# State Estimation Using UFIR and Kalman Filtering in Applications to Local Crystal and Master Clocks

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*Abstract:* To meet the needs of the IEEE Standard 1139-2008, this paper suggests using the recently designed iterative unbiased finite impulse response (UFIR) filtering algorithm to estimate clock state via measurement of the time interval error (TIE) on an interval of $N$ most recent past points. The algorithm has the Kalman filter (KF) form. But, unlike the KF, requires neither the noise statistics nor the initial values. Because noise in clocks has colored Gaussian components (not white, as required by the KF), the UFIR filter demonstrates higher robustness and becomes optimal when $N \gg 1$. Applications are given for state estimation in an ovenized crystal clock and error prediction in a masted clock. Based upon extensive experimental investigations, we show that the UFIR algorithm outperforms the KF, because the clock covariance matrix cannot be specified correctly in view of the colored noise.

*Key–Words:* Clock errors, unbiased FIR filter, Kalman filter, robustness.

## 1 Introduction

The IEEE Standard 1139-2008 [1] and International Telecommunication Union (ITU-T) recommendation G.810 [2] suggest that a clock has three states, namely the time interval error (TIE), fractional frequency offset, and linear frequency drift rate. The problem with accurate estimation of clock state is associated with noise, which has slow flicker (colored) components requiring special filtering and smoothing algorithms. Note that the Kalman filter (KF), which is a standard technique in clock state estimation, requires noise to be white Gaussian and time-uncorrelated. Furthermore, in the three-state clock model the high-order states are ignored that causes model errors.

The problem is often complicated by not white measurement noise and data uncertainties when the reference source of time is removed, which is the case of the Global Positioning System (GPS)-based timekeeping. Under such conditions, the KF may produce extra errors [3, 4, 5] and more robust solutions are required. There were attempts to use the $H_\infty$ filter with this aim. However, the $H_\infty$ often demonstrates an ability to diverge if the tuning factor is not set properly [6]. Other solutions relying on Gauss-Markov colored noise are still not developed for the clock flicker noise.

Jazwinski stated in [3] that the limited memory filter appears to be the only device for preventing divergence in the presence of unbounded perturbation in the system. The finite impulse response (FIR) filter, which has a limited memory, has an imbedded bounded input/bounded output (BIBO) stability, demonstrates higher robustness than the KF [5], and has lower sensitivity to noise and uncertainties [7, 8, 9]. Moreover, the optimal FIR (OFIR) [8] and unbiased FIR (UFIR) [10] filtering estimates converge on large horizons $N \gg 1$ [8] that is typical for highly oversampled measurements of the TIE.

An important feature of the UFIR filter derived by Shmaliy in [10] is that its estimate does not depend on the noise statistics and initial errors. Thus, the colored noise components do not affect the UFIR estimate as much as the KF estimate. Note that the IEEE Standard 1139-2008 [1] states that "an efficient and unbiased estimator is preferred" for clock applications. Most recently, the UFIR filter was developed to a computationally efficient iterative Kalman-like algorithm [8, 11] and compared to the KF [12] that makes it even more attractive for clock applications. Referring to these advantages and aimed at overcoming issues associated with the KF, the UFIR filter was used by many authors in diverse applications instead of the KF [13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23].

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## 2 Clock Model

The clock TIE $x(t)$ caused by oscillator instabilities can be modeled with the finite Taylor series as [2]

$$x(t) = x_0 + y_0 t + \frac{z_0}{2} t^2 + w_x(t)\,, \tag{1}$$

where $t$ is the continuous time, $x_0 = x(0)$ is the initial TIE (first state), $y_0 = y(0)$ is the initial fractional frequency offset (second state), and $z_0 = z(0)$ is the initial linear frequency drift rate (third state). Noise $w_x(t) = \varphi(t)/2\pi\nu_{\text{nom}}$ is completely defined by the clock oscillator random phase deviation component $\varphi(t)$ and nominal frequency $\nu_{\text{nom}}$ in Hz. The IEEE standard [1] states that $w_x(t)$ is affected by different kinds of phase and frequency fluctuations (white, flicker, and random walks).

