Error Correction and Equilibrium investigation in Random Access MAC Protocols for Wireless Networks

Mohamed Lamine Boucenna 1, Hadj Batatia 2 and Malek Benslama 1
1 Electromagnetism and Telecommunications Laboratory (LET), University Mentouri, 25000 Constantine, Algeria
2 IRIT/ INP-ENSEEIHT, University of Toulouse, 2, rue Charles Camichel, 31071 Toulouse Cedex 7, France.
boucenna.m.lamine@gmail.com

Abstract: - The present paper concentrates on the random access MAC protocols, where we examine the throughput of the classical Aloha, Slotted Aloha and the Multi-copy Aloha respectively to show the most performing among them in terms of throughput. The integration of Erasure coding in each of these techniques has ameliorate their reliability and transmission level. Here, since each node sends \(N\) coded packets instead of the \(k\) original packets, we have \((N-k)\) redundant packets. The introduction of redundancy and subsequently structuring it in an exploitable manner, allows serious errors injected by the channel to be corrected. In wireless networks and especially in Mobile ad hoc network (MANET), access at medium is managed by Distributed Coordination Function (DCF) method, which is based primarily on Carrier Sense Multiple Access (CSMA) protocol. The nodes in MANET are selfish and non-cooperative, where each node attempts to achieve its best benefit without regard for the other nodes' actions, this could affect overall system throughput. To analyze such conflicting situations where the action of one node has an impact on the other nodes’ actions, we present a redundant packets control game. A non-cooperative game is introduced and its different characteristics are defined. We investigate the network equilibrium by applying Nash Equilibrium (NE) theorem and we evaluate our results by appropriate simulations.

Key-Words: - Slotted-Aloha; CSMA; Erasure Coding; Game theory.

1 Introduction
In ad hoc networks like in MANET, when two users transmit their packets in the same time, only one packet is liable to pass and the packets sent simultaneously get mutually destroyed causing what we name a collision. The random access protocols present in general better delays than techniques of fixed access (we can reach up a maximum throughput of 1unit, i.e., a continuous transmission) because of an efficient resolution of collision [1]. In the case when a big number of sources are often inactive, the random access allows a more efficient use of the channel. Besides, the system is put in use with much simplicity thanks to the decentralization of random access protocols. In random access techniques many sources can attempt to transmit packets in the same time which can provoke collisions and a delayed transmission. access control (MAC) protocols were designed to avoid collisions in media access. Currently, almost all new wireless data networks still use random access protocols, like ALOHA and Carrier Sense Multiple Access (CSMA). The ALOHA system is one of the earliest random access systems, and in the slotted version, all nodes are synchronized with a global clock and only allowed to start transmission at the beginning of common time slots. The CSMA protocol adds important functionality to the MAC protocols, and, as its name suggests, CSMA (and its variants) senses the medium to determine whether any other node is transmitting. There are typically three types of CSMA protocols: persistent CSMA, p-persistent CSMA, and non-persistent CSMA, where the difference between them is the action (algorithm) that is taken after sensing the channel. Despite all these preventative techniques, these protocols still suffer from packet collisions, thus reducing throughput and affecting system performance. In an attempt to improve the efficiency of these protocols, several models and solutions have been proposed. A game theoretic approach for wireless networks was proposed in [2], a deep analysis of random MAC protocols was presented in [3-6], and considerable
improvement in throughput was achieved in [7]. In this paper, we proceed in the same context by integrating the erasure coding schema in the Aloha and CSMA protocols to recover collided packets, and accordingly increase network throughput also in CSMA protocols and with more results and simulations done in [4]. However, users wishing to transmit typically want to do so as soon as possible, and if multiple users try to transmit simultaneously, all accesses fail. Moreover, unsuccessful attempts to transmit may be costly. Thus, users trying to transmit have conflicting objectives, and the appropriate tool for examining interaction between selfish users with conflicting objectives is game theory. Therefore, we formulate a game model and investigate a network equilibrium in which all users are satisfied.

