Performance Analysis of TAS/MRC Systems over *η*-*μ* Fading Channels with Equal Correlation

RAJKISHUR MUDOI AND RUPABAN SUBADAR
Department of Electronics and Communication Engineering
North-Eastern Hill University
Shillong, Meghalaya, 793022
INDIA
rmudoi@nehu.ac.in, rsubadar@nehu.ac.in

Abstract: - This paper analyses the outage probability and average bit error rate (ABER) of a multiuser Transmit Antenna Selection/Maximal-ratio combining (TAS/MRC) system over equally correlated η-μ fading channels. In TAS/MRC system the best antenna, based on channel state information (CSI) is used for information transmission and MRC is performed in the receiver. The expressions of PDF for SNR of the system, outage probability and ABER have been derived considering equally correlated model of η-μ fading channels. Study of correlation effect and the number of diversity branches on the performance of the communication model have been carried out for different fading parameters and reported in the numerical result and discussion section. This can be a guideline to the design engineers to optimize their performance of communication model. The results are also validated with the computer simulation and found to be correct.

Key-Words: - ABER, Equal correlation, η-μ fading, Outage probability, Multiple-Input-Multiple Output (MIMO), TAS/MRC.

1 Introduction

MIMO is an efficient technique in a wireless fading environment to improve the performance of communication receiver and meet the requirement of high speed data rate in wireless networks [1, 2, 3]. MIMO system uses multiple antennas in transmitter and receiver, thereby increasing the system complexity considerably. Among the various MIMO systems, TAS/MRC has been used to reduce the huge RF chains present in a conventional MIMO system, which reduces the cost and complexity of the system. TAS/MRC reduces the complexity by doing the transmission using the best channel exist between transmitter and receiver, the best link is established based on the CSI data received from the receiver through a faithful link [4, 5, 6]. The η-μ describes a fading environment for small scale, when no line of sight path is available in a non-homogeneous environment with multiple clusters [7, 8]. With specific data of the fading parameters Nakagami-*q* (Hoyt) and Nakagami-*m* distributions can also be realized from this distribution [8]. The performance of MRC receiver over η-μ fading channels has been studied in [9, 10, 11, and 12]. In [13], analysis of SC diversity receiver has been presented over η-μ fading channels. Considering independent and identical distributions in all branches, various expressions of performance measures have been derived with integer μ. In practice it is difficult to realize independent fading path due to space constraints in the mobile devices. Therefore considering an independent fading channel in the analysis, is not having much importance from the practical point of view. It was found that the correlations have serious effects on the performance of multi-antenna system as it degrades the diversity gain of the system [14].

This paper deals with the mathematical analysis of the TAS/MRC receiver, considering dual MRC with correlated η-μ fading. We have derived closed-form analytical expressions for the Outage Probability and average BER of Binary phase-shift keying (BPSK) and Differential binary phase shift keying (DBPSK) modulation schemes for the MRC receiver. Equal correlation model is observed in an antenna array placed on an equilateral triangle or by a closely spaced set of antennas [15].

2 System and *η*-*μ* Fading Model

A K user MIMO system is considered here for analysis with N transmit antenna at the base station. Each user has two receive antennas and performs MRC to elevate the received signal. The communication channels are assumed as a slow flat fading with η-μ statistics. From the CSI data, the base station selects the best transmit antenna to transmit data for each user. The signal received by the l^{th} antenna of the user may be expressed as

$$r_l(t) = \alpha_l e^{j\phi_l} s(t) + n_l(t), \tag{1}$$

over one bit duration T_b . Where $s(t)$ is the signal of the transmitted bit with energy (E_b) and $n_l(t)$ is the noise with PSD $2N_0$. ϕ_l is a Random variable (RV) used to represent the instantaneous phase and RV α_l is the amplitude with η-μ statistics. The mathematical expression of the η-μ PDF is given by [8].