In state space, (1) can be represented with the differential equation

$$\frac{d}{dt}\mathbf{x}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{w}(t)\,, \tag{2}$$

in which $\mathbf{x}(t) = [x(t)\, y(t)\, z(t)]^T$ is the clock state vector and $\mathbf{w}(t) = [w_{\text{y}}(t)\, w_{\text{z}}(t)\, w_{\dot{\text{z}}}(t)]^T$ is the zero mean Gaussian noise vector having the covariance $\mathbf{Q}_w(t,\theta) = E\{\mathbf{w}(t)\mathbf{w}^T(\theta)\}$. In this vector, $w_{\text{y}}(t)$ is the frequency noise, $w_{\text{z}}(t)$ is the linear frequency drift noise, and $w_{\dot{\text{z}}}(t)$ is noise in the first time derivative of the linear frequency drift. Note that $\mathbf{w}(t)$ is nonstationary on a long time scale, although it is typically assumed to be stationary on a short time. The clock state transition matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}. \tag{3}$$

In discrete time $t_n$ with a time step $\tau = t_n - t_{n-1}$, model (2) can be represented as

$$\mathbf{x}_n = \mathbf{\Phi}(\tau)\mathbf{x}_{n-1} + \bar{\mathbf{w}}_n\,, \tag{4}$$

with the process matrix $\mathbf{\Phi} \triangleq \mathbf{\Phi}(\tau)$ is defined by the matrix exponential $\mathbf{\Phi} = e^{\mathbf{A}\tau}$ to be [24]

$$\mathbf{\Phi} = \begin{bmatrix} 1 & \tau & \frac{\tau^2}{2} \\ 0 & 1 & \tau \\ 0 & 0 & 1 \end{bmatrix}. \tag{5}$$

The zero mean Gaussian noise vector $\bar{\mathbf{w}}_n$ and its covariance $\mathbf{Q}_{\bar{w}}(i,j) = E\{\bar{\mathbf{w}}_i \bar{\mathbf{w}}_j^T\}$ are given by, respectively,

$$\bar{\mathbf{w}}_n = \int_{t_{n-1}}^{t_n} \mathbf{\Phi}(\theta)\mathbf{w}(\theta)\, d\theta\,, \tag{6}$$

$$\mathbf{Q}_{\bar{w}}(i,j) = \int_{t_{i-1}}^{t_i}\int_{t_{j-1}}^{t_j} \mathbf{\Phi}(\tau)\mathbf{Q}_w(\tau,\theta)\mathbf{\Phi}^T(\theta)\, d\theta\, d\tau \tag{7}$$

and we notice that (7) serves to any kind of phase and frequency noise sources. For white Gaussian approximation required by KF, this matrix was shown in [24, 25, 26].

For the measured clock first state $x(t)$, the observation equation can be written as

$$s_n = \mathbf{C}\mathbf{x}_n + v_n\,, \tag{8}$$

where $\mathbf{C} = [\,1\ 0\ 0\,]$, and $v_n$ is the zero mean measurement noise, which may not obligatorily be white and Gaussian, as in GPS timing receivers.

## 3 UFIR Estimator of Clock State

An idea behind the UFIR estimator originally derived in state space in [10] is the following. If we cannot specify correctly the noise covariance and measurement is highly oversampled, $N \gg 1$, then let us use a convolution-based FIR filter. Such filters suppress random noise by averaging and noise becomes insignificant when $N \gg 1$. Suppressed noise, the FIR estimator must be tuned to satisfy the unbiasedness condition and the remaining question is the error. Extensive investigations have shown [12] that this strategy often leads to smaller errors than in the KF even when $N$ is not large.

