Two Stage Op-Amp Design Verification and Optimization by Symbolic Computation

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Abstract: In the classical Op-Amp design some very simple expressions are used, based on a simplified circuit model. This could lead to a prototype whose response is not the desired one and the Op-Amp must be redesigned. Another possible case is when the designed Op-Amp's response is the desired one but only for a certain load, a different load could produce an undesirable behavior or an unstable response. In this paper a two stage Miller compensated Op-Amp, designed in 180nm CMOS technology is verified from the point of view of phase margin and is optimized from the settling time point of view. To this aim the phase margin symbolic expression of the Op-Amp in the open loop is computed starting from poles and zeros symbolic expressions. Using this expression, an analysis to evaluate the influence of the Miller and the load capacitance on phase margin is performed. This way, the designer can rapidly verify if the response of the Op-Amp is stable for various Miller and load capacitances. After that the symbolic expression of the time constant is estimated starting from the poles and zeros symbolic expressions of the Op-Amp in the closed loop, function of the Miller and load capacitances. The settling time is evaluated for various Miller and load capacitances values to find the optimum, smallest time response. The numerical results for phase margin and settling time obtained with this algorithm are compared with those computed with SPECTRE RF.

Key-Words: Op-Amp, pole/zero, phase margin, settling time, symbolic expressions, design verification, design optimization.

1 Introduction

In analog linear circuits designing, such as filters and amplifiers, the symbolic poles and zeros expressions of are very useful. These expressions can be used in circuit parameter identification or in solving stability problems.
As an example of this problem, the compensation capacitance calculation to assure the closed loop stability of a two stage operational amplifier (Op-Amp) for different capacitive loads can be considered. In the basic Miller compensation technique [1], a capacitance is connected between the two amplifier stages. The classical design procedure is based on a simplified circuit model having two left half plane poles and one right half plane zero frequency response for the amplifier in open loop configuration [2, 3, 4]. In figure 1 is presented a schematic of a typical two stage operational amplifier with Miller compensation, designed in 180nm CMOS technology. The compensation network is composed of the CM capacitance connected across the two stages of the amplifier. Some papers presents that actually the open loop Op-Amp transfer function is described by three poles and one zero [5, 6, 7]. Numerical pole/zero computation with software like SPECTRE RF and HSPICE [8] show that the number of poles and zeros is larger.
In this paper we compute the numerical values and the simplified symbolic expressions of all four poles and three zeros of an open loop and closed loop operational amplifier. Section 2 described in brief the algorithm used to compute the poles/zeros numerical and symbolical expressions based on LR iterations together with some simplifications This algorithm can be found in [9, 10, 11]. Also in this section a two stage Op-Amp with Miller compensation, used with a capacitive load is used to obtain the poles and zeros numerical values and the simplified symbolic expressions with our algorithm. The poles/zeros numerical values are compared with those given by SPECTRE RF. In Section 3 the symbolic transfer function for the Op-Amp in open loop is obtained. The symbolic phase margin (PM) expression is then obtained and is used to investigate the validity range for the compensation and for the load capacitance set of the Op-Amp

mentioned above. This Op-Amp was design for a PM around 90°. In Section 4 the symbolic transfer function for the Op-Amp in closed loop is computed. Starting from this expression the time constant is estimated and the settling time of the circuit is improved. Finally, Section 5 presents some conclusions.

2 Example

Consider a linear circuit for which the state equations in the normal form are:

$$\dot{x} = Ax + Bu + E\,\dot{u} \tag{1}$$

and the output equations are:

$$y = Cx + du \tag{2}$$

where *x* is the state variable vector, *A* is the state matrix, *u* is the input vector having only one component for this example, *y* is the output vector having only one component, *d* is a scalar, and the *B*, *E* and *C* are matrices having appropriate dimensions. The entries of all these matrices are ratios of polynomials in terms of the circuit parameters.

The state matrix *A* can be computed using the algorithm for the computation of the symbolic hybrid matrix of a resistive multiport [8] and improved in [10] and [11].

The poles are the eigenvalues of the *A* matrix, and the zeros are the eigenvalues of the "state-like" matrix *A'* and can be computed using the equation 3 or with the algorithm in [12] or [13].

$$A' = \left(A - \frac{BC}{d}\right)\left(1 + \frac{EC}{d}\right)^{-1} \tag{3}$$

Figure 1 presents a two stage Op-Amp with Miller compensation designed in the 180nm technology, and figure 2 presents the test-bench circuit. Using the DC operating point analysis from SPECTRE RF, the transistors small signal parameters have been extracted (Fig. 3).

