Feedback based Channel Estimation for Time Varying Multipath System

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Abstract: - Channel estimation is a crucial design issue for faithful reception of high data rate mobile applications in a wireless communication system. Channel estimation algorithms, mainly available in literature, include least square (LS) estimation, linear minimum mean squared error (LMMSE) estimation, and low rank linear minimum mean squared error (LRMMSE) estimation with varying degrees of performance. MMSE-based algorithms offer higher accuracy though at the cost of complexity in comparison to the low complexity LS algorithm. In this work, we propose an algorithm based on the LS channel estimation technique by using decision based feedback technique to improve upon its estimation capabilities and still retaining its simplicity. To examine its usefulness especially for wireless mobile applications, performance in terms of the symbol error rate is analyzed with varying Doppler frequency and delay parameters, which is found to be quite optimistic in comparison to LMMSE and LRMMSE algorithms.

Key-Words: - Pilots symbols, Symbol error rate, Doppler frequency shift, OFDM

1 Introduction

Orthogonal frequency division multiplexing (OFDM), a multicarrier transmission scheme with all subcarriers orthogonal to each other, provides high message rate transmission at reasonable complexity and precision. The concept of parallel-data transmission in OFDM scheme reduces multipath-fading influence and makes complex equalizers unnecessary by converting the frequency selective channel into a flat channel. The common issues in OFDM, such as co-channel interference and inter-channel interference (ISI) that degrade the system performance can be addressed by introducing guard bands and cyclic prefixes. Owing to its spectral efficiency, OFDM is a popular choice for high data rate next generation communication applications. However, Doppler frequency shifts and transmission impairments for wireless mobile communication may degrade system performance. Further, Doppler spreading destroys the orthogonality of the subcarrier producing inter-carrier interference (ICI) that reduces the signal to noise ratio [1]. Various types of channel estimators are proposed in literature to improve channel estimation accuracy by overcoming ICI and non-linear effects in the channel for multipath transmission systems [2][3][4]].

Pilot-based channel estimators and blind channel estimators are the most preferred channel estimation techniques. Accuracy of blind channel estimation depends on the statistics of the received signal, which increases its computational complexity and makes it as the major hindrance. In pilot based channel estimation, the pilot (training) data known to the receiver are multiplexed with the transmitted information at pre-determinate positions before transmission. Information of the transmitted data can be obtained by interpolating between different channels with the previous training data. Once the data is trained, prediction time can be very short for pilot-based channel estimation. However, error occurrences while doing interpolation may leads to inaccuracy in the received signal.

The LS channel estimation or Minimum Mean-Square (MMSE) estimation scheme is used for block type pilot arrangement. MMSE estimate shows 10-15dB gain in SNR over LS estimate of channel for same MSE [5]. Need for equalization in channel estimation has been resolved by using comb type pilot arrangement when the channel properties changes within one block [6]. LS, LMMSE and LRMMSE estimator is mainly used for pilot assisted OFDM channel estimation. Accuracy of LMMSE and low rank LMMSE estimators depend on the knowledge of signal to noise ratio and channel autocorrelation matrix. However, mismatching of delay spread and the Doppler frequency shift tends to deteriorate performance of these algorithms [7]. So, certain measures are required to compensate for mismatching of these parameters without affecting accuracy of the system as attempted in LMMSE and low rank LMMSE estimators. Since delay spread and Doppler frequency shift are the major impairments for high data rate mobile communication applications, so it is important to develop channel estimators that can cope with these impairments more efficiently. Presently designed estimators to overcome these impairments are quite complex and limiting their suitability for predicted mobile communication systems. In this work, we are proposing a channel estimation technique by employing feedback mechanism to overcome the Doppler frequency shift and delay spreading impairments. Proposed estimator is based on modifying the low complexity LS channel estimator and addresses the impairment variations through a decision based feedback mechanism.