The iterative UFIR filtering algorithm [19] requires two matrices:

$$\mathbf{\Lambda}_i = \left[\, (\mathbf{C}\mathbf{\Phi}^i)^T\ (\mathbf{C}\mathbf{\Phi}^{i-1})^T\ \ldots\ \mathbf{C}^T\, \right]^T, \tag{9}$$

$$\mathbf{S}_{n,m} = [\, s_n\ s_{n-1}\ \ldots\ s_m\,]^T, \tag{10}$$

where $m = n - N + 1$. Given the number of the clock states $K$, an averaging interval of $N$ points, $v = m + K - 1$, and an iterative variable $l$ ranging from $m + K$ to $n$, and auxiliary matrices

$$\mathbf{L} = (\mathbf{\Lambda}_{K-1}^T \mathbf{\Lambda}_{K-1})^{-1}\,, \tag{11}$$

$$\mathbf{G}_v = \mathbf{\Phi}^{K-1}\mathbf{L}(\mathbf{\Phi}^{K-1})^T\,, \tag{12}$$

where $\mathbf{G}_n$ is the generalized noise power gain (GNPG), then the algorithms produces an estimate in two phases:

1. Compute the clock state at $v$ given data from $m$ to $v$:
$$\tilde{\mathbf{x}}_{v|v} = \mathbf{\Phi}^{K-1}\mathbf{L}\mathbf{\Lambda}_{K-1}^{T}\mathbf{S}_{v,m}\,. \tag{13}$$
2. Update:
$$\mathbf{G}_l = \left[\mathbf{C}^T\mathbf{C} + (\mathbf{\Phi}\mathbf{G}_{l-1}\mathbf{\Phi}^T)^{-1}\right]^{-1}, \tag{14}$$
$$\tilde{\mathbf{x}}_{l|l} = \mathbf{\Phi}\tilde{\mathbf{x}}_{l-1|l-1} + \mathbf{F}_l\mathbf{C}^T \times (s_l - \mathbf{C}\mathbf{\Phi}\tilde{\mathbf{x}}_{l-1|l-1})\,. \tag{15}$$

Note that $\mathbf{G}_l\mathbf{C}^T$ in (15) is the bias correction gain of the UFIR filter, which does not depend on the noise statistics and is therefore does not equal to the Kalman gain. The algorithm updates values iteratively until an iterative variable reaches $l = n$ in each iterative cycle.

# 4 Applications to Clock Problems

Below, we will employ algorithm (11)–(15) to estimate the clock state in several practical situations.

## 4.1 Filtering of OCXO-based Clock State via GPS-Based TIE Measurement

The local clock current state can be estimated if to employ the GPS one pulse per second (1PPS) timing signals. The TIE measurements of an oven controlled crystal oscillator (OCXO)-based local clock imbedded in the Frequency Counter SR620 (Stanford Research System, Inc., Sunnyvale, CA) have been provided with another SR620. As a reference signal, we use the 1PPS output of the GPS SynPaQ III Timing Sensor (Synergy Systems, LLC, San Diego, CA). To obtain an actual TIE, simultaneous measurements were organized for the Cesium Frequency Standard CsIII (Symmetricom, Inc., San Jose, CA).

### 4.1.1 UFIR Filter

To estimate the OCXO state at $n$ for $K = 3$, we modify the UFIR algorithm as
$$\mathbf{L} = (\mathbf{\Lambda}_2^T\mathbf{\Lambda}_2)^{-1}\,, \tag{16}$$
$$\mathbf{G}_v = \mathbf{\Phi}^2\mathbf{L}(\mathbf{\Phi}^2)^T\,, \tag{17}$$
$$\tilde{\mathbf{x}}_{v|v} = \mathbf{\Phi}^2\mathbf{L}\mathbf{\Lambda}_2^T\mathbf{S}_{m+2,m}\,, \tag{18}$$
$$\mathbf{G}_l = \left[\mathbf{C}^T\mathbf{C} + (\mathbf{\Phi}\mathbf{G}_{l-1}\mathbf{\Phi}^T)^{-1}\right]^{-1}, \tag{19}$$
$$\tilde{\mathbf{x}}_{l|l} = \mathbf{\Phi}\tilde{\mathbf{x}}_{l-1|l-1} + \mathbf{G}_l\mathbf{C}^T(s_l - \mathbf{C}\mathbf{\Phi}\tilde{\mathbf{x}}_{l-1|l-1}) \tag{20}$$

with $\mathbf{\Lambda}_2$ specified by (9) as
$$\mathbf{\Lambda}_2 = \begin{bmatrix} 1 & 2\tau & 2\tau^2 \\ 1 & \tau & \frac{\tau^2}{2} \\ 1 & 0 & 0 \end{bmatrix} \tag{21}$$

and find optimal $N_{\mathrm{opt}} = 3500$ following [27].

