# A scheme of multi-type spectrum aggregation under constrained collision probability[1]

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*Abstract*: In cognitive radio networks, secondary users may temporarily occupy dynamic sub-channels originally authorized to primary users. With the increase of the number of aggregated dynamic channels for secondary users, there is an increase of collision probability leading to degradation of the overall system. In this paper, the effects of the number of aggregated dynamic sub-channels on the collision probability are studied, and the system capacity is analyzed. Based on this, an aggregation scheme is designed. By the scheme and under collision probability constraint, the optimal number of aggregated dynamic sub-channels is obtained. Simulation results show that the optimal number of aggregated channels and the maximum capacity are gained while the collision probability is bounded below the collision tolerable level.

*Key-Words:* multi-type spectrum, collision constraint, collision probability, dynamic sub-channel, spectrum aggregate

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# 1 Introduction

In CR (cognitive radio) networks, secondary users (i.e., unauthorized users) may temporarily occupy idle spectrum (namely spectrum hole) originally authorized to primary users (i.e., authorized users) through spectrum sensing technology [1]. For dynamic spectrum resources, a primary user has the absolute priority to use them at any time even these resources are being occupied by secondary users. That is to say, if the collision on a channel occurs between a primary user and a secondary user, the latter has to quit or switch to another new channel. The data packets must be left in the buffer queue until there is a new channel. So, the occurrence of collision is bound to cause extra delay. Short-time interference may affect the performance of not only secondary users but also primary users [2].

By aggregating more than one available channel, the performance of secondary users can be improved. [3]. Meanwhile, the cost of payments can be reduced. However，the occurrence of collision may lead to a decline in the performance of secondary users. The existing spectrum aggregation strategies of secondary users mainly aim at improving spectrum utilization without considering the impact of collision. In [4-5], the studies focused on how many dynamic sub-channels should be aggregated so as to accommodate much more secondary users and improve data speed. In [6-7], a spectrum aggregation scheme with limited hardware was investigated, and an aggregation scheme was proposed to obtain higher network throughput. In [8-11], the authors presented a spectrum allocation scheme with collisions, and further studied the collision probability. The works in aforementioned literatures were done under the situation that a secondary user only use a single channel. Researches on the aggregation of dynamic and static spectrum are not yet fully studied. In this paper, a scheme of aggregation with dynamic and static spectrum resources is proposed to optimize the system performance under the constraint of collision.

# 2 Analysis of Collision Probability and System Capacity

## 2.1 Collision Probability

The collision probability is defined as

$$P_C = \lim_{x\to\infty} \frac{N_C(T)}{N(T)} \tag{1}$$

where $N_c(T)$ is the total number of collisions, and $N(T)$ is the total number of transmissions for secondary users.

The tolerable maximum collision probability for secondary users is presented by the $\xi_{th}$. So, $P_c < \xi_{th}$ indicates that the spectrum resource is available. The value of $\xi_{th}$ is relevant to the performance of not only secondary users but also primary users. Here, a primary user's performance of QoS (Quality of Service) is represented by $\xi_{th}$ [12][13].

## 2.2 System Capacity

To meet the QoS requirement of a secondary user, it is assumed that the total number of sub-channels to be aggregated is $n$ including $n_d$ dynamic sub-channels and $n-n_d$ static sub-channels. The $SNR$ of each channel can be represented as

$$\gamma_i = \frac{|h_i|^2}{\sigma^2} * \frac{P_{\max}}{n_d}, i = \{1,2,\ldots,n_d\} \tag{2}$$

Where $P_{\max}$ is the maximum power of base station, $h_i$ is the gain coefficient of i$^{th}$ channel, which is subject to complex Gaussian random distribution where the average is 0 and the variance is $\sigma^2$.

The capacity of channel $i$ with bandwidth $B$ is defined as

$$C_i = B\log_2(1+\gamma_i) \tag{3}$$

Substituting $\gamma_i$ into Equ. (3), the $C_i$ can be expressed as

$$C_i\left(n_d, \frac{|h_i|^2}{\sigma^2}\right) = B\log_2\left(1+\frac{|h_i|^2}{\sigma^2} * \frac{P_{\max}}{n_d}\right) \tag{4}$$

Assuming that every channel has the same bandwidth, so the capacity of channel is only related to $SNR$ [14]. The bandwidth can be simplified as $B=1$, and then the sum of capacity of dynamic channels is

$$C_d = \frac{n_d}{N}\sum_{i=1}^{N} E\left[\log_2\left(1+\frac{|h_i|^2}{\sigma^2} * \frac{P_{\max}}{n_d}\right)\right]$$

The capacity of static channels is

$$C_0 = (n-n_d)\log_2\left(1+\frac{S_0}{N_0}\right) \tag{6}$$

Where $\frac{S_0}{N_0}$ is the SNR of static channels.