This paper is organized as follows. Section 2 provides overview of OFDM system model and describes mathematical expression showing Doppler spreading. Section 3 discusses basic concepts behind other available Channel estimation approaches that shall be used for comparison. Sections 4 presents the proposed feedback based channel estimation techniques. Section 5 shows the performance comparison with other techniques and finally, section 6 concludes the work done.

2 OFDM System Model

A typical OFDM system is shown in figure 1. The Stream of 'N' OFDM symbols is splitting into data blocks of block length 'L' each containing pilot symbols or data symbols or both depending on the nature of training patterns. Guard band are included in OFDM system to prevent the adjacent channel interference and considered as pilot symbols. Data symbols a(k) and pilot symbols p(k) of kth block are combined to get x(k) = a(k) + p(k). x(k) is added with cyclic prefix (CP) after that g(k) is transmitted with block size $N = L + L_{CP}$.(Here L_{CP} is the size of the CP). Mathematically, it is given as below

$$g[k] = T_{CP}W^{H}x[k] \qquad (1)$$

Here, W^{H} is the window function for Fast Fourier transform matrix. Parallel to serial converted $g[kN+n] \cong [g[k]]_n$ signal stream is transmitted over multipath channel assuming discrete impulse response of linear time varying channel. Response at the receiver end is given by

$$s[n] = \sum_{m=0}^{M-1} h[n,m][n-m] + w[n] \qquad (2)$$

Where $h(n,m)$ the discrete time impulse response, n is time index and m is time delay index. $w(n)$ is the additive white Gaussian noise (AWGN).The received sample of OFDM blocks are grouped to $s(k)$ vector, with cyclic property of $[s(k)]_n = s[kN+n]$ The received sample is given below

$$s[k] = H_0[k]s[k] + H_1[k]s[k-1] + w[k] \qquad (3)$$

Here $[H_0[k]_{n,m} = [h(kN+n, n-m)]$ and

$$[H_1[k]_{n,m} = [h(kN+n, N+n-m)]$$

For complete removal of ISI, assuming the duration of cyclic long enough

(L_{cp} (block length ≥ M-1 (channel order)).

Next, after removing cyclic prefix the received signal is converted to frequency domain by applying FFT and is given by

$$z(k) = H_F[k]x[k] + w[k] \qquad (4)$$

$H_F[k]$ is frequency domain channel matrix representing discrete Doppler spread and expressed as follows [8].

$$[H_F[k]]_{l+d,l} = \frac{1}{L}\sum_{n=0}^{L-1}\sum_{m=0}^{L-1}[H_T[k]]_{n,m}e^{-\frac{j2\pi(l(n-m)+\delta_n)}{L}}$$

$$= \frac{1}{L}\sum_{n=0}^{L-1}\sum_{m=0}^{L-1}[h(kN+L_{cp}]_{n,m}e^{-\frac{-j2\pi(l(n-m)+\delta_n}{L}} \qquad (5)$$

Here δ_n represents discrete Doppler index and 'l' subcarrier index, the l^{th} column of diagonal elements of $H_F[k]$ associated with the l^{th} subcarrier, which causes inter-carrier interference (ICI) due to other symbols on l^{th} symbol of the OFDM block.

3 Related Work

Wireless communication channels are practically time variant and frequency selective channels, which may lead to inaccurate signal reception for wireless mobile applications. Channel estimation is necessary to recover the original signal from channel irregularities [4] [9]. In pilot assisted channel estimation techniques, inclusion of some pilot symbols in place of OFDM symbols for the purpose of channel estimation, results in decreasing the OFDM symbols rate; whereas, in blind channel estimation, instead of pilot symbols, statistical information about the channel is used without sacrificing any data rate.

However, the probability of error with blind channel estimation techniques is higher and adds computational complexities and may be difficult to implement in case of real time applications [2][4][10]. Pilot channel estimation is further classified as block type and comb type pilot arrangements. Comb type pilot arrangement is preferred over block type for frequency selective and better resistance to fast fading channels [2][3][[4][11].