$$f_{\alpha_l}(\alpha_l) = \frac{4\sqrt{\pi}}{\Gamma(\mu)}\left(\frac{\mu}{\Omega_l}\right)^{\mu+\frac{1}{2}} \frac{h^{\mu}\alpha_l^{2\mu}}{H^{\mu-\frac{1}{2}}} e^{-\frac{2\mu h}{\Omega_l}\alpha_l^2}$$

$$\times I_{\mu-\frac{1}{2}}\left[\frac{2\mu H}{\Omega_l}\alpha_l^2\right] \tag{2}$$

where $\Omega_l = E[\alpha_l^2]$, $E[.]$ is the operation of statistical average. The parameter $\mu = \frac{E^2(\alpha_l^2)}{2V(\alpha_l^2)}\left[1+\left(\frac{H}{h}\right)^2\right]$ where $V(\cdot)$ is the variance operator and $I_M(\cdot)$ is the M^{th} order modified Bessel function of the first kind [16, (9.6)]. Two different formats are used to represent the physical environment of η-μ fading models [8] namely Format 1 and Format 2. In Format 1, the range of η is $0<\eta<\infty$ and represents the power ratio of the in-phase and quadrature components of every cluster. In this format, $h = \frac{2+\eta^{-1}+\eta}{4}$ and $H = \frac{\eta^{-1}-\eta}{4}$. In format 2, the range of η lies between -1 and 1, and represents the amount of correlation between in-phase and quadrature components of each cluster. h and H are given as $\frac{1}{1-\eta^2}$ and $\frac{\eta}{1-\eta^2}$, respectively. Mathematically, one format is related to the another format by the relation, $\eta_{Format2} = \frac{1-\eta_{Format1}}{1+\eta_{Format1}}$ or, $\eta_{Format1} = \frac{1-\eta_{Format2}}{1+\eta_{Format2}}$.

We consider that the receiving antennas are placed close to each other due to space constraints, hence correlations are observed among the received signals. For simplification in the representation we assume equal correlations (ρ) for each user.

Each user employs dual MRC to efficiently recover the received signals. MRC accomplish co-phasing and appropriately weighted each received signal based on their quality and add together to have a better output SNR. The mathematical expression of SNR γ for an L branch MRC receiver can be given by [3]

$$\gamma = \sum_{i=1}^{L}\gamma_i = \frac{E_b}{N_o}(\alpha_1^2+\alpha_2^2+\ldots+\alpha_L^2) \tag{3}$$

where γ_i represents the SNR of i^{th} branch, with $\gamma_i = \frac{E_b}{N_o}\alpha_i^2$. The average of γ_i is, $\bar{\gamma}_i = \frac{E_b}{N_o}E[\alpha_i^2] = \frac{E_b}{N_o}\Omega_i$.

3 CDF of Output SNR

We consider L=2 for each MRC receiver. The output SNR PDF of each user can be found by finding the PDF of $\alpha^2 = \alpha_1^2 + \alpha_2^2$ and multiplying it by a factor $\frac{E_b}{N_o}$ using the theory of RV transformation. α^2 can be obtained by using the theory of η-μ distribution [8] and sum of Gaussian square distribution [17].

Format 1 of η-μ distribution as mentioned in the previous section is considered here. The η-μ distributed RV α_l of the fading signal is given as [8]

$$\alpha_l^2 = \sum_{i=1}^{2\mu} X_{l,i}^2 + \sum_{i=1}^{2\mu} Y_{l,i}^2 \tag{4}$$

where $X_{l,i}$ and $Y_{l,i}$ are mutually independent Gaussian RVs with zero mean and variances σ_x^2 and σ_y^2, respectively. Here, the fading parameter $\eta = \frac{\sigma_x^2}{\sigma_y^2}$. From this, α^2 can be expressed as sum of X^2 and Y^2, with $X^2 = \sum_{i=1}^{2\mu} X_{1,i}^2 + \sum_{i=1}^{2\mu} X_{2,i}^2$ and $Y^2 = \sum_{i=1}^{2\mu} Y_{1,i}^2 + \sum_{i=1}^{2\mu} Y_{2,i}^2$. $X_{1,i}$ and $X_{2,i}$ ($Y_{1,i}$ and $Y_{2,i}$) are correlated with coefficient ρ.

