

An analysis of a BJT high frequency phase shift amplifier in a polynomial form

KITTIPONG TRIPETCH

Department of Electronic and Telecommunication Engineering
Rajamangala University of Technology Suvarnabhumi

7/1 Nonthaburi Road, Suan Yai, Muang, Nonthaburi, THAILAND

Kittipong.tripetch.mr@ieee.org <https://www.facebook.com/kittipong.tripetch>

Abstract: - An analysis of a BJT phase shift amplifier is proposed in a detail polynomial form. The numerator and denominator polynomial form are derived which can be seen as a symbolic coefficient which are multiplied with complex frequencies of the order of the polynomial. A detail analysis is very laborious but it is presented some of the coefficients which are a function of the small signal parameters of the equivalent circuit such as a transconductance, output resistor, base spreading resistor, pi resistor, myu resistor and two parasitic capacitances, passive resistor for biasing, passive capacitors and resistors which are the core circuit of the phase shift networks. After dc operating point are designed as a function of the current consumption. All small signal parameters of the transistors of the phase shift amplifier are designed as a function of the current consumption. The BJT phase shift frequency response can be plotted by a polynomial form which can be calculated by MATLAB. The BJT phase shift amplifier may not worked as a phase shift oscillator without the input impedance formulas.

Key-Words: - BJT amplifiers, BJT oscillators, transfer function of the input impedance, impedance matching function, MATLAB graphs,

1 Introduction

The phase shift oscillator design was existed and published before 1941 [1]. The extension of the operating frequency of the phase shift oscillator can be done by using the equivalent circuit of a resistance capacitance transmission line [2]. The generalized input impedance can be derived from passive phase shift network [3]. Its input impedance formula can be used with feedback to the input of vacuum tube amplifier. The application of signal flow graph to analyze the phase shift oscillator was done while the root locus methodology was used to plot the graph of the pole frequency of the transfer function [4]. The paper do not used the high frequency equivalent circuit of the BJT to calculate the input impedance of the phase shift amplifier [4]. To design other type of oscillator such as a cross couple oscillator. It can be shown in [5] that a parametric sweep of capacitor and inductor can be done with the polynomial form of the input impedance of the circuit. The resistor inductor oscillator is not a popular topology because it is bulky and expensive than a resistor capacitor oscillator. Since that time, the transfer function can be derived based on application of the control theory which separates phase shift oscillator circuit diagram into a network of passive circuit and a common emitter which composed of a resistor and a

capacitor. The gain necessary for oscillator can be derived by determining the ratio of the output voltage and input voltage, then substituting a complex frequency s with $j\omega$ so that the denominator polynomial is transformed into a symbolic complex number. The next step is to equate the symbolic real part and symbolic imaginary part with zero. After that, the oscillation frequency can be derived. The problem is that the tube amplifier is not well described in that paper, instead it used Thevinin voltage source and a series source resistors to represent a tube amplifier. In section 2, an original concept of the phase shift oscillator are reviewed with a matrix of Kirchhoff's current law. In section 3, we described how can we derived the input impedance transfer function of the phase shift amplifier. The small signal parameters which are mixed with a passive parameters in the phase shift oscillator circuit diagram. It can be grouped to make a new coefficients so that it can be written in a more compact matrix form. The input impedance transfer function of the BJT phase shift oscillator can be derived by a symbolic Gaussian elimination, by keeping only a single

node variable for each column in the matrix and eliminating all other node variable for the same column. The ratio of the input impedance can be derived in a symbolic polynomial form. The section4, we present the graph of the frequency response of the input impedance of the phase shift amplifier which all defined coefficients can be grouped as a function of the small signal parameters and all the passive parameters name which can be design iteratively because the exact factorization may not existed in the literature if the polynomial form is higher than 4th order.

A contribution of this paper is the novel input impedance formula in symbolic polynomial form of the phase shift amplifier and a typical frequency response graph of the input impedance at a typical three value of the collector current consumption and a typical passive value of the capacitors and resistors in the phase shift network.