Samarendra Nath Sur, Rabindranath Bera and Bansibadan Maji had focused on the design of decision feedback equalization-based receivers for MIMO systems in correlated Nakagami-m channel. Here, channel state information (CSI) is assumed to be present at the receiver side only. This work used QPSK modulation with DFE receiver that improved results over LE receiver. Proposed work contributed that pre-equalization combining DFE achieves optimal performance and can be implemented to overcome severe channel condition [25].

Roozbeh Mohammadian *et al.* had proposed an approach, which minimize the coherence of the associated Fourier submatrix to select the pilot subcarriers. In this method different pilot value has been assigned for each transmitter. Simple sparse recovery techniques were used if the channels are sparse enough in time domain. All transmitters were designed to share a same carrier frequency pilot overhead was reduced with channel estimation block at the receiver. Evaluation results achieved improvement in terms of bit error rate and mean-square channel estimation error [26].

4 Proposed LS Channel Estimation (FLS)

Mismatching of delay spread and the Doppler shift may degrade the system performance in LMMSE and LRMMSE channel estimation techniques. A modified technique is proposed that will be more robust against these impairments variations. To keep the complexity at lower side, we choose to work on the least complex LS channel estimator by using feedback mechanism as shown in figure 2.

The improvement is obtained using the feedback mechanism (de-mapping signal block) which shall be exactly the output of channel equalizer and shall use this information to compensate the impairment variations that will improve the system performance by reducing symbol error rate to its lowest value compare to LS, LMMSE and LRMMSE channel estimators. Working of proposed scheme is divided into four phases. First is initialization phase, second is channel equalization phase, third is modification of transfer function phase and fourth one is end phase. Detail is as follows.

Step I: - Initialization phase

In this phase of estimation channel transfer function $h_{m,k}$ (k^{th} iteration for m^{th} OFDM symbol frame) set to its initial status depending upon the OFDM symbol frame(m) and known as the initial status of the proposed estimation technique.

a) $m = 0$, channel estimation is obtained from pilot symbols and

b) $m > 0$, channel estimation is obtained from the previous symbols.

This is known as the initial status of the proposed estimation technique.

Step II: - Channel equalization phase

After initialization, channel equalization is depending upon the value of k^{th} iteration as follows

a) $k = 0$, channel equalization output is the current status of OFDM symbols and the initial status of channel estimation.

b) $k \geq 0$, channel equalization output is the current status of OFDM symbols with the output from previous OFDM symbol obtained after demapping.

Step III: - Modification of transfer function phase

Channel estimation inside the block can be updated using feedback k^{th} subcarrier can be described as follows.

a) Channel response from K^{th} subcarrier, $H_e(k)$ (from previous symbols) is used to get Xe (k) (estimated transmitted signal).

$$X_e(k) = \frac{Z(k)}{H_e(k)}$$

b) Demapper mapped signal to $\{X_e(k)\}$ binary data and then again mapper converted signal to $[X(k)_{decision}]$.

c) Updated channel estimation transfer function is obtained as

$$H(k)_{updated} = \frac{Z(k)}{X(k)_{decision}}$$

Updated $H(k)_{updated}$ proceed further for processing.

Step IV: - End phase

If K ≥ number if iterations exit, then do continues to next OFDM symbol Z_{K+1}. Start from step (I), otherwise move to step (II) and set K=K+1

5 Performance Comparison

The proposed feedback-based least square algorithm was implemented using MATLAB platform and its performance was analyzed using following parameters.

