Eigenstructure assignment by constant output feedback for generalized state-space systems

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Abstract: - In this paper the possibilities of constant output feedback in assigning the finite eigenstructure of closed-loop system obtained from a given linear time invariant generalized state-space system by constant output feedback are studied. In particular, an explicit necessary condition is given for the existence of a constant output feedback which assigns the desired finite eigenstructure of closed-loop system. The present method is simple and is based on properties of polynomial and rational matrices.

Key-Words: - invariant polynomials, eigenstrucure assignment, constant output feedback, linear systems.

1 Introduction

Can the eigenstructure of a given linear time invariant system be assigned by feedback? This simple question is known in linear systems theory as eigenstructure assignment or dynamics assignment or general pole placement, and has a long hisrory.

The complete solution of eigenstructure assignment by proportional state feedback for controllable regular state-space systems is given in [1]. This result is known as fundamental theorem of state feedback. The explicit necessary and sufficient condition given in [1] for the solution of eigenstructure assignment by proportional state feedback for controllable regular state-space systems, consists of inequalities which involve the controllability indices of the open – loop system and the degrees of invariant polynomials of the closed–loop system. Alternative proofs can be found in [2]–[4]. In [5] an explicit sufficient condition is obtained for the solution of eigenstructure assignment problem by dynamic output feedback for linear regular state-space systems. In [6] (as well as in [7]), and [8], explicit necessary conditions are established for the solution of eigenstructure assignment problem by constant output feedback for linear regular state-space systems. In [9] explicit necessary conditions are established for the solution of eigenstructure assignment problem by constant output feedback for linear generalized state-space systems. In [10]-[13], the Rosenbrock's fundamental theorem of state feedback is extended to generalized state-space systems.

In this paper, the finite eigenstructure assignment problem by constant output feedback for linear generalized state-space systems is examined. In particular, an explicit necessary condition is established for a list of nonunit monic polynomials having coefficients in field of real numbers to be invariant polynomials of a system obtained by constant output feedback from the given linear generalized state-space system. The list of invariant polynomials of closed-loop system is termed finite eigenstructure.

2 Problem Formulation

Consider a linear controllable and observable generalized state-space system described by the following state-space equations

$$\mathbf{E}\dot{\mathbf{x}}(t)=\mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \tag{1}$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) \tag{2}$$

with

$$\det[\mathbf{E}]=0 \text{ and } \det[\mathbf{E}s - \mathbf{A}] \neq 0 \tag{3}$$

and the control low

$$\mathbf{u}(t) = -\mathbf{F}\mathbf{y}(t) + \mathbf{v}(t) \tag{4}$$

where $\mathbf{F}$ is an $m \times p$ real matrix, $\mathbf{v}(t)$ is the reference input vector of dimensions $mx1$, $\mathbf{x}(t)$ is the state vector of dimensions $n \times 1$, $\mathbf{u}(t)$ is the vector of inputs of dimensions $m \times 1$ and $\mathbf{y}(t)$ is the vector of outputs of dimensions $p \times 1$ and $\mathbf{E}$, $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are real matrices of dimensions $n \times n$, $n \times n$, $n \times m$ and $n \times p$, respectively. The transfer function matrix of system (1) and (2) is given by

$$\mathbf{T}(\mathrm{s}) = \mathbf{C}(\mathbf{E}\mathrm{s} - \mathbf{A})^{-1}\mathbf{B} \qquad (5)$$

By applying the constant output feedback (4) to the system (1)and (2) the state-space equations of closed–loop system are

$$\mathbf{E}\dot{\mathbf{x}}(t)=(\mathbf{A}-\mathbf{BFC})\,\mathbf{x}(t) + \mathbf{B}\mathbf{v}(t) \qquad (6)$$

$$\mathbf{y}(t)=\mathbf{C}\mathbf{x}(t) \qquad (7)$$

Let $c_1(s)$, $c_2(s)$, …., $c_r(s)$ be arbitrary monic polynomials having coefficients in field of real numbers which satisfy the conditions

$$c_{i+1}(s) \text{ divides } c_i(s), \text{ for } i=1,2,\ldots,r-1, \qquad (8)$$

where r is the rank of the transfer function matrix (5) of system (1) and (2). The finite eigenstructure assignment problem considered in this paper can be stated as follows: Does there exist a constant output feedback (4) such that the closed-loop system (6) and (7) has invariant polynomials $c_1(s)$, $c_2(s)$,..., $c_r(s)$? If so, give conditions for existence.