### 4.1.2 Kalman Filter

To apply the KF, one needs to know the noise covariances and initial errors [28]. Noise in the OCXO embedded into SR620 is specified with three values of the Allan deviation: $\sigma_{\mathrm{y}}(1s) = 2.3 \times 10^{-11}$, $\sigma_{\mathrm{y}}(10s) = 1.0 \times 10^{-11}$, and $\sigma_{\mathrm{y}}(100s) = 4.2 \times 10^{-11}$. These values can be converted to the diffusion parameters [26] via
$$\sigma_{\mathrm{y}}^2(\tau) = \frac{q_1}{\tau} + \frac{q_2\tau}{3} + \frac{q_3\tau^3}{20} \tag{22}$$

and then $\mathbf{Q}_{\bar{w}}(\tau)$ specified in white Gaussian approximation using (7) as [25]
$$\frac{\mathbf{Q}_{\bar{w}}(\tau)}{\tau} = \begin{bmatrix} q_1 + \frac{q_2\tau^2}{3} + \frac{q_3\tau^4}{20} & \frac{q_2\tau}{2} + \frac{q_3\tau^3}{8} & \frac{q_3\tau^2}{6} \\ \frac{q_2\tau}{2} + \frac{q_3\tau^3}{8} & q_2 + \frac{q_3\tau^2}{3} & \frac{q_3\tau}{2} \\ \frac{q_3\tau^2}{6} & \frac{q_3\tau}{2} & q_3 \end{bmatrix}. \tag{23}$$

Because $\sigma_{\mathrm{y}}^2(\tau)$ is upper-bounded in the OCXO specification, we conventionally reduce this bound by the factor of 2, thinking that the KF will perform better. Finally, the variance of the sawtooth noise induced by GPS SynPaQ III Timing Sensor was found as $Q_v = 50^2/3$ ns$^2$ and the unknown initial states taken as $x_{10} = y_0$, $x_{20} = 0$, and $x_{30} = 0$.

Estimates of $x_n$ and $y_n$ are sketched in Fig. 1a and Fig. 1b, respectively, in line with the reference (actual) data (dashed) provided with CsIII.

Based upon these results, we acknowledge that efforts to describe $\mathbf{Q}_{\bar{w}}(\tau)$ via $\sigma_{\mathrm{y}}(\tau)$ were unsuccessful, as the KF produces worst estimates, especially for the second state (Fig. 1b). Errors can be reduced by decreasing values of $\sigma_{\mathrm{y}}^2(\tau)$. However, this drops the Allan deviation below our imagination about the stability of the OCXO exploited and we deduce that the KF does not suit well the clock model, at least its applications need further investigations. In contrast, with $N_{\mathrm{opt}} = 3500$, the UFIR filter produces much smaller errors and demonstrates higher robustness against GPS-time uncertainties, especially for the second state (Fig. 1b). The UFIR filter outperforms the KF even for $N = 2500$ and $N = 1500$.

## 4.2 Error Estimation and Prediction of the NIST MC

The National Institute of Standards and Technology (NIST) has published on the WEB site the UTC–UTC(NIST MC) time differences (285 points) measured each 10 days in 2002-2009, as issued monthly by BIPM. For this measurement, we form the time