The total capacity for aggregated channels is

$$C_{total} = C_d + C_0 = \frac{n_d}{N}\sum_{i=1}^{N} E\left[\log_2\left(1+\frac{|h_i|^2}{\sigma^2} * \frac{P_{\max}}{n_d}\right)\right] + (n-n_d)\log_2\left(1+\frac{S_0}{N_0}\right) \tag{7}$$

## 3 Aggregation scheme under collision constraint

To obtain the optimal value of $n_d$ and the maximum capacity of channels, the solution can be expressed as

$$maxC_{total} = max\left\{\frac{n_d}{N}\sum_{i=1}^{N} E\left[\log_2(1+\frac{|h_i|^2}{\sigma^2}*\frac{P_{\max}}{n_d})\right] + (n-n_d)\log_2(1+\frac{S_0}{N_0})\right\} \quad (8)$$

While the Equ. (9) holds

$$P_{c,n} \le \xi_{th} \quad (9)$$

Where $P_{c,n}$ is the collision probability between the primary system and the secondary system.

### 3.1 Effect of aggregated channel number on system capacity

Since the logarithmic is a convex function, the Jensen's inequality [15] is used to derive the capacity of dynamic sub-channels. The inequality equation is

$$C_d = \frac{n_d}{N}\sum_{i=1}^{N} E\left[\log_2(1+\frac{|h_i|^2}{\sigma^2}*\frac{P_{\max}}{n_d})\right] \le \frac{n_d}{N}\sum_{i=1}^{N}\log_2\left(1+\frac{\gamma_i}{n_d}\right) \quad (10)$$

For the sake of description, let

$$C^*_{total} = \frac{n_d}{N}\sum_{i=1}^{N}\log_2\left(1+\frac{\gamma_i}{n_d}\right) + (n-n_d)*\log_2\left(1+\frac{S_0}{N_0}\right),$$

then

$$C_{total} \le C^*_{total} \quad (11)$$

$C^*_{total}$ is obviously a continuous function of $n_d$, hence,

$$\frac{\partial C^*_{total}}{\partial n_d} = \frac{1}{N\ln 2}\sum_{i=1}^{N}\left[\ln\left(1+\frac{\gamma_i}{n_d}\right) - \frac{\gamma_i}{n_d+\gamma_i}\right] - \log_2\left(1+\frac{S_0}{N_0}\right) \quad (12)$$

$$\frac{\partial^2 C^*_{total}}{\partial^2 n_d} = \frac{1}{N\ln 2}\sum_{i=1}^{N}\left[\frac{-\gamma^2}{(n_d+\gamma_i)^2}\right] < 0 \quad (13)$$

Where $\gamma_i = \gamma, \forall i$. Equ. (13) shows that two order derivative to variable $n_d$ of the function is always less than 0, so the first order derivative function decreases monotonically. For $0 \le n_d \le n$, the following results are obtained.

$$\begin{cases} \dfrac{\partial C^*_{total}}{\partial n_d} > 0, n_d = 0 \\ \dfrac{\partial C^*_{total}}{\partial n_d} < 0, n_d = n \end{cases}$$

Obviously, the first derivative has zero point. Let variable $x_0$, $(0 < x_0 < n_d)$ represents the zero point. So, the maximum total capacity can be gained while $n_d = x_0$ holds.