3 Basic concepts and preliminary results

Let us first introduce some notations that are used throughout the paper. Let $\mathbf{R}$ be the field of real numbers also let $\mathbf{R}[s]$ be the ring of polynomials with coefficients in $\mathbf{R}$. Let $\mathbf{D}(s)$ be a nonsingular matrix over $\mathbf{R}[s]$ of dimensions $m \times m$ write $\deg_{ci}$ for the degree of column i of $\mathbf{D}(s)$. if

$$\deg_{ci}\mathbf{D}(s) \geq \deg_{cj}\mathbf{D}(s),\ i<j \qquad (9)$$

the matrix $\mathbf{D}(s)$ is said to be column degree ordered.
Denote $\mathbf{D}_n$ the highest column degree coefficient matrix of $\mathbf{D}(s)$.The matrix $\mathbf{D}(s)$ is said to be column reduced if the real matrix $\mathbf{D}_n$ is nonsingular. A polynomial matrix $\mathbf{U}(s)$ of dimensions $k \times k$ is said to be unimodular if and only if has polynomial inverse. The matrix $\mathbf{D}(s)$ is said to be column monic if its highest column degree coefficient matrix is the identity matrix. Two polynomial matrices $\mathbf{A}(s)$ and $\mathbf{B}(s)$ having the same numbers of columns are said to be relatively right prime if and only if there are matrices $\mathbf{X}(s)$ and $\mathbf{Y}(s)$ over $\mathbf{R}[s]$ such that

$$\mathbf{X}(s)\mathbf{A}(s) + \mathbf{Y}(s)\mathbf{B}(s)=\mathbf{I} \qquad (10)$$

where $\mathbf{I}$ is the identity matrix of dimensions $l \times l$, l is the number of columns of the polynomial matrices $\mathbf{A}(s)$ and $\mathbf{B}(s)$. Let $\mathbf{D}(s)$ be a nonsingular matrix over $\mathbf{R}[s]$ of dimensions $m \times m$, then there exist unimodular matrices $\mathbf{U}(s)$ and $\mathbf{V}(s)$ over $\mathbf{R}[s]$ such that

$$\mathbf{D}(s)=\mathbf{U}(s)\,\mathrm{diag}\,[a_1(s), a_2(s), \ldots a_m(s)]\mathbf{V}(s) \qquad (11)$$

where the polynomials $a_i(s)$ for $i=1,2, .., m$ are termed invariant polynomials of $\mathbf{D}(s)$ and have the following property

$$a_i(s) \text{ divides } a_{i+1}(s), \text{ for } i=1,2,\ldots m-1 \qquad (12)$$

Furthermore we have that

$$a_i(s) = \frac{d_i(s)}{d_{i-1}(s)},\ \text{for } i= 1,2,\ldots\ldots m \qquad (13)$$