In the remainder of this paper, we provide a brief introduction to erasure coding and game theory. Then, we present our game theoretic model for slotted Aloha, we discuss the utility function, existence of equilibrium, and the effect of equilibrium on throughput. Section 6 concludes the paper.

2 Aloha and its derivatives

In the Aloha system, the delay is a consequence of collision and the transmission is completely decentralized. At the end of transmission of each packet of each source, the source receives the information if the packet has been received or if there has been a collision, in which case there will be a delayed retransmission. The probability that n packets arrive in two different times is given as:

$$P(n) = \frac{2e^{-2\lambda}}{n!}. \quad (1)$$

Where $\lambda$ is the traffic load.

The probability $P(0)$ that the packet be successfully received without collision is:

$$P(0) = e^{-2\lambda}. \quad (2)$$

The throughput is given as follows:

$$Th = \lambda P(0) = \lambda e^{-2\lambda}. \quad (3)$$

2.1 Slotted Aloha

In the Slotted Aloha technique, time is discredited in time parts called slots, which refer to the maximum propagation time, where each station sends a packet at the beginning of the slot, and thus all the stations will be synchronized. This latter ameliorate the situation in comparison with the non-slotted Aloha technique because two superposed frames can only be possible on a maximum slot instead of two slots (as they start at the beginning of the same slot). By this technique throughput can reach the maximum value: 0.36 (bad use of the channel). We consider n as the number of users, $P_i$ as the probability that a given user send a packet, and $Th_i$ as the probability that a packet be successfully received. If there is no other user in a state of transmission at the beginning of the time slot, the probability function $Th_i$ will be according to [8]:

$$Th_i = \frac{P_i}{(1-P_i)} \prod_{i=1}^{n}(1-P_i). \quad (4)$$

When a throughput is $Th$ and the traffic is $\lambda$ we can write: $Th = \frac{Th_i}{n}$ and $P_i = \frac{\lambda}{n}$, then:

$$Th = \frac{\lambda}{n} \left[ \frac{1}{\prod_{i=1}^{n}(1-\frac{\lambda}{n})} \right] \prod_{i=1}^{n}(1-\frac{\lambda}{n}). \quad (5)$$

The throughput in Aloha (non-Slotted) present a weak immunity in front of collision, and a low level of transmission efficiency ($Th_{\text{max}}=0.185$) and also an unstable communication regime. However the Slotted Aloha leads to higher throughput ($Th_{\text{max}}=0.37$), better synchronization and bigger capacity in avoiding collisions.

2.1 Multi-copy Aloha

When we send m copies of a packet with Slotted Aloha technique (Multi-copy), the success probability of the transmission of this packet or the probability that a packet, out of the m sent copies will not collide; will be more important in comparison with the case where only one copy is sent (S.Aloha). This can be true only when the other packets are sent in one copy or when the channel traffic remains stable and free from perturbation. To maximize the probability of transmission success, we assume that all users send the same numbers of copies (m) [9]. The Multi-copy Aloha is generally conceived for the following systems; -satellite systems which offer a better probability of transmission success, -multi-channel Aloha systems and -Aloha reservation systems with a higher probability of success.

We consider that the packet arrival is a Poisson process. For the simple Slotted Aloha, we assume that the average delay of transmission is higher than 5 slots, whereas the average value of the delay for the m copies including the first transmission must be as high as 5 slots. We have;
\[ \lambda = \lambda_1 + \lambda_2 + \ldots + \lambda_n = \sum_{i=1}^{n} \lambda_i . \]  

When we have \( N \) copies;

\[ N = \lambda^{-1} \sum_{i=1}^{n} i \lambda_i . \]  

The probability that packet \( i \) be successfully received is:

\[ P_i = 1 - \text{prob}[\text{All the copies are collided}] = 1 - (1 - e^{-\lambda_i})^i \]  

For \( k \) copies, the throughput equals;

\[ Th_k = \lambda P_k = \lambda \left[ 1 - (1 - e^{-k \lambda})^k \right] . \]  