Sum of L equally correlated Gaussian square RV is given in [17, (4)]. Considering $\lambda = \mu$ and applying theory of RV transformation in the expression, the PDF of X^2 is obtained as:

$$f_{X^2}(x) = \frac{x^{2\mu-1} e^{-\frac{x}{2\sigma_x^2(1-\rho)}} {}_1F_1\left(\mu; 2\mu; \frac{2\rho x}{2\sigma_x^2(1-\rho^2)}\right)}{\Gamma(2\mu)\left(2\sigma_x^2\right)^{2\mu}\left(1-\rho^2\right)^{\mu}} \tag{5}$$

Where $\Gamma(.)$ and ${}_1F_1(.;.;.)$ are the Gamma and confluent hypergeometric functions, respectively. A similar approach is followed to obtain, the PDF of Y^2. Now the PDF of α^2 can be obtained from the sum of PDF of X^2 and Y^2 using the method of convolution [18, (6.45)] as they are independent and can be expressed as :

$$f_{\alpha^2}(\alpha) = \frac{e^{-\frac{\alpha}{2\sigma_y^2(1-\rho)}}}{\left(4\sigma_x^2\sigma_y^2\right)^{2\mu}\left(1-\rho^2\right)^{2\mu}\Gamma^2(2\mu)}$$

$$\times \sum_{t_1=0}^{\infty}\sum_{t_2=0}^{\infty} \frac{(2\rho)^{t_1+t_2}(\mu)_{t_1}(\mu)_{t_2}\,\alpha^{4\mu+t_1+t_2-1}}{t_1!t_2!(2\mu)_{t_1}(2\mu)_{t_2}\,2^{t_1+t_2}\sigma_x^{2t_1}\sigma_y^{2t_2}}$$

$$\times \frac{B(2\mu+t_2, 2\mu+t_1)}{\left(1-\rho^2\right)^{t_1+t_2}}$$

$$\times {}_1F_1\left(2\mu+t_1; 4\mu+t_1+t_2; \frac{\sigma_x^2-\sigma_y^2}{2\sigma_x^2\sigma_y^2(1-\rho)}\alpha\right) \tag{6}$$

Where $(.)_n$ and $B(\cdot,\cdot)$ are Pochhammer and Beta function, respectively. This expression can be further used to obtain the PDF of output SNR γ in (3) by applying the theory of RV. For the simplification of the PDF of γ, we assume that $\sigma_y^2 = 1$ without loss of generality, thus the fading parameter results in $\eta = \sigma_x^2$ and $\frac{E_b}{N_o} = \frac{\bar{\gamma}}{2\mu(1+\eta)}$. The obtained expression is given by:

$$f_\gamma(\gamma) = \left[\frac{1-\rho}{1+\rho}\right]^{2\mu}\left[\frac{\mu(1+\eta)}{\bar{\gamma}(1-\rho)\sqrt{\eta}}\right]^{4\mu}$$

$$\times \sum_{t_1=0}^{\infty}\sum_{t_2=0}^{\infty} \frac{(\mu)_{t_1}(\mu)_{t_2}}{t_1!t_2!\eta^{t_1}\Gamma(4\mu+t_1+t_2)}$$

$$\times \left[\frac{2\rho\mu(1+\eta)}{\bar{\gamma}\left(1-\rho^2\right)}\right]^{t_1+t_2} \gamma^{4\mu+t_1+t_2-1} e^{-\frac{\mu(1+\eta)}{\bar{\gamma}(1-\rho)}\gamma}$$

$$\times {}_1F_1\left(2\mu+t_1; 4\mu+t_1+t_2; \frac{\mu\left(\eta^2-1\right)}{\eta(1-\rho)\bar{\gamma}}\gamma\right) \tag{7}$$

Writing the hypergeometric function in infinite series, the CDF of the above PDF in (7) can be derived. This obtained CDF can be simplified using [19, (3.381.1)] as shown below.