2 An Original Concept of a BJT phase shift oscillator topology

Because it is usually simple phase response which can be shifted as a function of the series resistor and capacitors multiple times. Sometimes, the phase response can be changed as a function of the input frequency. Thus, it may be obvious to see the phase shift oscillator and phase shift amplifier as an original circuit diagram according to the reference [1]. There are four topologies of the phase shift oscillator in the original paper which is published since 1941 [1]. But this paper has four section of RC network which are composed of four capacitors and three resistors, the two bias resistors are not included in the RC network. It is described in [1] that the phase shift oscillator has a magnitude response or voltage gain and phase response of an amplifier stage and feedback stage which are written as a function of a passive network. Its multiplication of the magnitude response of the amplifier and feedback network polynomial form should be higher or equal to one while the addition of the phase response should be zero at an oscillation frequency. After Kirchhoff's current law (KCL) is written by analyzing the circuit diagram of the phase shift oscillator in figure1. By

assigning the direction of the current which should be flow from high dc voltage level to lower dc voltage level which are biasing according to forward active region condition ($V_{BC} < 0, V_{BE} > 0.7$). There are six nodes in this circuit diagram. Therefore, all of the total equations in this circuit should be equal to six equations. The matrix which is a system of the current equations at all nodes after using KCL and grouping of the small signal parameters and passive element variables. The polynomial form could not finished easily without a definition of the grouping by alphabet variable which have a subscript numbers so that the variable name which are defined only 26 names. It is not enough novel coefficients without subscript number and subscript alphabet. The six equations of KCL can be written as a characteristics matrix which is shown in equation 1 as follows.

$$\begin{bmatrix} -a_1 & a_2 & 0 & 0 & 0 & a_3 \\ a_4 & -a_5 & a_6 & 0 & 0 & 0 \\ 0 & a_7 & -a_8 & a_9 & 0 & 0 \\ 0 & 0 & a_{10} & -a_{11} & a_{12} & 0 \\ 0 & 0 & 0 & a_{13} & -a_{14} & a_{15} \\ a_{16} & 0 & 0 & 0 & a_{17} & -a_{18} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = \begin{bmatrix} -I_{in} R_6 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

There are eighteen variable names after grouping the coefficients which is multiplied by six node voltages and six equation of the KCL. They are defined as following.

$$\begin{aligned} a_1 &= s(C_3 + C_4)R_6 + 1, a_2 = (sC_4R_6), a_3 = (sC_3R_6), a_4 = (sC_4) \\ a_5 &= \left[sC_4 + \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{r_b} \right) \right] = [sC_4 + a_{52}] \\ a_6 &= \left(\frac{1}{r_b} \right), a_7 = \left[\frac{1}{r_b} \right], a_8 = \left[\frac{1}{r_b} + \frac{1}{r_\pi} + sC_\pi + \frac{1}{r_\mu} + sC_\mu \right] \\ a_9 &= \left[\frac{1}{r_\mu} + sC_\mu \right], a_{10} = \left[sC_\mu + \frac{1}{r_\mu} - g_m \right], a_{11} = \left[sC_\mu + \frac{1}{r_\mu} + \frac{1}{r_o} + sC_1 \right] \\ a_{12} &= sC_1, a_{13} = sC_1, a_{14} = \left[sC_1 + \frac{1}{R_4} + sC_2 \right] \\ a_{15} &= (sC_2), a_{16} = (sC_3), a_{17} = (sC_2), a_{18} = \left[sC_2 + \frac{1}{R_5} + sC_3 \right] \end{aligned} \tag{2}$$

The phase shift amplifier can be shown in figure 1 (a) in the next page. The circuit diagram shows four capacitors and six resistors. The three resistors are for biasing circuit and three resistors are for phase shift network. The high frequency equivalent circuit is shown in figure 1 (b) as follow