For \( k=1,2,3,\ldots \) and \( k = \ln 2 / \lambda \); then

\[ Th \approx \frac{\ln 2}{k} \left[ 1 - \left(\frac{1}{2}\right)^k \right] = Th_k . \]  

Fig.1 Throughput of Multi-copy Aloha

By Multi-copy Aloha technique, we can ameliorate the performance of slotted Aloha throughput by sending multiple copies of packet when the traffic charge is not very high (less than 0.48). The original packet is successfully received if one of these copies is correctly received, where this method is going to increase \( m \) times the throughput of the system. Here, we propose to adopt more of \((N,K)\) coding diagram, like the Reed Solomon codes, to ameliorate the performances of slotted Aloha.

3 Erasure coding

We add to the message to be sent additional information which allows the message to be reconstructed at the receptor. The error corrector codes constitute a device which aims at ameliorating the transmission reliability on a noisy channel, the method they use consists of sending on the channel more data, exceeding the amount of information to be sent. A redundancy is thus introduced. If this redundancy is structured in an exploitable manner, it is then possible to correct possible errors introduced by the channel. We can thus despite the noise, find the whole information sent at the beginning. The purpose of erasure coding is to recover lost packets if their position is known in \((N-k)\) erasure codes. An \((N-k)\) code word consists of \( N \) code packets, with \( k \) original packets and \( (N-k) \) redundant packets. The \( k \) original packets can be recovered successfully if we receive \( N' \) packets out of the \( N \) coded packets. If \( N \) is sufficiently large compared to the loss rate, we can achieve high reliability without retransmission. Figure 2 illustrates the mechanism of erasure coding, when \((N-k)\) redundant packets are generated.

Fig.2 Erasure coding model description

The classic block codes for erasure correction are called Reed–Solomon codes. An \((N-k)\) Reed-Solomon code (over an alphabet of size \( q = 2^l \), where \( l \) is the length of a packet) has the ideal property that if any \( k \) of the \( N \) transmitted symbols are received, the original \( k \) source symbols can be recovered. But Reed-Solomon codes have the disadvantage that they are practical only for small \( N, k, \) and \( q \); standard implementations of encoding and decoding have a cost of \( C \) packet operations (the expected number of arithmetic operations to compute the coded packets, or to recover the original packets), where;

\[ C = k (N - k) \log_q N . \]  

Furthermore, with a Reed–Solomon code, as with any other block code, the erasure probability \( f \) must be estimated and the code rate \( R = k / N \) selected before the data are transmitted. For example, the first redundant packet is generated via the following equation;
With Erasure coding, it would be easy to recover the k original packets when N’ out of N coded packets are received. On the other hand, we remark the use of erasure coding is more efficient at high traffic load. When the throughput is low enough, the collision probability is low as well and this implies that Erasure coding is not necessary for recovering the collided packets. However when the throughput is high, then the collision is already serious and thus the use of all techniques of error control become necessary to ameliorate the situation. The results we obtained are similar to those found in [7] as far as Erasure coding application on Aloha technique is concerned.