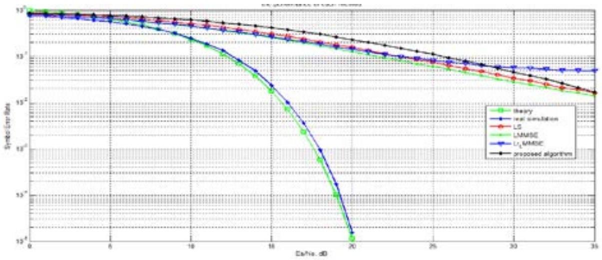
OFDM symbols	100
Channel model	Rayleigh
Doppler Spread	Jake model
Number of subcarrier	128
Cyclic prefix	16
Modulation Technique	16 QAM
Time interval of the pilot signal	5
Delay (seconds)	Del-1=[0 2e-6 4e-6 8e-6 12e-6] Del-II=[0 2e-4 4e-4 8e-4 12e-4] Del-III=[0 2e-2 4e-2 8e-2 12e-2]
Number of Multipath	5
Average time delay of the multiple channel	4μs
Sample time	1μs

Table 1: Simulation parameters

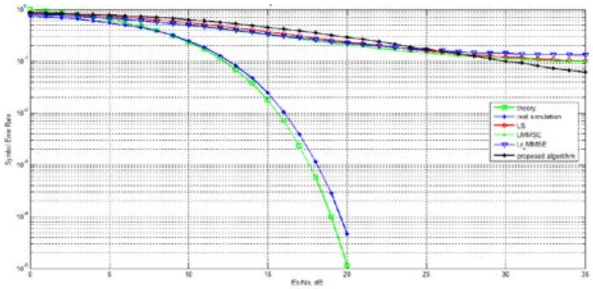
Randomly generated data sequences and pilot sequences are transmitted with 16 QAM modulation technique. The performance of proposed scheme is analyzed in terms of symbol error rates in comparison to LS, LMMSE, LRMMSE channel estimators for time varying multipath channel. Keeping in view the upcoming high data rate mobile applications, the performance is examined with varying Doppler frequency shifts and delay time with different E_b/N_0.

5.1 Performance with varying Doppler frequency shift

To analyze the impact of relative mobility of the receiver and transmitter w.r.t. each other on system's performance, Doppler frequency is varied from 20Hz to 200Hz, and results are shown in Figure-2 and Table-2 in terms of SER Vs E_b/N_o. The multipath delay is fixed at **[0, 2e-6, 4e-6, 8e-6, 12e-6]**.



(a)



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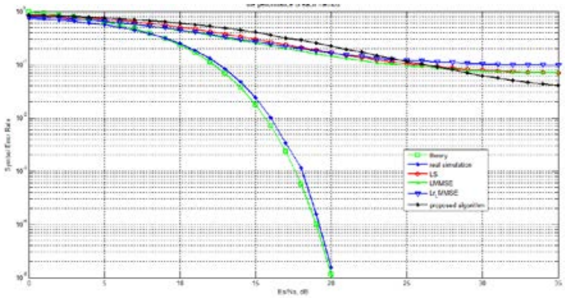

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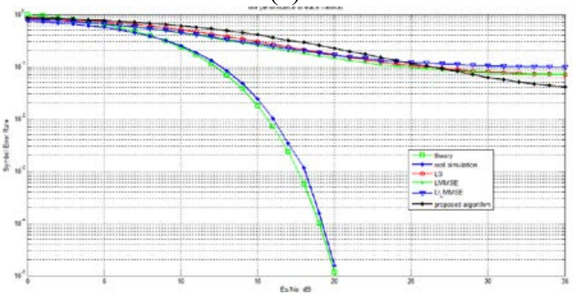

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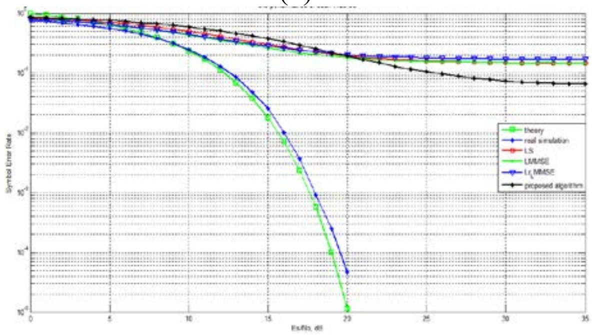