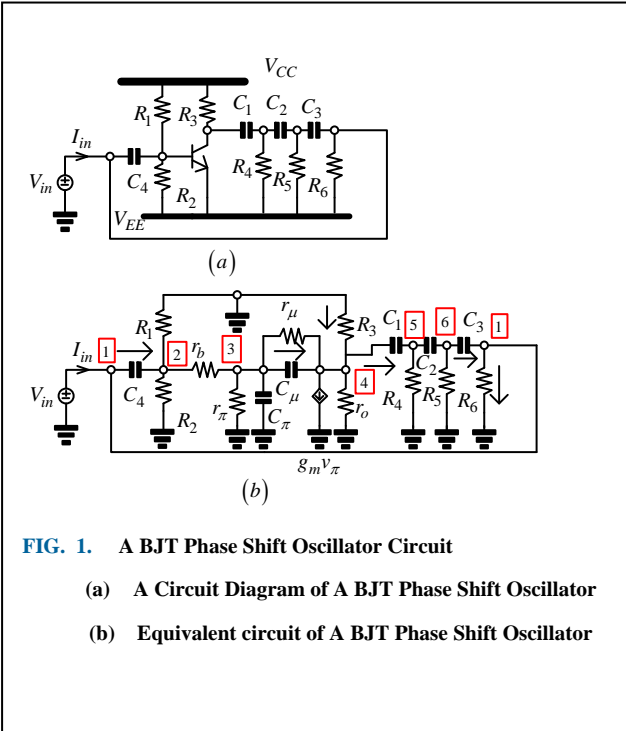


FIG. 1. A BJT Phase Shift Oscillator Circuit
 (a) A Circuit Diagram of A BJT Phase Shift Oscillator
 (b) Equivalent circuit of A BJT Phase Shift Oscillator

3 A Derivation of a BJT phase shift amplifier input impedance

By substituting the small signal high frequency equivalent circuit of a BJT transistor in fig. 1 (a). As a results, the whole circuit diagram of the small signal circuit of the BJT phase shift oscillator can be drawn in figure 1 (b). It can be seen that the voltage source is injecting at node1 in fig. 1 (b). The total of 6 nodes can be counted and assigned in the equivalent circuit of fig. 1 (b). After performing the KCL at all nodes. The system of equations are written as a matrix in frequency domain so that everyone can understand that any node ratio can be derived by symbolic Gaussian elimination. After admittance grouping are defined as the short coefficients. The last step of the derivation is written as a third equation for the description of the closed form which must be manipulate until the last equation which is shown in equation (7).

$$Z_m = \frac{V_1}{I_{in}} = \frac{f_{17}}{a_{17}} \tag{3}$$

The intermediate coefficients of an input impedance analysis of the BJT phase shift oscillator is to write a function of the node voltage V_1 of the first row of the matrix as a function of the other node voltage variables so that the same variable can be eliminated for the purpose of the next column has no recursive of the same variable in the first column when someone want to eliminate the variable V_2 with the second row equation which is written as a function of the other node voltage. The same procedure can be repeated until the next and the last column has one variable left. As a results, the derivation will be finished as a closed form solution as in equation (3).

The last group of intermediate coefficients are shown in equation (4) as follow.

$$\begin{aligned} f_{17} &= \left[f_{13} - \frac{f_{16}f_{14}}{f_{15}} \right], f_{16} = \left[f_{12} - \frac{f_9f_{10}}{f_7} \right], f_{15} = \left[\frac{f_8f_{10}}{f_7} + f_{11} \right] \\ f_{14} &= \left[d_{15} - \frac{f_8d_{14}}{f_7} \right], f_{13} = \left[d_{13} + \frac{f_9d_{14}}{f_7} \right], f_{12} = \left[\frac{d_9d_{11}}{d_7} - d_{10} \right] \\ f_{11} &= \left[-\frac{d_8d_{11}}{d_7} + d_{12} \right], f_{10} = \left[\frac{a_{12}d_{11}}{d_7} + a_{17} \right], f_9 = \left[\frac{d_9a_{13}}{d_7} \right] \\ f_8 &= \left[\frac{d_8a_{13}}{d_7} + a_{15} \right], f_7 = \left[\frac{a_{12}a_{13}}{d_7} + a_{14} \right], f_6 = \left[b_{10} - \frac{d_9a_9}{d_7} \right] \\ f_5 &= \left[\frac{d_8a_9}{d_7} - b_{12} \right], f_4 = \left[\frac{a_{12}a_9}{d_7} \right], f_3 = \left[d_6 - \frac{d_9d_4}{d_7} \right] \\ f_1 &= \left[\frac{a_{12}d_4}{d_7} \right], f_2 = \left[\frac{d_8d_4}{d_7} + d_5 \right] \end{aligned}$$

(4)