To obtain the same frequency and time response with a circuit having the minimum number of circuit elements, two CMOS equivalent circuit models have been tested (Fig. 3 and Fig. 4). The frequency response obtained with this two models (the AC analysis from SPECTRE RF) presented in figure 5, are almost the same.

The Op-Amp in figure 1 has been simulated with SPECTRE RF and the algorithm presented in [11]. In this picture in red is the response of the Op-Amp using the first CMOS model (the magnitude in dB and the phase in degree of the output voltage), and in green is the response using the second model.

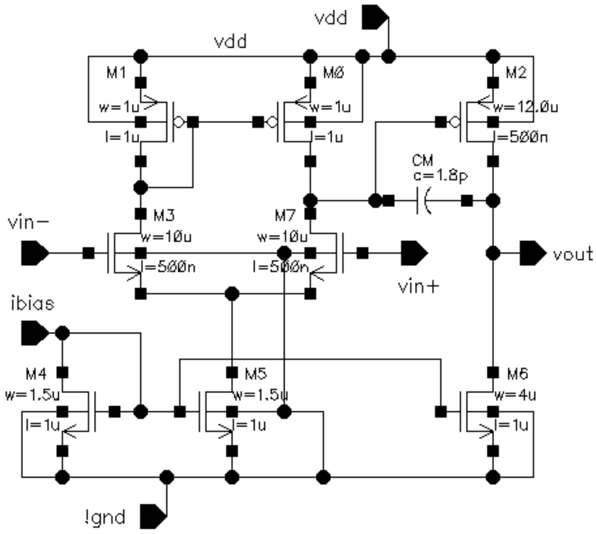

Fig. 1 A two stage Op-Amp Miller compensated

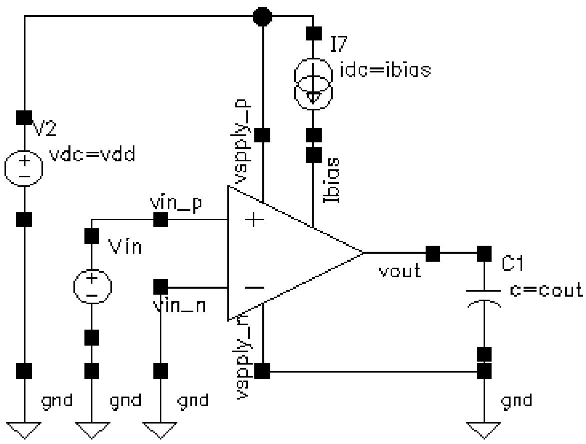

Fig. 2 The test-bench circuit

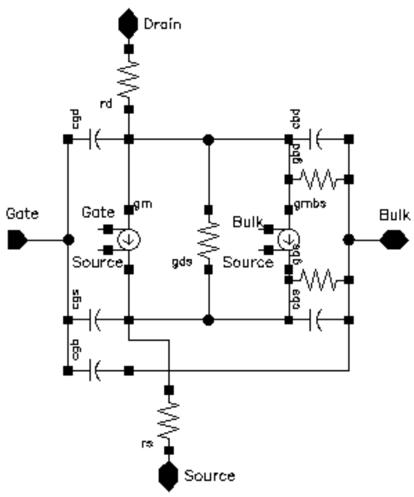

Fig. 3 The first CMOS equivalent circuit used

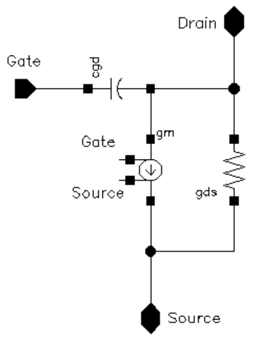

Fig. 4 The second CMOS equivalent circuit used

It can be seen that for a reasonable wide frequency range, the two models have the same frequency response. For this reason, for further analysis the second, simple, CMOS model will be used.

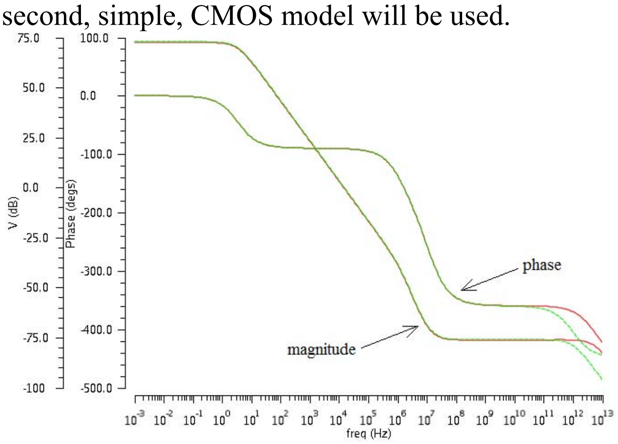

Fig. 5 The Op-Amp frequency response computed with SPECTRE RF using models in Fig. 3 and 4

Table 1 and Table 2 presents the numerical results for the poles and zeros obtained with the proposed algorithm and with SPECTRE RF.