### 3.2 Effect of aggregated channel number on collision probability

For dynamic sub-channels, the collision probability is $P_c$, i.e., $P_c(i) = P_c, \forall i \in n_d$. The collision probability of each static sub-channel equals zero, i.e. $P_c(j) = 0, \forall j \in (n-n_d)$. $P_{c,n}$ can be further simplified as.

$$\begin{aligned} P_{c,n} &= 1-\prod_{i=1}^{n_d}\left[1-P_c(i)\right]\cdot\prod_{j=1}^{n-n_d}\left[1-P_c(j)\right] \\ &= 1-\prod_{i=1}^{n_d}[1-P_c(i)] = 1-(1-P_c)^{n_d} \end{aligned} \quad (14)$$

$P_{c,n}$ increases with $n_d$ and is independent of the number of static sub-channels. Assuming that one data packet occupies one channel, the collision probability of a single channel during a transmission cycle $T_d$ is

$$P_c = \frac{\lambda_p * T_d}{N_{total} - \lambda_p * T_s} \quad (15)$$

For $n$ aggregated sub-channels,

$$P_{c,n} = 1-\left(1-\frac{\lambda_p * T_d}{N_{total} - \lambda_p * T_s}\right)^{n_d} \quad (16)$$

Where $T_d$ is the transmission periodic of secondary system. During this period, at least one data packet of primary users arrives. $T_s$ is the system's sensing cycle.

Based on the above discussions, the flowchart of the scheme is as follows.

**Step 1:** Initializing parameters, such as $\lambda_p$, $T_p$, $T_s$, $n$, $n_s$, $\xi_{th}$, $R^{req}$, etc.

**Step 2:** Calculating the collision probability $P_{c,n}$

**Step 3:** If $P_{c,n} \le \xi_{th}$ holds, the process ends and $n_s^{'} = n_d^*$ is the optimal solution, otherwise,

$$\begin{cases} n_d^{'} = n_d^{'} - 1 \\ n_s^{'} = n_s^{'} + 1 \end{cases}$$

If $0 \le n_d^{'} \le n_d$ , $0 \le n_s^{'} \le n_s$ and $n_s^{'} + n_d^{'} \le n$ hold, it returns to Step 2, otherwise, the process ends.

### 3.3 Solution of optimal number of aggregated dynamic channels

$C_{total}$ and $P_{c,n}$ are the functions of the number of aggregated channels. Given the threshold of collision probability $\xi_{th}$, there may exists an optimal solution to meet Equ. (8) and Equ. (9). Meanwhile, the variable $n_d$ must be an integer. From Equ. (9) and Equ. (14), we can get

$$1-(1-P_c)^{n_d} \le \xi_{th} \tag{17}$$

The inequality equation is rewritten as

$$n_d \le \frac{\ln(1-\xi_{th})}{\ln(1-P_c)}, 1 \le n_d \le N \tag{18}$$

The optimal solution $n_d^*$ must be the largest integer to meet the inequality Equ. (9). so the optimal solution is

$$n_d^* = \left\lfloor \frac{\ln(1-\xi_{th})}{\ln(1-P_c)} \right\rfloor, 1 \le \left\lfloor \frac{\ln(1-\xi_{th})}{\ln(1-P_c)} \right\rfloor \le N \tag{19}$$

Where $\lfloor x \rfloor$ represents the maximum number of integers that are no more than $x$.

Substituting $P_c$ into the Eq. (19), here is

$$n_d^* = \left\lfloor \frac{\ln(1-\xi_{th})}{\ln(1-\frac{\lambda_p * T_d}{N_{total} - \lambda_p * T_s})} \right\rfloor \tag{20}$$

The maximum system capacity obtained by aggregated dynamic channels is

$$C_d = \left\lfloor \frac{\ln(1-\xi_{th})}{\ln\left(1-\frac{\lambda_p * T_d}{N_{total} - \lambda_p * T_s}\right)} \right\rfloor \cdot \log_2\left(1+\frac{\gamma}{\frac{\ln(1-\xi_{th})}{\ln\left(1-\frac{\lambda_p * T_d}{N_{total} - \lambda_p * T_s}\right)}}\right)$$

Then the total system capacity is

$$C_{total} = C_o + C_d = (n - n_d^*) \cdot \log_2\left(1+\frac{S_0}{N_0}\right) + C_d$$

$$= \left\lfloor n - \left\lfloor \frac{\ln(1-\xi_{th})}{\ln\left(1-\frac{\lambda_p * T_d}{N_{total} - \lambda_p * T_s}\right)} \right\rfloor \right\rfloor \cdot \log_2\left(1+\frac{S_0}{N_0}\right) + C_d \tag{12}$$

## 4 Simulations and Performance Evaluation

### 4.1 Analysis of collision probability

In this section, the performance of the proposed aggregation scheme is evaluated through MATLAB R2010 simulation platform. Specific simulation parameters are set as shown in Table 1.