where $d_o(s)=1$ by definition and $d_i(s)$ is the monic greatest common divisor of all minors of order i in $\mathbf{D}(s)$, for $i=1,2,\ldots., m$. Two polynomial matrices $\mathbf{A}(s)$ and $\mathbf{B}(s)$ of appropriate dimensions are equivalent over $\mathbf{R}[s]$ if and only if there exist unimodular matrices $\mathbf{P}_1(s)$ and $\mathbf{P}_2(s)$ over $\mathbf{R}[s]$, such that $\mathbf{A}(s) = \mathbf{P}_1(s)\ \mathbf{B}(s)\ \mathbf{P}_2(s)$. If two polynomial matrices are equivalent over $\mathbf{R}[s]$, then they have the same invariant polynomials.
The relationship (12) is known as the Smith – McMillan form of $\mathbf{D}(s)$ over $\mathbf{R}[s]$. Let $\mathbf{R}(s)$ be the field of rational functions in s. A matrix $\mathbf{W}(s)$ whose elements are rational functions is called rational matrix. A conceptual tool for the study of rational matrices is the following standard form. Every rational matrix $\mathbf{W}(s)$ of dimensions $p \times m$ with rank $[\mathbf{W}(s)]=r$, can be expressed as

$$\mathbf{W}(s) = \mathbf{U}_1(s)\,\mathbf{M}(s)\,\mathbf{U}_2(s) \qquad (14)$$

The matrices $\mathbf{U}_1(s)$ and $\mathbf{U}_2(s)$ are unimodular matrices over $\mathbf{R}[s]$ and the matrix $\mathbf{M}(s)$ is given by

$$\mathbf{M}(s) = \begin{bmatrix} \mathbf{M}_r(s) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \qquad (15)$$

The nonsingular matrix $\mathbf{M}_r(s)$ of dimensions $r \times r$ is given by

$$\mathbf{M}_r(s) = \mathrm{diag}[b_1(s)/c_1(s), \ldots, b_r(s)/c_r(s)] \qquad (16)$$

(a) The polynomials $c_i(z)$ and $b_i(z)$ are relatively prime for every $i = 1, 2, \ldots, r$.
(b) The polynomial $c_{i+1}(z)$ divides polynomial $c_i(z)$ for every $i = 1, 2, \ldots, r - 1$.
(c) The polynomial $b_i(z)$ divides polynomial $b_{i+1}(z)$ for every $i = 1,2, \ldots, r - 1$.

The relationship (15) is the Smith-McMillan form of rational matrix $\mathbf{W}(s)$ over $\mathbf{R}[s]$.

Definiton 1. Relatively right prime polynomials matrices $\mathbf{D}(s)$ and $\mathbf{N}(s)$ of dimensions $m \times m$ and $p \times m$ respectively with $\begin{bmatrix} \mathbf{N}(s) \\ \mathbf{D}(s) \end{bmatrix}$ to be column reduced and column degree ordered such that

$$\mathbf{C}(\mathbf{E}s - \mathbf{A})^{-1}\mathbf{B} = \mathbf{N}(s)\mathbf{D}^{-1}(s) \qquad (17)$$

are said to form a standard right matrix fraction description of system (1).

The column degrees of the matrix $\begin{bmatrix}\mathbf{N}(s)\\ \mathbf{D}(s)\end{bmatrix}$ are the controllability indices of system (1) and (2).
The system (1) and (2) is controllable if and only if

$$\text{rank}\,[\mathbf{E}s - \mathbf{A}\,,\, \mathbf{B}] = n \tag{18}$$

for all complex numbers s. For more details see [14] and [15] and references given therein.
The system (1) and (2) is observable if and only if

$$\text{rank}\begin{bmatrix}\mathbf{C}\\ \mathbf{E}s-\mathbf{A}\end{bmatrix} = n \tag{19}$$

for all complex numbers s.
The following Lemmas are needed to prove the main theorem of this paper.

Lemma 1. Let $\mathbf{D}(s)$, $\mathbf{N}(s)$ be a standard right matrix fraction description of controllable and observable system (1). Then for every $m \times p$ real matrix $\mathbf{F}$, the polynomial matrices $\mathbf{N}(s)$ and $[\mathbf{D}(s)+\mathbf{FN}(s)]$ are relatively right prime over $\mathbf{R}[s]$.