$$F_\gamma(\gamma) = \sum_{t_1=0}^{\infty}\sum_{t_2=0}^{\infty}\sum_{t_3=0}^{\infty} \frac{(\eta-1)^{t_3}(\mu)_{t_1}(\mu)_{t_2}(2\mu+t_1)_{t_3}}{t_1!t_2!t_3!\eta^{2\mu+t_1+t_3}}$$

$$\times \frac{(2\rho)^{t_1+t_2}(1-\rho)^{2\mu}}{\Gamma(4\mu+t_1+t_2+t_3)[1+\rho]^{2\mu+t_1+t_2}}$$

$$\times g\left(4\mu+t_1+t_2+t_3, \frac{\mu(1+\eta)}{\bar{\gamma}(1-\rho)}\gamma\right), \tag{8}$$

where $g(m,n) = \int_0^n x^{m-1}e^{-x}dx$ is the gamma function (lower incomplete).

4 Outage Probability

Outage probability is considered as one of the widely used performance measures in communication engineering and defines the probability of receiver SNR falls below threshold γ_{th}, $P(\gamma<\gamma_{th})$. The outage probability can easily be obtained from the CDF expression by replacing the SNR value with the threshold γ_{th}. Since, in the system we have considered that total $N\times K$ number of communication links are present, hence from (8) outage probability can be obtained as

$$P_{out}=\left[\frac{1}{\eta^{2\mu}}\left[\frac{1-\rho}{1+\rho}\right]^{2\mu}\sum_{t_1=0}^{\infty}\sum_{t_2=0}^{\infty}\sum_{t_3=0}^{\infty}\frac{(\mu)_{t_1}(\mu)_{t_2}}{t_1!t_2!t_3!}\right.$$
$$\times\frac{(2\mu+t_1)_{t_3}(\eta-1)^{t_3}}{\eta^{t_1+t_3}\Gamma(4\mu+t_1+t_2+t_3)}\left[\frac{2\rho}{1+\rho}\right]^{t_1+t_2}$$
$$\left.\times g\left(4\mu+t_1+t_2+t_3,\frac{\mu(1+\eta)}{(1-\rho)\overline{\gamma}_N}\right)\right]^{NK} \tag{9}$$

where $\overline{\gamma}_N$ represents the normalized average SNR given by the ratio of $\overline{\gamma}$ and γ_{th}.

5 Average Bit Error Rate

The average bit error rate (ABER) of a digital communication system for various modulations can be obtained by averaging the derivative of the conditional error probability (CEP) of the modulation scheme used over the CDF of the output SNR [20]. It can be given as

$$\overline{p_e}=-\int_0^{\infty}p_e'(\gamma)F_\gamma(\gamma)d\gamma \tag{10}$$

Where, $p_e'(\gamma)$ is the derivative of the CEP [20], mathematically

$$p_e'(\gamma)=\frac{-a^{\eta_1}\gamma^{\eta_1-1}e^{-a\gamma}}{2\Gamma(\eta_1)} \tag{11}$$

Where, the constants a and η_1 are [20]: $(a,\eta_1)=(1,0.5)$ for BPSK, and $(a,\eta_1)=(1,1)$ for DBPSK. Putting the values of (8) and (11) to (10), the ABER can be given as

$$\overline{p_e}=\frac{a^{\eta_1}}{2\Gamma(\eta_1)}\frac{1}{\eta^{2\mu NK}}\left[\frac{1-\rho}{1+\rho}\right]^{2\mu NK}$$
$$\times\sum_{h_i|_{i=0}^{NK}}^{\infty}\sum_{k_i|_{i=0}^{NK}}^{\infty}\sum_{t_i|_{i=0}^{NK}}^{\infty}\frac{\eta^{-\sum_{i=1}^{NK}(h_i+t_i)}\prod_{i=1}^{NK}(\mu)_{h_i}\prod_{i=1}^{NK}(\mu)_{k_i}}{\prod_{i=1}^{NK}h_i!\prod_{i=1}^{NK}k_i!\prod_{i=1}^{NK}t_i!}$$
$$\times\frac{\left\{\prod_{i=1}^{NK}(2\mu+h_i)_{t_i}\right\}\left[\frac{2\rho}{1+\rho}\right]^{\sum_{i=1}^{NK}(h_i+k_i)}(\eta-1)^{\sum_{i=1}^{NK}t_i}}{\prod_{i=1}^{NK}\Gamma(4\mu+h_i+k_i+t_i)}$$
$$\times\int_0^{\infty}\gamma^{\eta_1-1}e^{-a\gamma}\prod_{i=1}^{NK}g\left(4\mu+h_i+k_i+t_i,\frac{\mu(1+\eta)}{\overline{\gamma}(1-\rho)}\gamma\right)d\gamma \tag{12}$$