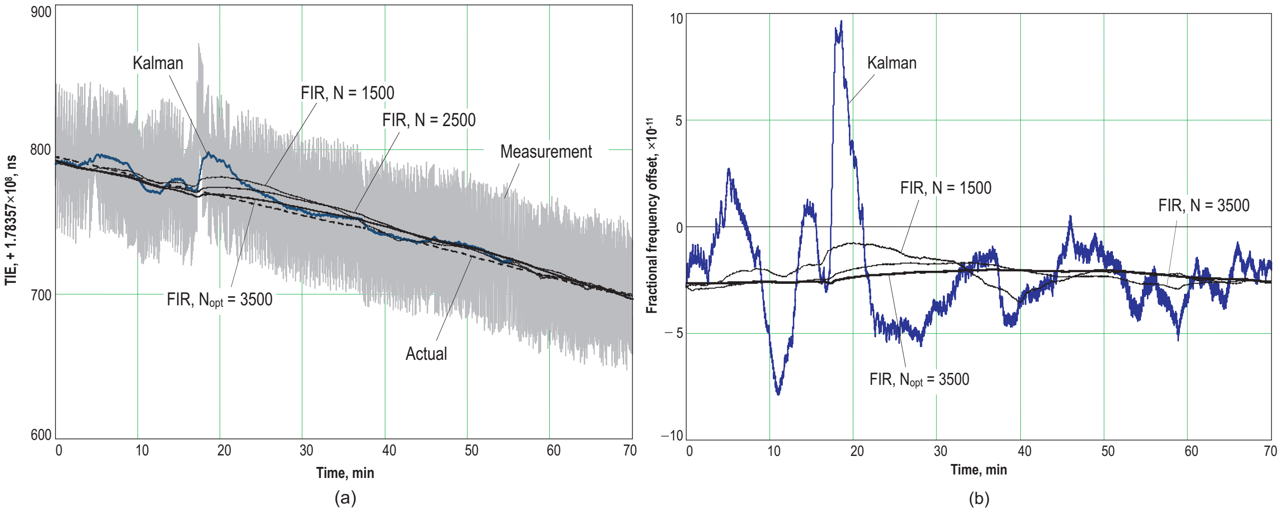


Figure 1: GPS-based TIE measurement and state estimation of the OCXO-based clock in the presence of the GPS time temporary uncertainties: (a) TIE and (d) fractional frequency offset.

scale starting with $n = 0$ (52279 MJD) and finishing at $n = 284$ (55129 MJD). The measurement is shown in Fig. 2a.

The Master Clocks (MCs) are commonly modeled to have two states, $K = 2$. Because the NIST MC is error-corrected, we suppose that the process is stationary and employ all data available. Accordingly, we let $N = n + 1$ and use the full-horizon $p$-step predictive UFIR algorithm [8]

$$\mathbf{G}_1 = \mathbf{\Phi}(\mathbf{\Lambda}_1^T\mathbf{\Lambda}_1)^{-1}\mathbf{\Phi}^T\,, \tag{24}$$

$$\tilde{\mathbf{x}}_{1+p|1} = \mathbf{\Phi}^{1+p}(\mathbf{\Lambda}_1^T\mathbf{\Lambda}_1)^{-1}\mathbf{\Lambda}_1^T\mathbf{S}_{1,0}\,, \tag{25}$$

$$\mathbf{G}_n = \left[\mathbf{C}^T\mathbf{C} + (\mathbf{\Phi}\mathbf{G}_{n-1}\mathbf{\Phi}^T)^{-1}\right]^{-1}, \tag{26}$$

$$\begin{aligned}\tilde{\mathbf{x}}_{n+p|n} &= \mathbf{\Phi}\tilde{\mathbf{x}}_{n-1+p|n-1} + \mathbf{\Phi}^p\mathbf{G}_n\mathbf{C}^T \\ &\quad\times(s_n - \mathbf{C}\mathbf{\Phi}^{1-p}\tilde{\mathbf{x}}_{n-1+p|n-1})\,,\end{aligned} \tag{27}$$

with $n \geqslant 2$ and

$$\mathbf{\Lambda}_1 = \begin{bmatrix} 1 & \tau \\ 1 & 0 \end{bmatrix}. \tag{28}$$

A remarkable property of this algorithm is that it needs only $p$ for interpolation or extrapolation. Because the NIST MC exhibits excellent etalon properties, we set zero initial conditions to the KF, $x_0 = 0$ and $y_0 = 0$. For the resolution of 0.1 ns in the published data, the uniformly distributed digitization noise has the variance $\sigma_v^2 = 0.05^2/3$ ns$^2$ $= 8.33 \times 10^{-4}$ ns$^2$.