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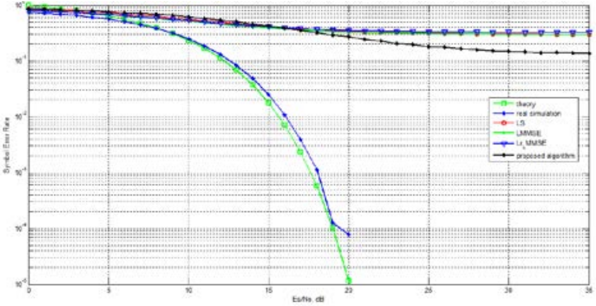

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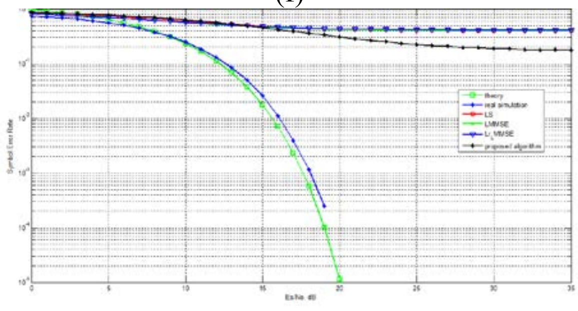


(g)

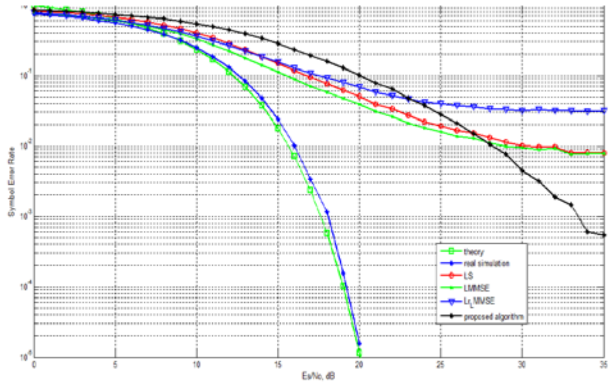
Figure 3:- Performance comparison of channel estimators (Doppler frequencies (a) 20 Hz (b) 30 Hz (c) 40 Hz (d)50 Hz (e) 100 Hz (f) 150Hz (g) 200 Hz)

The proposed scheme is better able to adjust the changing Doppler frequency shift effect (especially at higher E_b/N_o values) due to $X(k)_{decision}$ and $H(k)_{updated}$ parameters in channel estimator. SER of proposed scheme is better than other estimators under consideration beyond certain E_b/N_o and below that performance of all are comparable.

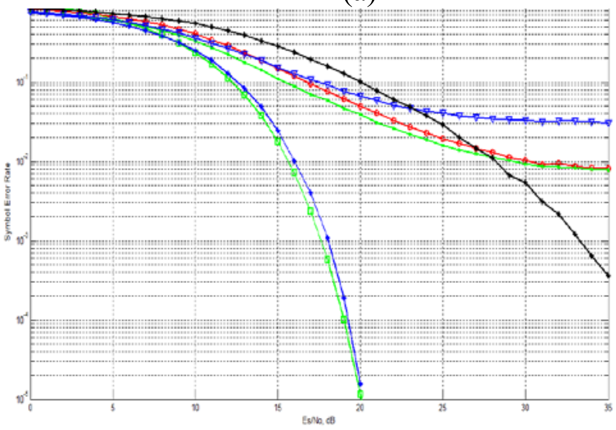
As shown in Table-2, the results are much improved for the proposed estimator as compared to the baseline LS, LMMSE and LRMMSE channel estimators. Typically above 10 dB (Eb/N_0) the order of merit is **Proposed (FLS)) > LMMSE > LS > LRMSSE.** Due to Doppler spreading, signal undergoes frequency dispersion leading to distortion. Our proposed feedback based LS (FLS) estimator perform better at higher Doppler frequency shift and results are much improved as E_b/N_o (dB) moves from lower to higher value. For example at E_b/N_o = 25dB, SER for the proposed FLS is 0.2773, whereas for the LS, LMMSE, and LRMMSE algorithms it is 0.4706, 0.4651, and 0.4856 respectively at 200 Hz Doppler shift.