Expressing $g(m,n)$ in ${}_1F_1(.;.;.)$ [16, (6.5.12)] and applying [21, (C.1)] the integral can be solved, which gives the final expression of ABER as

$$\overline{p_e}=\frac{a^{\eta_1}(\overline{\gamma})^{\eta_1}}{2\Gamma(\eta_1)}\sum_{h_i|_{i=0}^{NK}}^{\infty}\sum_{k_i|_{i=0}^{NK}}^{\infty}\sum_{t_i|_{i=0}^{NK}}^{\infty}\frac{\prod_{i=1}^{NK}(\mu)_{h_i}\prod_{i=1}^{NK}(\mu)_{k_i}}{\prod_{i=1}^{NK}h_i!\prod_{i=1}^{NK}k_i!\prod_{i=1}^{NK}t_i!}$$
$$\times\frac{\mu^Z(2\rho)^{\sum_{i=1}^{NK}h_i+\sum_{i=1}^{NK}k_i}[1+\rho]^{\sum_{i=1}^{NK}t_i}\left\{\prod_{i=1}^{NK}(2\mu+h_i)_{t_i}\right\}}{\left\{\eta^{\sum_{i=1}^{NK}(h_i+t_i)}\right\}\left\{\prod_{i=1}^{NK}(4\mu+h_i+k_i+t_i)!\right\}(\eta-1)^{-\sum_{i=1}^{NK}t_i}}$$
$$\times\frac{\Gamma(\eta_1+4\mu NK+Z)}{(1+\eta)^{-(4\mu NK+Z)}}\left(\frac{(1-\rho)}{NK\mu(1+\eta)+a\overline{\gamma}(1-\rho)}\right)^{\eta_1+4\mu NK+Z}$$
$$\times F_A\left(\eta_1+4\mu NK+Z;\underbrace{1,.,1}_{NK,times};\right.$$
$$\left.U+h_1+k_1+t_1,.,U+h_{NK}+k_{NK}+t_{NK};\underbrace{V,.,V}_{NK,times}\right) \tag{13}$$

Where $Z = \sum_{i=1}^{NK} h_i + \sum_{i=1}^{NK} k_i + \sum_{i=1}^{NK} t_i$; $U = 1 + 4\mu$; $V = \frac{\mu(1+\eta)}{NK\mu(1+\eta) + a\overline{\gamma}(1-\rho)}$ and $F_A(.;.;.;.)$ is the Appell hypergeometric function.

6 Numerical Results and Discussion

Obtained mathematical expressions for performance measures have been numerically evaluated and plotted to study the behavior of the system with different parameters. We have considered two users system (K=2) for evaluations. In Fig. 1, P_{out} vs. $\overline{\gamma}_N$ has been shown for various values of ρ, η, μ and the number of transmit antennas N=2. The correlation effect on the outage is studied by relating outage of various values of ρ with $\rho=0$. We observed increment in ρ also increases the outage probability of the system, for a fixed value of $\overline{\gamma}_N$ and fading parameters. In Fig. 2, P_{out} vs. $\overline{\gamma}_N$ has been plotted for ρ=0.8 and different values of η, μ and transmit antennas N. Increase in transmit antennas on the base station (N), increases the total RF links of the system, so reduces the outage probability. The effect of outage at ρ=0.8 is less for N=3 than for N=2. For BPSK and DBPSK modulation schemes, ABER vs. $\overline{\gamma}$ curves have been plotted in Figs. 3 and 4 respectively, considering various values of ρ, η, μ and the number of transmit antennas N=2. Effect of ρ on ABER has been studied and it is established that for constant L, N, μ the ABER increases with an increment in ρ. The parameter μ, which define the number of cluster in the fading model, improve ABER with addition in cluster. Power difference in the in-phase and quadrature-phase (rise in η) degrades ABER. Validations of the obtained results have been done by means of simulation using the Monte Carlo method.