3 The Game and its Model

In this section, we formulate the Slotted ALOHA game when using the Erasure coding. We began by defining the network model with some assumptions, and then we identify a game and its basic elements. After that, we provide our rationale for selecting an appropriate type of Erasure coding. We consider M wireless nodes that are willing to transmit data (active nodes) to a designated receiver. Nodes use a slotted Aloha or non-persistent CSMA based protocol to resolve contention at the MAC layer and to reduce the number of collisions among packets by always rescheduling a packet that upon arrival finds the channel to be busy. A slotted version of the Aloha protocol can be considered where the time axis is slotted with a slot size of \((\text{tau}, \text{sec})\). All nodes are synchronized and are forced to start transmission only at the beginning of a slot. Slot duration is set to the maximum signal propagation time of \(\tau = d_{\text{max}}/C\) sec, where \(d_{\text{max}}\) is the maximum separation distance between nodes, and \(C\) is the speed of light. This ensures that after transmission stops, all nodes will find the channel to be clear after one slot time. Thus, each transmission must be preceded by an idle slot. We assume also that all packets are of constant length and the number of bits per packet \(l\) satisfies the condition for Reed-Solomon coding, \(E < q , (q = 2^t)\). Thus, each transmission must be preceded by an idle slot. We assume also that all packets are of constant length and the number of bits per packet \(l\) satisfies the condition for Reed-Solomon coding, \(E < q , (q = 2^t)\). We assume that the nodes are the players in the game. A player enters the game when he has a packet to transmit (active player), and leaves the game when all his packets have been successfully transmitted. The game in this paper is a repeated non-cooperative game, i.e., there is no coordination.
between players and they act as free agents, and each player attempts to maximize his own payoff according to his strategy. Here the strategy is to select a suitable type of erasure coding, and the utility is the player’s throughput. When in the game, the player chooses the type of coding that minimizes the cost of coding, but at the same time maximizes his utility (throughput). After each transmission failure, the player repeats his strategy until transmission succeeds. Figure 4 illustrates the game model where each node selects \( \rho \) in coding before transmitting via a medium in a wireless network. The strategy of each player involves setting the number of redundant packets \( \rho \) to maximize player’s expected utility \( u \).

A game consists of a principal and a finite set of players \( \{1, 2, ..., M\} \), each of which selects a strategy \( s \in S \) with the aim of maximizing his utility \( u \). The utility function \( u_i(s) : S \rightarrow R \) represents each player’s sensitivity to the actions of the other players. Thus, our game can be modeled as a triple, \( G = (M, S, u) \) where \( M = \{1, 2, ..., M\} \) denotes the set of players in the game (action set), \( S \) denotes the set of strategies for the players \( (s_i \text{ denotes the strategies for player } i \text{ and } s_{-i} \text{ denotes the strategies for all players except player } i, s = \{s_1, s_2, ..., s_M\} \) = \{\( \rho_1, \rho_2, ..., \rho_M \} \) and \( u \) denotes the payoff assigned to each player (where \( U_i \) is the payoff utility assigned to player \( i \)).

To measure the utility (throughput) function, we follow the probability concept based on that in [7]. However in our study, we take into consideration the number of nodes forming the network, as well as the likelihood of choosing a type of encoding (decoding), and also the cost of encoding/decoding, which greatly influences the speed of the system. Next we derive the expressions for throughput for slotted Aloha protocol.

### 4 The Slotted Aloha Game

In our game we have \( M \) nodes, \( \lambda \) (packet/time slot) traffic load with the transmission of packets following a Poisson process with mean \( \lambda \), and throughput \( Th \) defined as:

\[
Th = \lambda P_S .
\]  
(2)

Where

\[
P_S = P_k + \sum_{n=1}^{k-1} \sum_{m=1}^{n} m/k P_{n,m} .
\]  
(3)

The probability that a packet is successfully transmitted is given by

\[
p = (e^{-\lambda N/k})(1-e^{-\lambda N/k})^{M-1}.
\]  
(4)

For simplicity, we set \( G = \lambda N/k = \lambda(1+\rho/k) \), where \( \rho \) is the number of redundant packets, then

\[
P_{\rho} = (e^{-\lambda k(1+\rho/k)})(1-e^{-\lambda k(1+\rho/k)})^{M-1}.
\]  
(5)

Let \( Q_{\rho} = 1-P_{\rho} \). Then, the probability that at least \( k \) encoded packets are received successfully is:

\[
P_k = \sum_{i=0}^{N} \left(\begin{array}{c} N \\
 i \end{array}\right) P_{\rho}^i Q_{\rho}^{N-i} .
\]  
(6)

\( P_{n,m} \) is the probability that only \( n(n \leq k) \) encoded packets are received where \( m \) out of \( n \) are original packets. Then, we have:

\[
P_{n,m} = \binom{k}{m} \binom{N-k}{n-m} P_{\rho}^m Q_{\rho}^{N-n} .
\]  
(7)