Proof: Let $\mathbf{D}(s)$ and $\mathbf{N}(s)$ be a standard right matrix fraction description of (1), [16]. Then for the transfer function matrix of closed – loop system (6), (7) we have that

$$\mathbf{C}[\mathbf{E}s - \mathbf{A} + \mathbf{BFC}]^{-1}\mathbf{B} = \mathbf{N}(s)\,[\mathbf{D}(s) + \mathbf{FN}(s)]^{-1} \tag{20}$$

It is assumed that $\det[\mathbf{E}s - \mathbf{A}+\mathbf{BFC}] \neq 0$.We can write

$$\begin{bmatrix}\mathbf{N}(s)\\ \mathbf{D}(s)+\mathbf{FN}(s)\end{bmatrix} = \begin{bmatrix}\mathbf{I}_p & \mathbf{O}\\ \mathbf{F} & \mathbf{I}_m\end{bmatrix}\begin{bmatrix}\mathbf{N}(s)\\ \mathbf{D}(s)\end{bmatrix} \tag{21}$$

Since the matrix

$$\begin{bmatrix}\mathbf{I}_p & \mathbf{O}\\ \mathbf{F} & \mathbf{I}_m\end{bmatrix} \tag{22}$$

is unimodular and the matrices $\mathbf{N}(s)$ and $\mathbf{D}(s)$ are by assumption relatively right prime over $\mathbf{R}[s]$, we have from (21) that the matrices $[\mathbf{D}(s) + \mathbf{FN}(s)]$ and $\mathbf{N}(s)$ are relatively right prime over $\mathbf{R}[s]$ for every $m \times p$ real matrix $\mathbf{F}$ and the proof is complete.

Lemma 2. Let $\mathbf{D}(s)$, $\mathbf{N}(s)$ be a standard right matrix fraction description of system (1). Let $b_i(s)$ for $i = 1,2,\ldots,r$ be the invariant polynomials of polynomial matrix $\mathbf{N}(s)$. Then for every $m \times p$ real matrix $\mathbf{F}$; the Smith–McMillan form over $\mathbf{R}[s]$ of the transfer function matrix $\mathbf{T}_c(s)$ of closed-loop system (6), (7) has the following form

$$\mathbf{T}_c(s) = \mathbf{U}_1(s)\begin{bmatrix}\mathbf{T}_r(s) & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\mathbf{U}_2(s) \tag{23}$$

The matrices $\mathbf{U}_1(s)$ and $\mathbf{U}_2(s)$ are unimodular matrices over $\mathbf{R}[s]$ and the nonsingular matrix $\mathbf{T}_r(s)$ of dimensions $r \times r$ is given by

$$\mathbf{T}_r(s) = \text{diag}[b_1(s)/c_1(s), \ldots, b_r(s)/c_r(s)] \tag{24}$$

Proof: Let $\mathbf{D}(s)$ and $\mathbf{N}(s)$ be a standard right matrix fraction description of (1). Then for the transfer function matrix $\mathbf{T}_c(s)$ of closed – loop system (6) and (7) we have that

$$\mathbf{T}_c(s) = \mathbf{C}[\mathbf{E}s - \mathbf{A} + \mathbf{BFC}]^{-1}\mathbf{B} = \mathbf{N}(s)[\mathbf{D}(s) + +\mathbf{FN}(s)]^{-1} \tag{25}$$

It is assumed that $\det[\mathbf{E}s - \mathbf{A}+\mathbf{BFC}] \neq 0$. Since the transfer function matrix $\mathbf{T}_c(s)$ of closed-loop system (6) and (7) is rational matrix, there exist unimodular matrices $\mathbf{U}_1(s)$ and $\mathbf{U}_2(s)$ over $\mathbf{R}[s]$ such that

$$\mathbf{T}_c(s) = \mathbf{U}_1(s)\begin{bmatrix}\mathbf{T}_r(s) & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\mathbf{U}_2(s) \tag{26}$$