All of the intermediate coefficients are defined as an addition, negation, multiplication or division of at least three prior intermediate coefficients as follows

$$\begin{aligned} d_1 &= \left[b_7 + \frac{b_{10}b_8}{b_{11}} \right], d_2 = \left[\frac{a_9b_8}{b_{11}} \right], d_3 = \left[\frac{b_{12}b_8}{b_{11}} + b_9 \right] \\ d_4 &= \left[\frac{a_9a_6}{b_{11}} \right], d_5 = \left[b_3 - \frac{b_{12}a_6}{b_{11}} \right], d_6 = \left[\frac{b_{10}a_6}{b_{11}} - b_1 \right] \\ d_7 &= \left[\frac{a_9a_{10}}{b_{11}} - a_{11} \right], d_8 = \left[\frac{b_{12}a_{10}}{b_{11}} \right], d_9 = \left[\frac{b_{10}a_{10}}{b_{11}} \right] \\ d_{10} &= \left[b_{13} + \frac{b_{10}b_{14}}{b_{11}} \right], d_{11} = \left[\frac{a_9b_{14}}{b_{11}} \right], d_{12} = \left[b_{15} + \frac{b_{12}b_{14}}{b_{11}} \right] \\ d_{13} &= \left[d_1 - \frac{d_9d_2}{d_7} \right], d_{14} = \left[\frac{a_{12}d_2}{d_7} \right], d_{15} = \left[\frac{d_8d_2}{d_7} - d_3 \right] \end{aligned}$$

(5)

The parameter name d_i are coefficients name in equation (5) which can be written as a function of other intermediate coefficients names b_i and a_i and prior intermediate coefficients which have prior subscript which is lower number such as d_{15} is a function of d_8 multiplied with d_2 and divide with d_7 .

$$\begin{aligned}
 b_1 &= \left(\frac{R_6 a_4}{a_1} \right), b_2 = \left(\frac{a_2 a_4}{a_1} - a_5 \right), b_3 = \left(\frac{a_3 a_4}{a_1} \right) \\
 b_4 &= \left(\frac{a_2 a_{16}}{a_1} \right), b_5 = \left(\frac{a_3 a_{16}}{a_1} - a_{18} \right), b_6 = \left(\frac{R_6 a_{16}}{a_1} \right) \\
 b_7 &= \left[R_6 - \frac{b_1 a_2}{b_2} \right], b_8 = \left(\frac{a_6 a_2}{b_2} \right), b_9 = \left(a_3 - \frac{b_3 a_2}{b_2} \right) \\
 b_{10} &= \left(\frac{b_1 a_7}{b_2} \right), b_{11} = \left(\frac{a_6 a_7}{b_2} + a_8 \right), b_{12} = \left(\frac{b_3 a_7}{b_2} \right) \\
 b_{13} &= \left(b_6 - \frac{b_1 b_4}{b_2} \right), b_{14} = \left(\frac{a_6 b_4}{b_2} \right), b_{15} = \left(b_5 - \frac{b_3 b_4}{b_2} \right)
 \end{aligned}
 \tag{6}$$

After this step, the polynomial form can be start manipulated by substituting the coefficients from equation (2) into equation (6) with intermediate coefficients b_1, b_2, \dots until b_{15} . Then some of the polynomial form of b_1 can be substituted into intermediate coefficients which is called b_{10} to make a polynomial form.

$$Z_m = \frac{V_i - I_{17}}{I_m - a_{17}} = \frac{\begin{aligned} & s^{35} h_1 + s^{34} h_2 + s^{33} h_3 + s^{32} h_{38} + s^{31} h_{39} + s^{30} h_{70} + s^{29} h_{71} + s^{28} h_{72} + s^{27} h_{73} + s^{26} h_{74} \\ & + s^{25} h_{75} + s^{24} h_{76} + s^{23} h_{77} + s^{22} h_{78} + s^{21} h_{79} + s^{20} h_{80} + s^{19} h_{81} + s^{18} h_{82} + s^{17} h_{83} + s^{16} h_{84} \\ & + s^{15} h_{85} + s^{14} h_{86} + s^{13} h_{87} + s^{12} h_{88} + s^{11} h_{89} + s^{10} h_{90} + s^9 h_{91} + s^8 h_{92} + s^7 h_{93} + s^6 h_{94} \\ & + s^5 h_{95} + s^4 h_{96} + s^3 h_{97} + s^2 h_{98} + s h_{99} + h_{100} \end{aligned}}{\begin{aligned} & s^{35} h_{101} + s^{34} h_{102} + s^{33} h_{103} + s^{32} h_{104} + s^{31} h_{105} + s^{30} h_{106} + s^{29} h_{107} + s^{28} h_{108} + s^{27} h_{109} + s^{26} h_{110} + s^{25} h_{111} \\ & + s^{24} h_{112} + s^{23} h_{113} + s^{22} h_{114} + s^{21} h_{115} + s^{20} h_{116} + s^{19} h_{117} + s^{18} h_{118} + s^{17} h_{119} + s^{16} h_{120} + s^{15} h_{121} \\ & + s^{14} h_{122} + s^{13} h_{123} + s^{12} h_{124} + s^{11} h_{125} + s^{10} h_{126} + s^9 h_{127} + s^8 h_{128} + s^7 h_{129} + s^6 h_{130} + s^5 h_{131} \\ & + s^4 h_{132} + s^3 h_{133} + s^2 h_{134} + s h_{135} + h_{136} \end{aligned}}
 \tag{7}$$