An original packet can be successfully received or recovered if at least \( k \) out of \( N \) encoded packets are correctly received or \( n(n \leq k) \) out of \( N \) encoded packets are received, \( m \) out of \( n \) received packets are original, and the packet under consideration is among these packets (with probability \( m/k \)). Thus, from (3) we conclude:
\[ P_S = \sum_{i=1}^{N} \left( \sum_{i=1}^{N} P_{\rho_i} Q_{\rho_i}^{N-i} + \sum_{n=1}^{k-i} \sum_{m=1}^{n} \left( m/k \right) \left( \sum_{i=1}^{n} P_{\rho_i} Q_{\rho_i}^{N-n} \right) \right) . \]  

From (2), the throughput can be expressed as:

\[ Th = \lambda \left( P_{\rho} + \sum_{n=1}^{k-1} \sum_{m=1}^{n} \left( m/k \right) P_{\rho} Q_{\rho}^{N-n} \right) . \] (9)

In addition,

\[ Th(\rho) = \lambda \left( \sum_{i=1}^{N} \left( P_{\rho_i} Q_{\rho_i}^{N-i} + \sum_{n=1}^{k-1} \sum_{m=1}^{n} \left( m/k \right) P_{\rho_i} Q_{\rho_i}^{N-n} \right) \right) . \] (10)

Accordingly, the utility function for an active player \( i \) is defined as:

\[ Th(\rho_i) = \lambda \left( N \sum_{i=1}^{N} P_{\rho_i} Q_{\rho_i}^{N-i} + \sum_{n=1}^{k-1} \sum_{m=1}^{n} \left( m/k \right) P_{\rho_i} Q_{\rho_i}^{N-n} \right) . \] (11)

### 4.1 Existence of a Nash Equilibrium (NE)

**Theorem 1 [11]:** (Debreu, Glicksberg and Fan, 1952) Consider a strategic-form game whose strategy spaces \( S_i \) are nonempty compact convex subsets of a Euclidean space. If the payoff functions \( U_i \) are continuous in \( S_i \) and quasi-concave in \( S_i \), there exists a pure strategy Nash equilibrium (NE).

**Theorem 2 [11]:** An equilibrium exists for every concave \( n \)-person game.

For such a case, Rosen proved the existence of equilibrium for a concave utility function. It is thus clear that since our utility (11) satisfies all preceding conditions \( \partial U_i / \partial \rho_i = 0 \), both theorems are applicable to this game. Thus, a pure NE exists. To find the NE of the game we analyze the player’s best response function [12]. The best response of player \( i \) is the number of redundant packets that maximizes the utility function.

**Definition 1 [13]:** An action \( s \) is the best reply to \( s_i \), if \( u_i(s, s_{-i}) \geq u_i(s', s_{-i}) \) for all \( s' \in S \). Let \( BR(s_i) \) denote the set of best replies to \( s_i \). An NE is an action profile \( s = (s_1, s_{-i}) \) in which \( s_i \in BR(s_i) \) for all \( i = 1 \ldots M \).

First, we investigate whether there exists a value of \( \rho \) such that a better throughput can be achieved. We consider the scenario where all nodes use the same value \( \rho \) and modify this in synchronization with the other nodes in the network. Note that all our simulations were carried out using the MATLAB simulator.

We investigated the behavior of the conventional system throughput (before integrating erasure coding) by varying \( \lambda \). According to Figures 5 and 6, throughput is maximized when \( \lambda \in [0.7, 1.5] \). Figure 6 plots in 3D the average throughput obtained by the network for different values of \( \lambda \) and \( \alpha \). For better and more precise results, we use \( \alpha = 0.5 \) in the reminder of this work (like the conventional model).