5.2 Performance with varying delay time

Reception of transmitted signal depends on reflections from nearby obstacles (which cause reflection with a short excess delay), remote high-rise buildings cause strong reflections with large excess delay. The combined effects often result in multiple clusters of reflections. Keeping these effects into consideration delay time of system is varying continuously. To analyse the performance of FLS estimator under this environment and compare it with LS, LMMSE and LRMMSE channel estimator's simulations were carried and results are shown in figure-3



(a)



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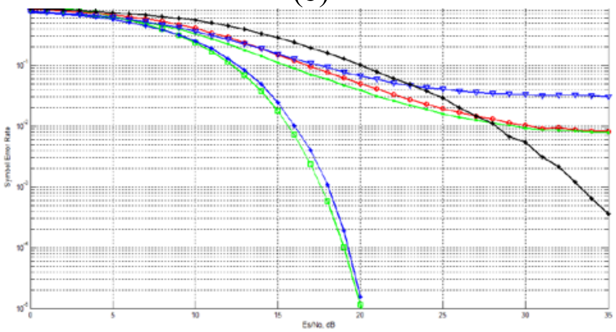

(c)

Figure 4:- Performance of channel estimators (delay effects (a) [0, 2e-4, 4e-4, 8e-4, 12e-4] (b) [0, 2e-3, 4e-3, 8e-3, 12e-3] and (c) [0, 2e-2, 4e-2, 8e-2, 12e-2])

Reception of transmitted signals causes pulse broadening due to impairments of multipath transmission. In this section, we examined the FLS algorithm under different delay environments as shown in figure-3. Results show that FLS performance is much better as delay time varies in figure 4 (a) to figure 4 (c). Thus proposed algorithm shows consistence performance under multiple clusters of reflections whereas existing techniques degrade the performance of system due to mismatching of its impairment parameters.

Table-3 comparing all four estimators at three delay times Del-1, Del-II and Del-III. Shows that below 20db performance is comparable while beyond 20dB **E_b/N_o,** performance order is **FLS>LS>LRMMSE=LMMSE** Results show that with the proposed channel estimator, symbol error rate (SER) is almost 50% lower at E_b/N_o (25 dB) as compared to LS, LMMSE and LRMMSE and it keeps on reducing further with increasing E_b/N_o.

5.3 Performance at constant E_b/N_o

Here performance of FLS is compared with LS, LMMSE and LRLMMSE at Doppler frequency shift from 25 Hz to 200Hz. Performance evaluation for channel estimators, symbol error rate criteria based on estimation error versus Doppler frequency shift.

Performance comparison shown in figure-4, we see that the proposed FLS scheme performs better at all Doppler frequency shifts and under multiple clusters of reflections compared to existing techniques.

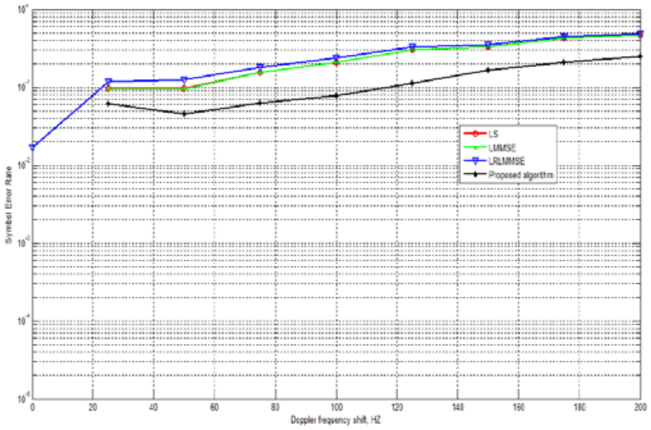

Figure 5:- Performance of channel estimator's (30 dB E_b/N_o).