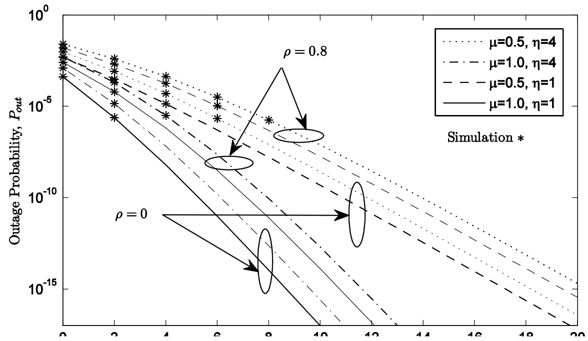

Fig.1: Outage Probability vs. $\overline{\gamma}_N$

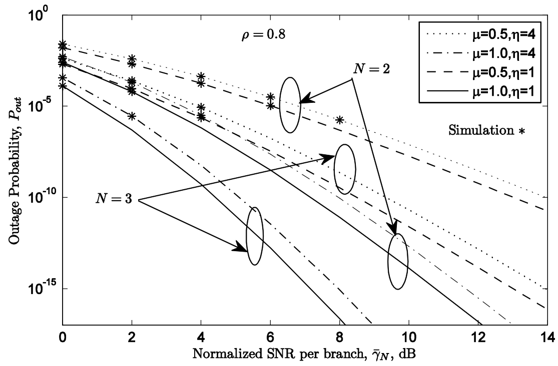

Fig. 2: Outage Probability vs. $\overline{\gamma}_N$ for N=2 and N=3

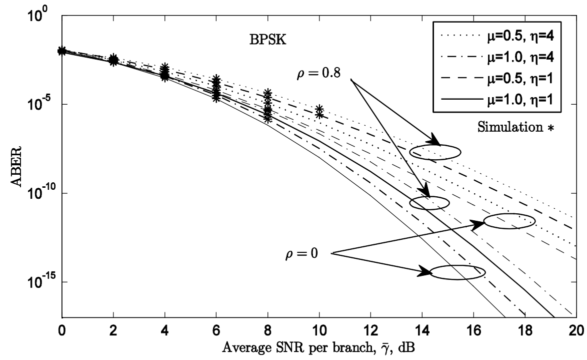

Fig. 3: ABER vs. $\overline{\gamma}$ for BPSK modulation scheme

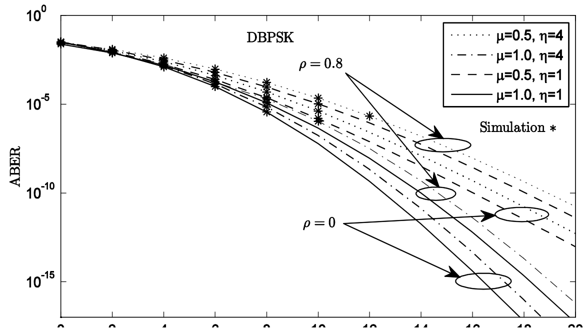

Fig. 4: ABER vs. $\bar{\gamma}$ for DBPSK modulation scheme

7 Conclusion

In this paper, we analyze the outage probability and ABER of TAS/MRC system over η-μ fading channels considering correlation in the MRC receiver. Outage probability and ABER have been derived using the methods of probability theory. The outage probability expression is obtained in terms of incomplete Gamma function whereas the ABER expression has been obtained in terms of Appell hypergeometric function. Effect of correlation and the other parameters on the system performance has been studied.

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