Table 1 Simulation parameters

| Parameters | Value |
|---|---|
| Bandwidth of a sub-channel $B$ | 200kHz |
| Request rate of Unauthorized user $R^{req}$ | 1 |
| Number of static channels for unauthorized user | 10 |
| Number of sub-channels for an unauthorized request $n$ | 20 |
| Total number of dynamic sub-channels of authorized user $N_{total}$ | 30 |
| Data packet arrival rate of authorized user $\lambda_p$ | 0~3/s |
| Data packet arrival rate of service time $1/\mu_p$ | 1s |
| TTI $= T_d$ | 0.1s |
| Base station sensing period $T_s$ | 0.08s |
| Collision Threshold $\xi_{th}$ | 0.005 |
| Total transmit power $P_{max}$ | 1 |
| Transmission cycle number | 10 |

The collision probability is evaluated with different number of dynamic sub-channels: 10, 11, 12, 13, and 14. The simulation results are shown in Fig. 1.

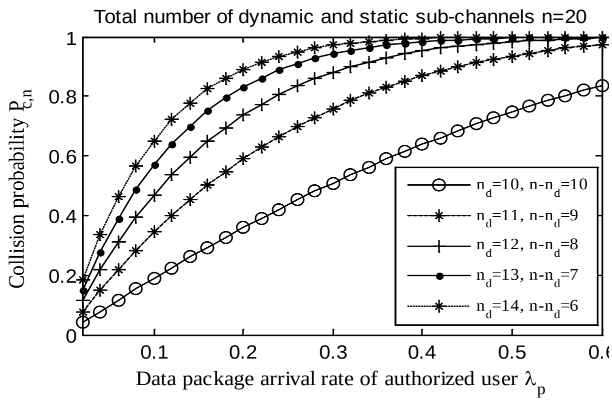

Fig.1 The comparison graph of collision probability

As shown in Fig.1, the collision probability $P_{c,n}$ increases with the increase of $n_d$, which is consistent with the theoretical analysis by Equ. (14).

### 4.2 Analysis of system performance
The performance of users can be improved by channel aggregation. The specific simulation parameters are set as shown in Table 2:

Table 2 Simulation parameters

| Parameter | property |
|---|---|
| Simulation time length | 50TTIs |
| Scheduling mode | Round Robin |
| TX antenna number | 2 |
| RX antenna number | 2 |
| Static sub-channel number | 12、11、10、9、8 |
| Dynamic sub-channel number | 8、9、10、11、12 |

The proposed scheme obtains the optimal number of aggregation sub-channels. Simulation results for three cases are shown in Fig. 2. From Fig.2, it can be seen that the static resource is fully utilized and the whole capacity of the system is improved by the optimal aggregation scheme with 10 dynamic sub-channels and 10 static sub-channels.

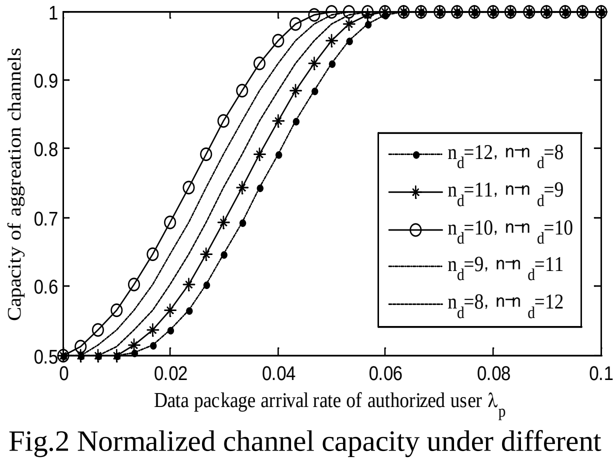


Fig.2 Normalized channel capacity under different proportion sub-channels

## 5 Conclusions and Future works
In this paper, the spectrum resource aggregation is studied. Considering the effect of the number of aggregated sub-channels on collision probability and the performance of primary and secondary system, an aggregation scheme under collision constraints is proposed. By the scheme and under collision probability constraint, the optimal number of aggregated dynamic sub-channels is obtained. The Simulation results show the effectiveness of the scheme in reducing collision probability and improving system capacity. Our future works will focus on the designs of schemes not only to improve the system capacity but also to guarantee time delay and fairness among users.

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