6 Conclusion

Channel estimation is crucial for correctly estimating the received data especially for high speed mobile data communications. Owing to the time varying and frequency selective channel characteristics, correct estimation of the channel becomes highly imperative along with the lower design complexity of such channel estimators. The main contribution of this paper is in offering a new feedback based Least Square channel estimator (FLS) for time varying multipath channels that exploits low complexity features of LS algorithm and improves upon channel estimation by means of a feedback mechanism. The performance of proposed scheme in comparison to the existing channel estimation schemes (LS, LMMSE and LRMMSE), shows that with increasing Doppler frequency shift and under multiple clusters of reflections FLS can effectively reduce the channel estimation error exhibiting superior performance. The proposed FLS algorithm could effectively overcome the ill effects of mismatching of delay spread and the Doppler frequency shift making it a more robust against the variation of doppler shift and delay time specially at higher E_b/N_o.

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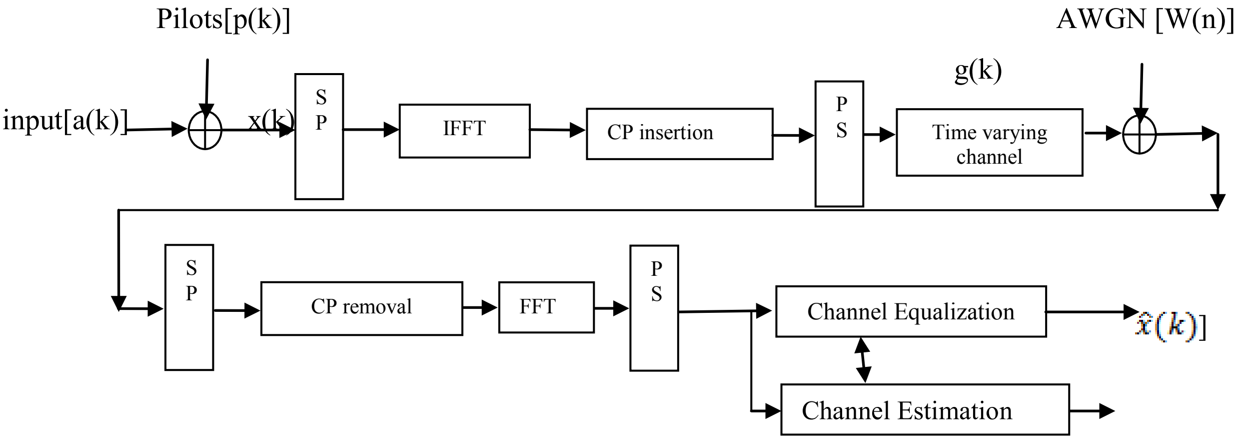

Figure 1:- Pilot channel Estimation for time Varying channel OFDM system.