Moreover, Figure 7 confirms that operating with \( \rho = 0 \) leads to network collapse. However, there exists an optimal point of operation (\( \rho = 2 \) packets) at which throughput is maximized \( Th(\rho = 2) = \text{argmax} Th \) for every node in the network. We refer to this optimal point of operation as \( \rho^* \), which corresponds to the NE for our scheme. In other words, when \( \rho \leq \rho^* \), congestion dominates resulting in lower throughput, whereas when \( \rho \geq \rho^* \) throughput decreases. Thus, \( Th(\rho, \rho_{-i}) \geq Th(\rho^*, \rho_{-i}) \), for all \( \rho \in \rho \), and \( \rho = BR(\rho_i) \).

![Fig. 5 Conventional model Throughput versus \( \lambda \)](image-url)
Fig. 6  Conventional model $T_h$ with varying values of $\lambda$ and $\alpha$. When $\lambda \in [0.7, 1.5]$, $T_h$ is maximized.

4.2 Investigation of Equilibrium

Now, we investigate the importance of the obtained results. We begin this assessment with the cost of coding employed in our model. The cost of coding denotes the number of arithmetic operations performed during encoding, or decoding operations to recover encoded packets. In other words, it represents the speed of the system. The fastest system is the one with the lowest cost. Figure 7 shows that cost decreases at equilibrium.

In Figure 8 we evaluate the cost while varying the number of redundant packets as well as the number of original packets used during erasure coding. At equilibrium, we have the lowest cost $[0.9, 2.2\%]$; in this case, the encoder (decoder) carried out 90 arithmetic operations to generate two redundant packets (or recover eight original packets). Furthermore, it performed 950 arithmetic operations for ten redundant packets. This difference greatly affects the speed and also system performance. Figure 9 shows a comparison of throughput at equilibrium and without equilibrium (conventional system). In Figures 9, 10 and 13, we can see the improvement due to equilibrium, which becomes even clearer when we mimic reality by using a greater number of active nodes ($M=20$). In Figure 9, there is a small difference in throughput, which is possible only for a small $\lambda$: $\lambda \in [0.04, 0.14]$. For values beyond this interval, use of the NE model becomes ineffective. Figure 10 shows a remarkable improvement in throughput when using the NE model, where the difference can reach 0.02 packets/sec, with $\lambda=2$. Note that when we increase the number of active nodes (users), the advantage of equilibrium becomes more important.

At equilibrium (NE), throughput increases quickly to reach its maximum, after which it begins to deteriorate slowly until it reaches its lowest level at $\lambda=10$ packets/sec. It should be noted that, by increasing the number of active nodes, network throughput decreases; this is due to the number of collisions, which increases when increasing the number of active nodes (users).

4.3 Performance improvement in S-Aloha

The use of the erasure coding scheme and game theory analysis in the slotted Aloha protocol results in a stable and more efficient network. However, to confirm this advantage in general, in the following figures we compare the game model throughput with conventional model. In Figure 13, we can observe the difference in throughput between both game and conventional model. As in NE, the network acquire a remarkable progress in terms of throughput, and all the nodes are satisfied with their
best benefit without the need to change unilaterally their behaviour when access at MAC layer.

![Graph](image1.png)

**Fig. 9** Throughput versus $\lambda$, with $M=1$

![Graph](image2.png)

**Fig. 10** Throughput versus $\lambda$, with $M=20$

![Graph](image3.png)

**Fig. 12** Successful and failure reception probability.

6 Conclusion

We propose in this paper two complementary solutions to improve network performances in mobile ad hoc networks. It has been shown in the first section that the integration of Erasure coding in slotted Aloha protocol allows to recover all original packets and improve the system throughput. But the selfish behaviour of network users make the system instable and create more collisions. For that conflicting situation, we propose in the second section another solution based on Game theory. Our game model leads to acquire more advantage results. At Nash equilibrium, network throughput is maximized and all nodes are satisfied, without the need to change their strategies, which makes the network stable and more efficient. Use of our proposed solutions based on erasure coding and game theory makes the slotted Aloha MAC protocol stronger and able to resist many collisions.

References:


