

# Use of genetic algorithm to Tuning a Lyapunov Adaptive Controller applied to a temperature system controlled environment.

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*Abstract:* - This work presents the use of a Lyapunov based adaptive controller optimized by a Genetic Algorithm applied to tuning the parameter that define a fall rating of the error between a reference value (set point) and the output of the system in a simulation environment. The system has been modeled with certain degree of uncertainty with regard of its parameters. The complete system, which include the the Transfer Function of the system being utilized was simulated in Scilab software being the results of the simulations presented.

*Key-Words:* - Genetic algorithm, Lyapunov-based controller, temperature controller, Optimization, adaptive control, control

## 1 Introduction

According to [1] an Adaptive Control System is a system which automatically adjust on-line the parameters of its controller. Among the great quantity of technique in the branch of adaptive control it is worth noting the use of the Lyapunov-based techniques (i.e., the controller and the adaptive update law are designed based on a Lyapunov analysis). Parks [2] was one of the first researchers who used Lyapunov's second method to design a stable adaptive controller for single input single output systems (SISO). [3] uses two Lyapunov based adaptive controllers for a vibrational gyroscope to compensate for uncertainty in the natural frequencies, mode coupling and damping. For the other hand, there is a very significant presence of the works relating to optimization process in literature in order to improving the results of the controllers. In [4] it had been employed Lyapunov Theory and Particle Swarm in order to obtain an optimization process.[5] have compared a Mamdani Fuzzy controller optimized by genetic algorithm with a Neural Network controller in a system consisted of a two axis robot.

In this work is shown the results obtained using an optimized Lyapunov-based controller to drive a system represented by a one order Transfer Function equation. In order to optimized the controller it was used a genetic algorithm as presented by [6]. The method is validated with the presentation of simulation results. This work is organized in the following way: In this first section is presented the

overall view of the subject treated. In section two is presented the Problem Formulation, where is shown the Lyapunov-based controller and is shown the results obtained by such controller when one uses the parameter that updating the controller gains by means of random choice. Section three describes the design and the implementation of the Lyapunov-based controller with the gain updating with a parameter previously determined by the genetic algorithm. Finally, in the fourth section, are drawn the conclusions and perspectives for future work.

## 2 Problem Formulation

In this work, it has been used a Transfer Function equation representing a real plant identified by the ARX identification process. The identification process method of the plant followed the algorithm presented by [7]. The model plant that was used as a benchmark for the system consisted of another first order equation with optimized parameter related by the overall performance of the model plant. The main motivation to determining the parameters of the model was minimal overshoot and minimal settling time. The definition of these parameters can be viewed in [8].

In order to accomplish the trajectory imposed by the output signal of the model plant it has implemented a controller composed by two parts  $\theta_1$  e  $\theta_2$ , the first applied to the reference signal and the second one is applied to the feedback signal coming from the controlled system output.

The output of the model plant is compared with the output of the controlled plant forming the error

signal which must has its fall down toward zero established by a parameter named gamma. The controlled system can be seen in Fig. 1

Fig. 1: Lyapunov-based controlled System and Model Reference System

In this research we simulated a first order plant with the transfer function given by (1) being  $y$  the output  $u$  the input signal and  $a$  and  $b$  the parameters of the plant. The overall trajectory of the plant when it is applied the Lyapunov-based Model Reference Controller must follow with great accuracy the trajectory performed by a model system when both, the system being controlled and the model plant are stimulated by the same input. In this case the input chosen was the unit step. The difference equation that representing the plant is presented by Eq. (2). The reference model and its difference equation is presented by Eq. (3) and (4).