In Fig. 1, we sketch estimates of $x_n$ and $y_n$ obtained by (24)–(28) and with the standard KF. Current estimates of the first state are shown in Fig. 1a along with the prediction of time errors (arrows) provided from the points indicated by increasing $p$. Since the UFIR estimator is unbiased, the first arrow coincides in the direction with several initial measurement points. Increased $n$, the prediction vectors show possible behaviors extrapolated at $n+p$, $p > 0$, over measurement from 0 to $n$. Although, measurement in Fig. 1a looks like a stationary process, the estimator reveals a small positive angle resulting in the frequency offset shown in Fig. 1b.

Observing Fig. 2, one may conclude that the KF is less suitable for MCs than the UFIR filter. In fact, due to the transients, the KF does not provide a real picture with a small number of the measurement points. In the intermediate region (about the point of $1 \times 10^3$ days), both filters produce consistent estimates. On a longer baseline, the full-horizon UFIR filtering algorithm keeps filtering noise out, while the KF inherently exhibits qualitatively the same error picture.

## 5 Conclusions

In this correspondence, we have investigated error in local and Master clocks using the UFIR filtering algorithm with and without the prediction option and in applications to state filtering in the OCXO-based clock and error prediction (extrapolation) in the NIST MC. What has been revealed is that the UFIR algorithm, which completely ignores the noise statistics and initial values, produces smaller errors and has better robustness than the standard KF due to the following reasons. The Allan deviation is commonly specified for oscillators at 1 s, 10 s, and 100 s that is insufficient for the specification of the noise covariance matrix even in white Gaussian approximation. There-

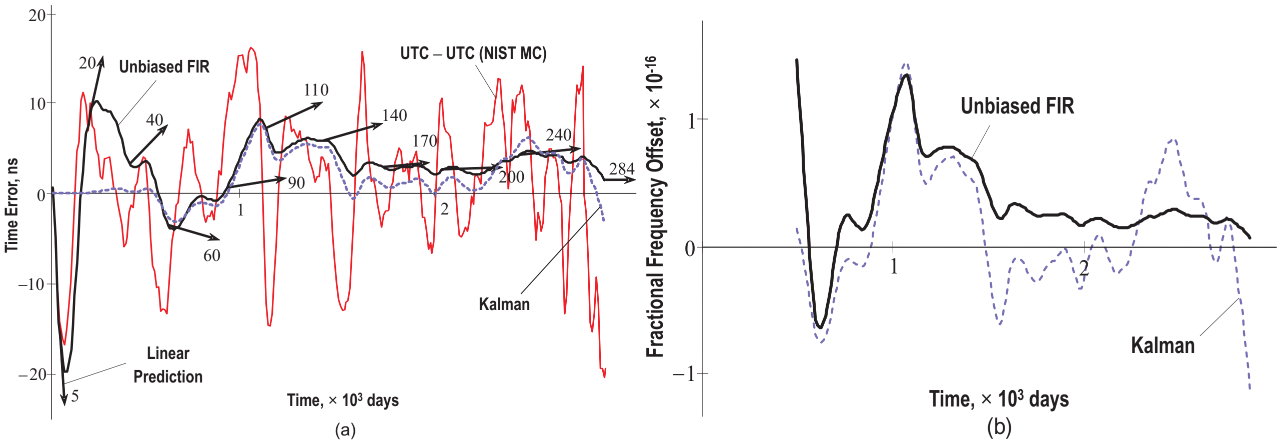


Figure 2: Estimates of the NIST MC current state via the UTC–UTC(NIST MC) time differences (285 points) measured in 2002–2009 each 10 days: (a) TIE and (b) fractional frequency offset. Digits indicate the time points from which the arrows come out.

fore, the KF may produce poor suboptimal estimates and its applications to clocks need thus further investigations. On the other hand, the UFIR ignores the noise covariance and is thus more suitable for clocks.

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