on different size of MIMO system while other parameters are settled the same.Fig.3 is simulated under the 4×4 MIMO system and Fig.4 is simulated under the 8×8 MIMO system.

Because of the uncertainty of channel condition, only one way transmission may not guarantee at a certain SNR. In both Fig.3 and Fig.4, three single channel transmissions are plotted. For example in Fig.3, when settle SNR=24, BER of the third channel only reaches 1.7×10^{-3} .This bit error rate may not be satisfied. Sometimes the reliability in single transmission scheme is unknown. So we introduce the multiplex transmission scheme. In both 4×4 and 8×8 MIMO system, majority logic decoding can improve the coding gain comparing to other three single way of transmission. The novel majority logic decoding scheme can achieve respectively 2.61dB and 1.07dB coding gain at BER=10^{-4} compared to the best receiving in both 4×4 and 8×8 systems(we can see from the Fig.3 and Fig.4 that the first transmission is the best in each system).

With the number of transmission antennas and receiving antennas increasing, coding gain between the majority logic decoding and single transmission will be reduced. How to design a better scheme to achieve higher coding gain in large MIMO system is needs further researches. Higher size of system will influence the effect of detection. If the channel condition is better, coding gain might be dropped and if the channel condition is poor, that majority logic decoding will be more efficient. For example in Fig.3, the second channel is worse than the first one, but it can improve more coding gain by using multiplex transmission majority logic decoding.

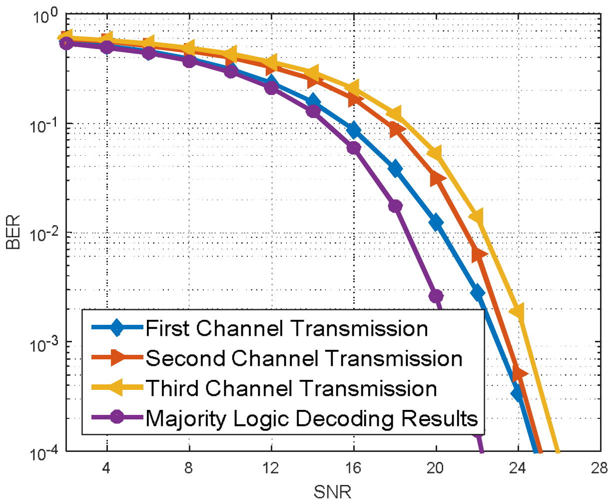

Fig 4 Simulation results of majority logic decoding and other original single transmission in a 4×4,16QAM system, 100000 bits

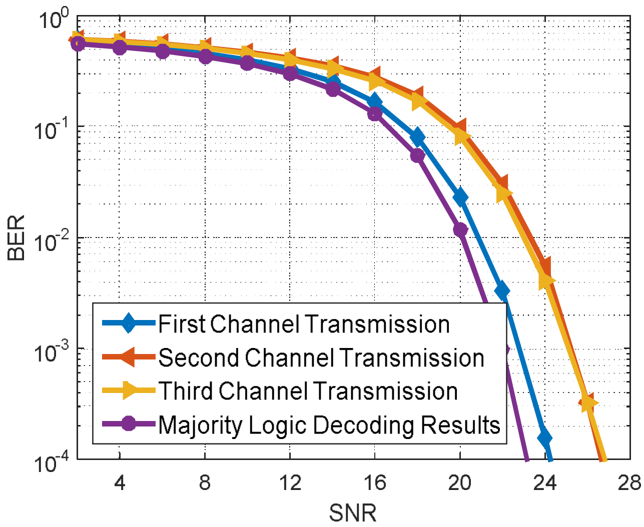

Fig 5 Simulation results of majority logic decoding and other original single transmission in an 8×8,16QAM system, 100000 bits

5 Conclusions

In this paper, we introduce a multiplex transmission scheme based on hard decision majority logic decoding. Original single transmission scheme usually go through poor channel condition, so this may be a challenge for signal detection and receiving. But for the multiple transmissions, after detecting of each bit stream, we apply hard decision majority logic decoding to them. By majority logic decoding, another coding gain can be obtained so that at certain level of BER, multiplex transmission can save a lot of transmission power.

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