$$\frac{dy}{dt} = -ay + bu \quad (1)$$

$$y_{(k)} = ay_{(k-1)} + bu_{(k-1)} \quad (2)$$

$$\frac{dym}{dt} = -a_m ym + b_m u \quad (3)$$

$$ym_{(k)} = aym_{(k-1)} + bum_{(k-1)} \quad (4)$$

The complete algorithm to implementing the Lyapunov-based controller can be found in [9]. In the case of Lyapunov-based have utilized the Genetic Algorithm. The fall down rate of the error is given by its derivative, as can be seen by the Eq. (5), (6) and (7)

$$\frac{de}{dt} = \frac{dy}{dt} - \frac{dym}{dt} \frac{de}{dt} = -a \cdot y + b(\theta_1 \cdot uc - \theta_2 \cdot y) + am \cdot ym - bm \cdot uc \quad (5)$$

$$\frac{de}{dt} = -am \cdot e - (b \cdot \theta_2 + a - am) \cdot y + (b \cdot \theta_1 - bm) \cdot uc \quad (6)$$

The Lyapunov function chosen is shown by Eq. (8).

$$v(e, \theta_1, \theta_2) = \frac{1}{2} \left[ e^2 + \frac{1}{b\gamma} (b\theta_2 + a - am)^2 \right] + \frac{1}{2} \frac{1}{b\gamma} (b\theta_1 - bm)^2 \quad (8)$$

The derivative of such function, given by Eq. (9) must be negative.

$$\frac{dv}{dt} = e \frac{de}{dt} + \frac{d}{dt} \left[ \frac{1}{2b\gamma} (b^2\theta_2 + 2(a - am)^2) \right] + \frac{d}{dt} \frac{1}{2b\gamma} [b^2\theta_1^2 - 2b\theta_1 b_m + b_m^2] \quad (9)$$

$$\frac{dv}{dt} = e \frac{de}{dt} + \frac{b}{\gamma} \theta_2 \frac{d\theta_2}{dt} + \frac{(a - am)}{\gamma} \frac{d\theta_2}{dt} + \frac{(b\theta_1)}{\gamma} (b\theta_1 - b_m) \frac{d\theta_1}{dt} \quad (10)$$

After some developments one reach the Eq. (11)

$$\frac{dv}{dt} = -a_m e^2 + \frac{1}{\gamma} (b\theta_2 + a - am) \left[ \frac{d\theta_2}{dt} - ye\gamma \right] + \frac{1}{\gamma} (b\theta_1 - b_m) \left[ \frac{d\theta_1}{dt} + \gamma u_c e \right] \quad (11)$$

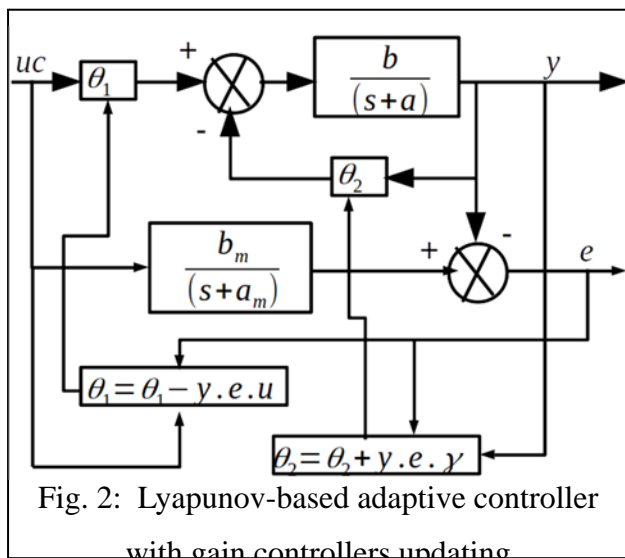
For the Equation (11) knowing that  $am$  is always positive as a result to the constraint that the model must be stable and considering that the term  $e^2$  it is of course always positive so the derivative of the Lyapunov function is negative since the following conditions is applied.

$$\frac{d\theta_2}{dt} = ye\gamma, \frac{d\theta_1}{dt} = -yeu_c \tag{12}$$

Thus, we formulated a script that imposing the rate of variation of the two parameters of the controller as shown by Eq. (12). In order to do this we simulated a control system as depicted by Fig. 2 applying at each sample time -represented by the indice  $k$  - the following update of the terms  $\theta_1$  e  $\theta_2$ .