$a_{17} = (sC_2)$

The last equation of this paper is shown to make evidence that the polynomial form is finished in the derivation report [6].

4 A Simulation results of the input impedance of a BJT phase shift amplifier with MATLAB

All of the necessary equations which are defined and typed in the text file of MATLAB platform so the polynomial form of the input impedance function of the BJT phase shift amplifier can be defined accurately. The BJT phase shift amplifier dc operating point can be designed by setting the collector current consumption such as a 1mA, 5mA

and 10 mA because the transistor 2N3904 operates these ranges of a collector current. The three values of the biasing resistors can be computed by using Kitchoff's current law at the base terminal of the figure 1b. Base current can be computed by dividing the collector current by a dc current gain which can be seen from a data sheet of the transistor at a typical value of the operating temperature. If the feasible room temperature is 40 degree Celcius because the room has no air conditioning. The thermal voltage can be computed by a Boltzmann' constant multiplied by temperature and divided by electron charge. The base emitter voltage can be computed by multiplied thermal voltage by a natural logarithm of the ratio of the collector current and the saturation current. A saturation current can be seen from SPICE model of the transistor datasheet. For the circuit diagram of the phase shift amplifier, the emitter terminal is not connected with other passive elements because it is connected with a ground plane or a negative supply voltage

The other passive components in the phase shift network which are three resistors and four capacitors can be designed by equate them with numerical values. So that if the input impedance of the phase shift amplifier are defined and computed with all of the coefficients by basic programing. It means something similar with the first group of the coefficients which is label by the parameter a_i , the second group of the coefficients until the last group which are a function of the small signal parameters and all passive components in the circuit diagram. The magnitude and phase response at the various collector current consumption can be plotted as shown in fig. 2

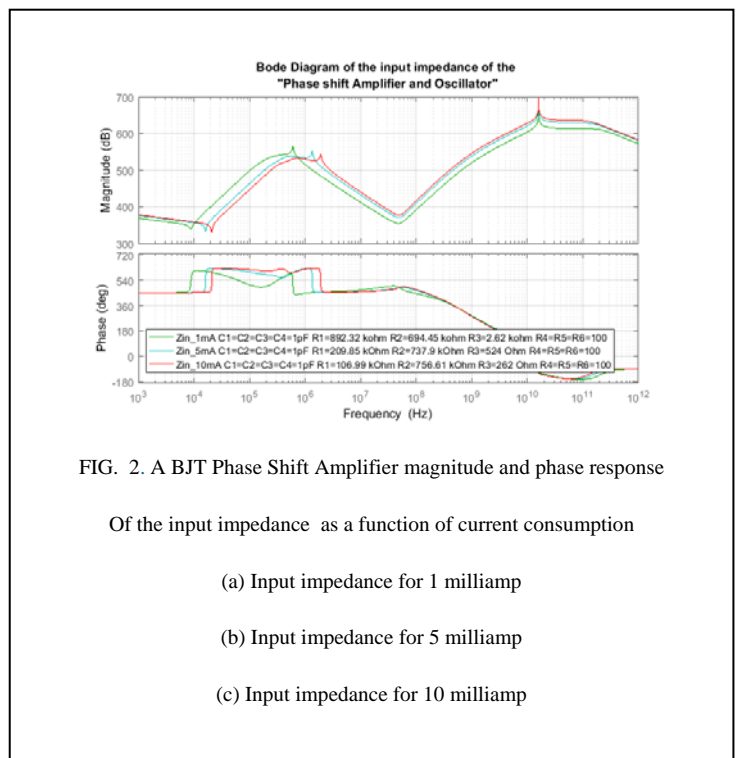


FIG. 2. A BJT Phase Shift Amplifier magnitude and phase response

Of the input impedance as a function of current consumption

- (a) Input impedance for 1 milliamp
- (b) Input impedance for 5 milliamp
- (c) Input impedance for 10 milliamp