Table 1. Poles values [Hz]

	Our algorithm	SPECTRE RF
p1	-3.68	-3.43
p2	$-1.31\ 10^{6}$	$-1.39\ 10^{6}$
p3	$-2.32\ 10^{7}$	$-2.32\ 10^{7}$
p4	$-1.02\ 10^{12}$	-

TABLE 2. Zeros values [HZ]

	Our algorithm	SPECTRE RF
z1	$7.86\ 10^{6}$	$8.65\ 10^{6}$
z2	$1.50\ 10^{7}$	$1.50\ 10^{7}$
z3	$-2.38\ 10^{7}$	-

Both frequency characteristic, computed with SPECTRE RF and with our algorithm (figure 5 and figure 6) exhibit a pole around the 10^{12} Hz. The PZ analysis from SPECTRE cannot compute it even though the analysis accuracy is increased (Table 1).

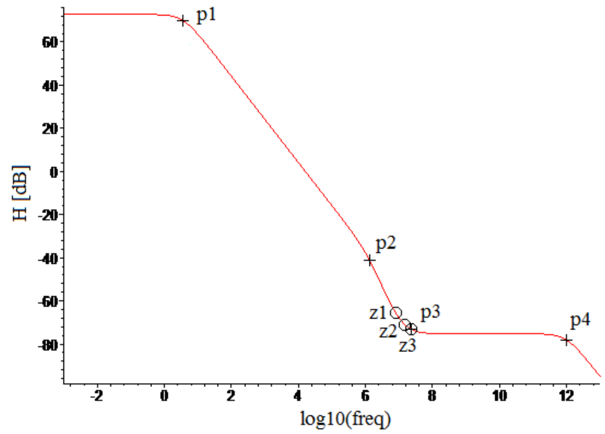

Fig. 6 The Op-Amp frequency response obtained with our algorithm

The poles approximate symbolic expressions obtained with the algorithm presented in [11] are given in Table 3 together with the relative error to the numerical poles:

Table 3. Approximate symbolic pole expressions

pole	The simplified expression	Error [%]
p1	$-\frac{6.8710^{-15}C_M+1.2610^{-26}}{1.8510^{-12}C_M g_{m2}+C_M^2 g_{m2}}$	3.06
p2	$-\frac{g_{m2}-5.2410^{-6}}{C_{out}}$	0.36
p3	$-\frac{1.00}{\left(r_{ds1}\vert\vert\frac{1}{g_{m1}}\right)c_{gd0}}$	0.86
p4	$-\frac{1.02}{r_{Vin}c_{gd7}}$	1.79

Table 4 contain the zeros approximate symbolic expressions obtained with the same algorithm, together with the relative error to the numerical zeros.

Only the symbols with the greatest differential sensitivities are kept in these expressions, the others being replaced with their numerical values. Having these expressions is interesting to observe what part of the circuit gives each of the pole and zero.

All poles/zeros simplified expressions are compared with the eigenvalues numerically computed for a $\pm 30\%$ range of the nominal parameter value. There is a very good agreement between these simplified expresions and the numerical values, the maximum error in the nominal point being less than 4%.

Table 4. Approximate symbolic zero expressions

zero	Simplified expression	Error [%]
z1	$\dfrac{6.44\,10^{-16} g_{m7}^{2} + 4.11\,10^{-8} c_{gd7} g_{m7}}{4.18\,10^{-8} c_{gd7}^{2} + 7.63\,10^{-16} c_{gd7} g_{m7}}$	0.95
z2	$\dfrac{g_{m2}}{C_M}$	0.32
z3	$-\dfrac{1.00}{\left(r_{ds1} \vert\vert \dfrac{1}{g_{m1}}\right) c_{gd0}}$	3.68

3 The phase margin verification

The transfer function of the Op-Amp in open loop is computed with the following formulae using the simplified pole/zero expressions:

$$H_{op}(s) = K \frac{(s+z_1)(s+z_2)(s+z_3)}{(s+p_1)(s+p_2)(s+p_3)(s+p_4)} \quad (4)$$