The nonsingular matrix $\mathbf{T}_r(s)$ of dimensions $r \times r$ is given by

$$\mathbf{T}_r(s) = \text{diag}[f_1(s)/c_1(s), \ldots, f_r(s)/c_r(s)] \tag{27}$$

The relationship (26) is the Smith–McMillan form over $\mathbf{R}[s]$ of the transfer function matrix $\mathbf{T}_c(s)$ of closed-loop system (6) and (7).
We define the matrices $\mathbf{K}(s)$ and $\mathbf{L}(s)$ over $\mathbf{R}[s]$ of dimensions $m \times p$ and $m \times m$ respectively

$$\mathbf{K}(s) = \mathbf{U}_1(s)\,[f_1(s), \ldots, f_r(s), 0, \ldots, 0] \tag{28}$$

$$\mathbf{L}(s) = \mathbf{U}_2^{-1}(s)[c_1(s), \ldots, c_r(s), 1, \ldots, 1] \tag{29}$$

Since the relationship (26) is Smith–McMillan form over $\mathbf{R}[s]$ of the transfer function matrix $\mathbf{T}_c(s)$ of closed-loop system (6), (7), it is obvious that the matrices $\mathbf{K}(s)$ and $\mathbf{L}(s)$ over $\mathbf{R}[s]$ are relatively right prime over $\mathbf{R}[s]$. Using (26), (27), (28) and (29) the relationship (25) can be rewritten as follows

$$\mathbf{T}_c(s) = \mathbf{N}(s)[\mathbf{D}(s) + +\mathbf{FN}(s)]^{-1} = \mathbf{K}(s)\mathbf{L}^{-1}(s) \tag{30}$$

Since by Lemma 1 the polynomial matrices $\mathbf{N}(s)$ and $[\mathbf{D}(s)+\mathbf{FN}(s)]$ are relatively right prime over $\mathbf{R}[s]$ for every $m \times p$ real matrix $\mathbf{F}$ and the matrices $\mathbf{K}(s)$ and $\mathbf{L}(s)$ over $\mathbf{R}[s]$ are also relatively right prime over $\mathbf{R}[s]$, there exists a unimodular matrix $\mathbf{V}(s)$ over $\mathbf{R}[s]$ such that

$$[\mathbf{D}(s) + \mathbf{FN}(s)] = \mathbf{L}(s)\,\mathbf{V}(s) \tag{31}$$

$$\mathbf{N}(s) = \mathbf{K}(s)\,\mathbf{V}(s) \tag{32}$$

From (32) it follows that the matrices $\mathbf{N}(s)$ and $\mathbf{K}(s)$ are equivalent over $\mathbf{R}[s]$ and therefore they have the same invariant polynomials. We have that

$$f_i(s) = b_i(s) \text{ for every } i = 1, 2,\ldots, r \tag{33}$$

Substituting relationship (33) to relationship (27) we have

$$\mathbf{T}_r(s) = \text{diag}[b_1(s)/c_1(s), \ldots, b_r(s)/c_r(s)] \tag{34}$$

Relationship (23) follows from (26) and (34) and the proof is complete.

4 Problem Solution

The following theorem is the main result of this paper and gives an explicit necessary condition for the solution of the finite eigenstructure assignment problem by constant output feedback for controllable and observable generalized state-space systems.

Theorem 1. Let **D**(s), **N**(s) be a standard right matrix fraction description of controllable and observable system (1). Also let $b_i(s)$ for $i = 1, \ldots, r$ be the invariant polynomials of polynomial matrix **N**(s). Further let $c_1(s)$, $c_2(s)$, …., $c_r(s)$ be arbitrary monic polynomials having coefficients in field of real numbers subject, however, to condition (8). Then the finite eigenstructure assignment problem by constant output feedback for controllable and observable generalized state-space systems has a solution only if

$$\deg b_i(s) = 0 \text{ for every } i = 1,2, \ldots, r \quad (35)$$

Proof: Suppose that there exists a constant output feedback (4) such that the finite eigenstructure of closed-loop system (6) and (7) is given by the list of arbitrary monic polynomials having coefficients in field of real numbers $c_1(s)$, $c_2(s)$, …., $c_r(s)$ subject, however, to condition (8). According to Lemma 2 the transfer function matrix $\mathbf{T}_c(s)$ of closed-loop system (6) and (7) has the following form:

$$\mathbf{T}_c(s) = \mathbf{U}_1(s)\begin{bmatrix} \mathbf{T}_r(s) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}\mathbf{U}_2(s) \quad (36)$$