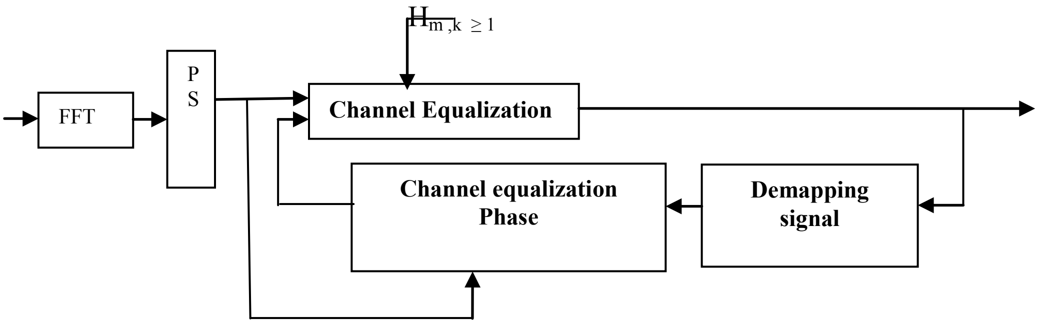

Figure 2: - Overview of proposed feedback-based LS channel estimator

	Estimation Scheme	SER at different E_b/N_o (dB)							
		1	**5**	**10**	**15**	**20**	**25**	**30**	**35**
Doppler Frequency 200 HZ	**LS**	0.8336	0.7440	0.6401	0.5210	0.4828	0.4706	0.4650	0.4650
	LMMSE	0.7965	0.7056	0.5868	0.5505	0.4775	0.4694	0.4651	0.4647
	LRMMSE	0.7977	0.7107	0.6000	0.5223	0.4947	0.4856	0.4826	0.4825
	Proposed	0.8506	0.7724	0.6378	**0.4718**	**0.3419**	**0.2773**	**0.2459**	**0.2355**
Doppler Frequency 100 HZ	**LS**	0.8206	0.7079	0.5163	0.3499	0.2608	0.2205	0.2078	0.2046
	LMMSE	0.7716	0.6532	0.4641	0.3202	0.2465	0.2152	0.2068	0.2040
	LRMMSE	0.7737	0.6597	0.4863	0.3486	0.2755	0.2458	0.2370	0.2329
	Proposed	0.8393	0.7566	0.5985	**0.3907**	**0.2179**	**0.1080**	**0.0762**	**0.0617**

Table-2: Comparison of channel estimators with delay time [0 2e-8 4e-8 8e-8 12e-8]

	Estimation Scheme	**SER at different E_b/N_o (dB)**							
		1	**5**	**10**	**15**	**20**	**25**	**30**	**35**
Del-I **0 2e-2 4e-2 8e-2 12e-2]**	**LS**	0.82083	0.71023	0.51634	0.36114	0.29511	0.27370	0.26786	0.26484
	LMMSE	0.75775	0.63825	0.45411	0.33316	0.28444	0.27045	0.26695	0.26422
	LRMMSE	0.75775	0.63825	0.45411	0.33316	0.28444	0.27045	0.26695	0.26422
	FLS (proposed)	0.84028	0.75480	0.59366	0.37511	***0.21591***	***0.14813***	***0.12370***	***0.11459***
Del-II **[0 2e-4 4e-4 8e-4 12e-4]**	**LS**	0.81778	0.71050	0.51442	0.36311	0.29486	0.27658	0.26802	0.26775
	LMMSE	0.75688	0.60172	0.49028	0.35027	0.28523	0.27350	0.26725	0.26755
	LRMMSE	0.75688	0.63925	0.45591	0.33177	0.28523	0.27350	0.26725	0.26755
	FLS (proposed)	0.83856	0.75739	0.58900	0.37548	***0.21839***	***0.14809***	***0.12250***	***0.11530***
Del-III **[0 2e-6 4e-6 8e-6 12e-6]**	**LS**	0.82072	0.70814	0.51700	0.36108	0.29736	0.27398	0.26730	0.26597
	LMMSE	0.75692	0.63736	0.45222	0.33255	0.28628	0.27123	0.26525	0.26602
	LRMMSE	0.75692	0.63736	0.45222	0.33255	0.28628	0.27123	0.26525	0.26602
	FLS (proposed)	0.84047	0.75528	0.58900	0.37542	***0.21906***	***0.14623***	***0.12302***	***0.11461***

Table-III:- Comparison at different delay time (80Hz Doppler frequency)