$$\begin{aligned} \theta_1(k) &= \theta_1(k - 1) - yeu_c \\ \theta_2(k) &= \theta_2(k - 1) + ye\gamma \end{aligned} \tag{13}$$

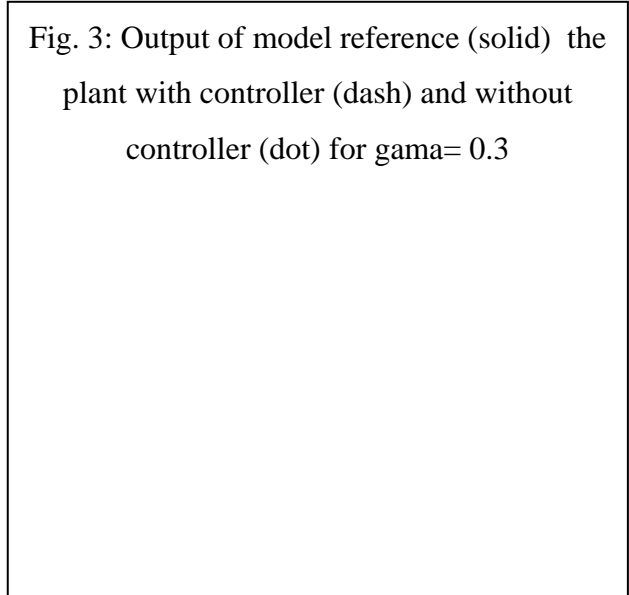
As can be seen, at each sample time is computed an error between the outputs of the model plant and the real plant. This error signal, as well the output of the plant  $y$  and the reference signal  $u$  is used as feedback to the block that perform the calculation of the new values of the controller gain  $\theta_1$  and  $\theta_2$ .



We fixed the value  $\gamma$  as 0.3. The parameters of the plant is  $a=0.9$  and  $b=0.1$ . The parameters of the reference model is  $am=1$  and  $bm=0.2$  It was applied a unit step as input. At each sample time the controllers gains are updating according to Fig. 2.

Fig. 3 shows the output of the controlled plant with the structure shown in Fig. 2. As it can be seen the outcome of the controlled system follow the reference signal but has some discrepancy in its initial trajectory regarding to the model output. The value of the parameter  $\gamma$  was adopted as 0.3 and was chosen in the random way. The curve show that the permanent regimen error is zero, demonstrating the good performance of the controller, but, for the other hand, it is seen that there is different values between the controlled plant and the model plant outputs in the transitory region.

In another simulation we have imposed  $\gamma$  as 0.05. The result is shown by Fig. 4. Clearly, the



performance in this last case is poorest than the previews results, despite the fact that there is the good approximation of the curves in the transitory region of the curves. In the permanent regimen the error is greater than in the precedent case.

In another simulation we have imposed  $\gamma$  as 0.05. The result is shown by Fig. 4.

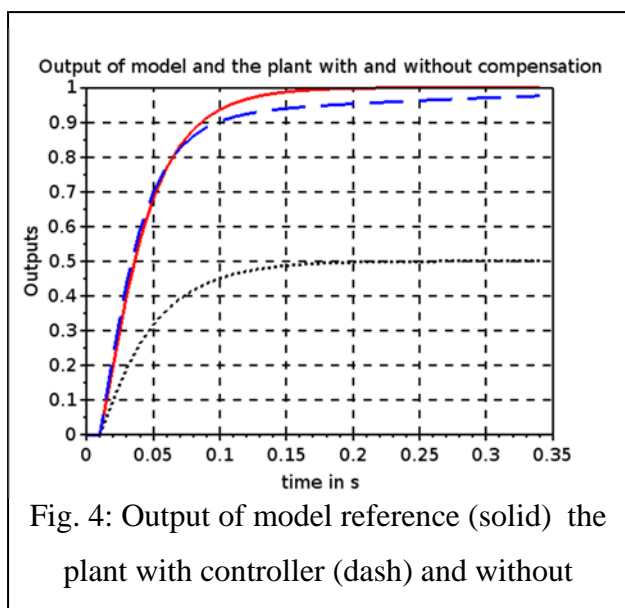


Fig. 4: Output of model reference (solid) the plant with controller (dash) and without

In this case, we can view that the controlled plant output follow the model output with huge delay reaching this ultimate signal in a time much longer than in the previous simulation. Clearly, the performance in this last case is poorest than the previews results, despite the fact that there is a good approximation of the curves in the transitory region of the curves. In the permanent regimen the error is greater than in the precedent case.