The matrices $\mathbf{U}_1(s)$ and $\mathbf{U}_2(s)$ are unimodular matrices over **R**[s] and the nonsingular matrix $\mathbf{T}_r(s)$ of dimensions $r \times r$ is given by

$$\mathbf{T}_r(s) = \mathrm{diag}[b_1(s)/c_1(s), \ldots, b_r(s)/c_r(s)] \quad (37)$$

Since relationship (36) is the Smith–McMillan form over **R**[s] of the transfer function matrix $\mathbf{T}_c(s)$ of closed-loop system (6) and (7), the polynomials $c_i(z)$ and $b_i(z)$ must be relatively prime for every $i = 1, 2, \ldots, r$. Since by assumption the list of polynomials $c_1(s)$, $c_2(s)$, …., $c_r(s)$ are arbitrary monic polynomials having coefficients in field of real numbers subject, however, to condition (8), the polynomials $b_i(z)$ and $c_i(z)$ are relatively prime for every $i = 1, 2, \ldots, r$ if and only if

$$\deg b_i(s)=0 \text{ for every } i = 1,2, \ldots, r \quad (38)$$

This completes the proof.

Remark . The eigenstructure assignment problem by constant output feedback for linear time invariant systems is probably one of the most prominent open questions in linear systems theory [17]. There are only a few explicit necessary conditions in the literature for the solvability of the eigenstructure assignment problem by constant output feedback for generalized state-space systems, namely the explicit necessary conditions in [9].
The main theorem of this paper adds yet another necessary condition to the existing necessary conditions. This clearly demonstrates the originality of the contribution of main theorem of this paper with respect to existing results.

Example: Consider a controllable and observable system of the form of equations (1) and (2) whose transfer function matrix is given by

$$\mathbf{T}(s) = \begin{bmatrix} \frac{s^2}{s+1} & 0 \\ 0 & s \end{bmatrix}$$

The task is to check if the finite eigenstructure assignment problem by constant output feedback has a solution. We define the matrices

$$\mathbf{D}(s) = \mathrm{diag}[(s+1),\ 1]$$

$$\mathbf{N}(s) = \mathrm{diag}\,[s^2, s]$$

It is obvious that matrices **D**(s) and **N**(s) are relatively right prime over **R**[s] and the matrix $\begin{bmatrix} \mathbf{N}(s) \\ \mathbf{D}(s) \end{bmatrix}$ is column reduced and column degree ordered. Since

$$\mathbf{T}(s) = \mathbf{N}(s)\mathbf{D}^{-1}(s)$$

we conclude that the polynomial matrices **D**(s), **N**(s) form a standard right matrix fraction description of the given system. The Smith–McMillan form over **R**[s] of polynomial matrix **N**(s) is given by

$$\mathbf{N}(s) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \mathrm{diag}\,[s,\ s^2] \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

The invariant polynomials of polynomial matrix **N**(s) are

$$b_1(s) = s \text{ and } b_2(s) = s^2$$

Since $\deg b_1(s)=\deg b_2(s)=1$, it follows from the main theorem of this paper that finite eigenstructure assignment problem by constant output feedback, has no solution.

5 Conclusions

The properties of polynomial and rational matrices are used to obtain an explicit necessary condition for the assignment of the finite eigenstructure of generalized state-space systems by constant output feedback. In particular, it has been proven that the finite eigenstructure problem of generalized state-space systems by constant output feedback has a solution only if the open-loop system has no finite zeros. The main theorem of this paper generalizes the main result of [6] (as well as [7]) to linear generalized state-space systems.

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