The magnitude response can be described as following. The magnitude response is increased at the input frequency higher than 10 kilo Hertz until it is reach the first peak of the magnitude response around 600 kilo Hertz at a current consumption of 1 milli ampere. Then, the magnitude response is decreased after it pass the first peak. The highest peak of the magnitude response is around 16 giga Hertz. Different current consumption has some similar magnitude response such as a maximum magnitude response from the 3 values of current consumption should be 20 milliamp at approximately 700 dB.

is decreased to 320 dB at the frequency approximately 35 mega Hertz.

4 Conclusion

The description of how to make polynomial form of the input impedance of the phase shift amplifier is well described. But the detail derivation report is not shown because it has many pages [6]. According to the theory of the root locus of the complex poles of the denominator after factorization of the input impedance in the frequency domain. If it is impossible for poles to move from complex poles to imaginary pole in the complex pole plot. It means that the phase shift oscillator may not practical without input impedance matching. In microwave oscillator theory, it is possible that any transistor network can be oscillate as a periodic waveform if it is matched with impedance matching network so that its input impedance can be factorized to have the real part less than or approximately zero ohm.

The novelty of this paper is to derive an exact input impedance of the phase shift amplifier in a polynomial form with a high frequency equivalent circuit. Then, the basic programing is used to type all grouping of the coefficients into the text file. The MATLAB is used to plot a magnitude and a phase response graph to represent the biasing condition and the Bode plot results.

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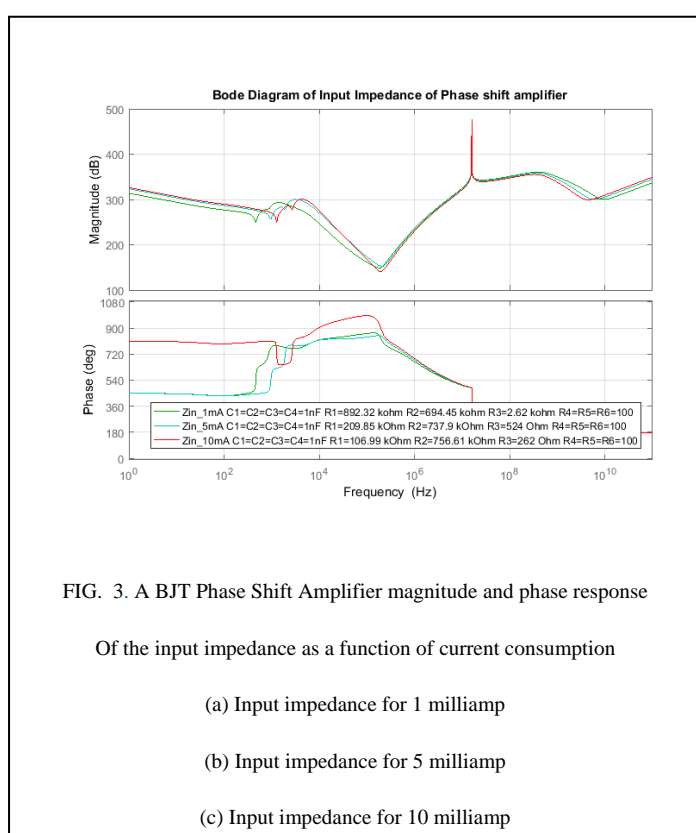


FIG. 3. A BJT Phase Shift Amplifier magnitude and phase response

Of the input impedance as a function of current consumption

- (a) Input impedance for 1 milliamp
- (b) Input impedance for 5 milliamp
- (c) Input impedance for 10 milliamp

All of the capacitors in the circuit diagram of the phase shift amplifier in figure1 were changed from 1 picofarad to 1 nanofarad in the program file which is used in MATLAB. The magnitude response and phase response which is plotted in figure3 is the result of execution of the program file inside MATLAB software. The magnitude response was decrease from approximately 320 dB at 1 Hz until it reached the minimum magnitude response at approximately 200 kilo Hertz. Then, the magnitude response is increased to the maximum point approximately 470 dB at 20 mega Hertz to 30 mega Hertz. After this frequency, its magnitude response