## 2.1 Optimization

In order to optimizing the Lyapunov-based controller it was used a genetic algorithm. The main purpose of the genetic algorithm was determining the factor gamma which could be utilized by the controller with the aim to minimizing the energy of the error between the output of an ideal system regarded as a benchmark and the output system of the controlled plant. To do this it has been chosen Integral of squared error (ISE) of the tracking error. The energy involved, expressed by this equation is the cost function utilized by the genetic algorithm.

$$e = \sum e^2 \quad (14)$$

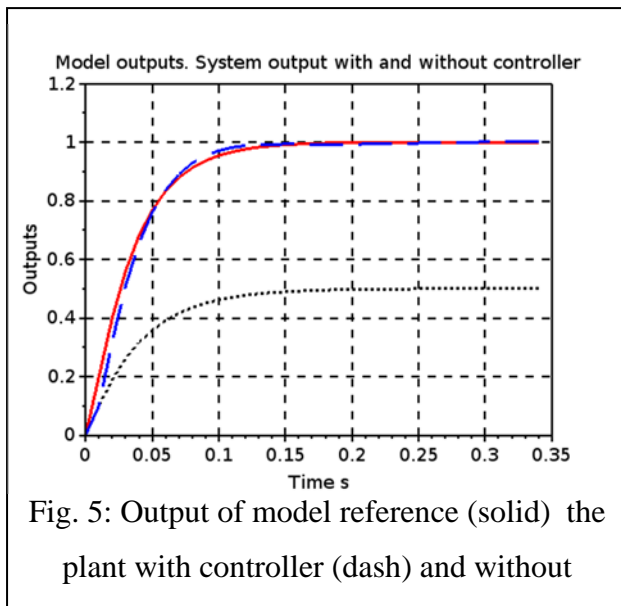
The genetic algorithm works in accordance with the following steps: gamma, the value which is placed in the Eq. (13) is codified by binary number with 12 bits. Initially, it has been created a population comprised by 12 individuals, each of them is actually one value of gamma. For each time the script was processed the algorithm performed the steps related the crossover and mutation between the most fitted individuals, since the least fitted individuals have been got rid of the overall population. It was then executed a simulation of the plant excited by a unity step input with the Lyapunov-based controller with the gains values updating using Eq. (13) to obtaining the control output for the Transfer Function equation representing the plant.

After obtaining the trajectory of the output signal of the plant it was calculated the error signal for each sample time and then it was computed the energy used in the robot movement according Eq. (14). If the energy is the minimal value, the optimization process is finished and the final optimum values gamma were found. If the energy is not the minimal value, it was selected the first four chromosomes in the crescent order of energy consumption, these four chromosomes are the parents of the new population while the last four chromosomes is discarded. With the four chromosomes that is chosen to be the parents it was performed the crossover and mutation to obtain the four last chromosomes of the new population.

## 3 Obtained Results

Fig. 5 show the result obtained with the gain provided by the genetic algorithm optimization process. It can be noted an overall improvement of the the controller using the gain obtained by this process. The genetic algorithm issued the value 0.263 for the parameter gamma.

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#### 4 Conclusion

The results showed the the importance of searching the parameter used to updating the gain controller in the Lyapunov-based adaptive controller by means of genetic algorithm rather than by means of random choice of this parameters. It was used a very simple and well know form of genetic algorithm already intensively known by the literature and it have been obtained good results as can be seen viewed by the figures shown in this paper. In the aftermath of this work we are planning to implement this controller in a real plant which consist of the an enviromental which its temperature must be precisaly controlled.

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