Keeping as symbols only the Miller and the load capacitances it result:

$$H_{op}(s) = \frac{3.3\,10^{19} C_{out}\left(C_M + 1.9\,10^{-12}\right)\left(sC_M - 1.7\,10^{-4}\right)}{\left(1.6\,10^{-13} s + 1.0\right)\left(6.9\,10^{-9} s + 1\right)\left(sC_{out} + 1.6\,10^{-4}\right)} \cdot$$
$$\cdot \frac{\left(8.2\,10^{-37} s - 4.0\,10^{-29}\right)\left(6.9\,10^{-9} s - 1.0\right)}{\left(1.7\,10^{-4} sC_M^2 + 3.1\,10^{-16} sC_M + 6.9\,10^{-15} C_M + 1.3\,10^{-26}\right)} \quad (5)$$

The symbolic phase margin expression is obtained using the well known formulae [14]:

$$PM = 180^{\circ} + arctg\left(\frac{\mathrm{Im}\left(H\left(j\omega_{gc}\right)\right)}{\mathrm{Re}\left(H\left(j\omega_{gc}\right)\right)}\right) \quad (6)$$

where:

$$\mathrm{Im}\left(H\left(j\omega_{gc}\right)\right) = -3.0\,10^{-25} C_{out} + 2.3\,10^{-12} C_M C_{out} - -1.2\,10^{-18} C_M - 6.7\,10^{-7} C_M^2 - 1.7\,10^{-36} + +1.8 C_M^2 C_{out} + 2.3\,10^{11} C_M^3 C_{out} + 1.3 C_M^3$$

and:

$$\mathrm{Re}\left(H\left(j\omega_{gc}\right)\right) = 5.1\,10^{-34} - 3.9\,10^{-21} C_M - -7.4\,10^{-10} C_M C_{out} - 3.9\,10^{2} C_M^2 C_{out} - 2.9\,10^{-9} C_M^2 - -3.9\,10^{2} C_M^3 - 1.0\,10^{-27} C_{out} - 7.6\,10^{8} C_M^3 C_{out}$$

The ω_{gc} is the pulsation for the unit magnitude, computed solving:

$$\left|H_{op}\left(j\omega_{gc}\right)\right|^2 = 1 \quad (7)$$

or:

$$\left|N_{op}\left(j\omega_{gc}\right)\right|^2 = \left|D_{op}\left(j\omega_{gc}\right)\right|^2 \quad (8)$$

where N_{op} is the nominator and D_{op} is the denominator of the transfer function.
Using the nominal parameters (C_{out} = 20pF, C_M = 1.8pF), the values in Table 5 are obtained for PM. Having the symbolic expression is very useful to verify the influence of various circuit parameters on the PM. For example in figure 7 the phase margin variation with C_M and C_{out} can be observed. The stability range for PM is between 45° and 135° and the Op-Amp was design to have a phase margin around 90°. In green is the phase margin computed with the symbolic expression, in blue is the phase margin computed by SPECTRE RF and the red dot is the value of phase margin in the nominal design point.

Table 5. Numerical phase margin

	Our algorithm	SPECTRE RF
PM	89.04°	89.24°

In Table 6, using the PM symbolic expression, some values are computed showing the validity range for C_M, C_{out} pair, and the relative errors ε_{rel} to numerical values obtained with the PZ analysis from SPECTRE RF are given in Table 7.

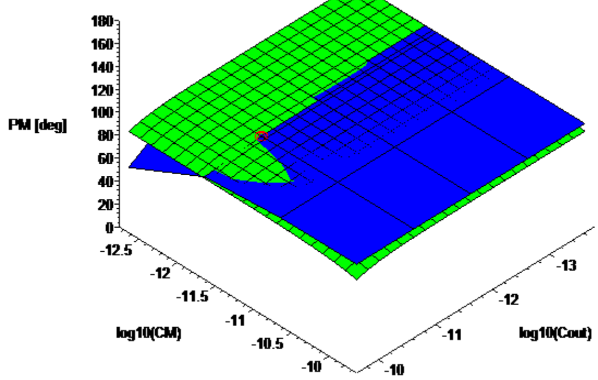

Fig. 7 PM as a function of C_M and the C_{out} capacitances

Table 6. PM for various C_M, C_{out} computed with the symbolic expression and with SPECTRE RF.

C_{out} \ C_M	20 fF	0.2 pF	2.0 pF	20 pF	0.2 nF
0.2 pF	89.92	89.91	89.85	89.24	83.17
	89.05	89.00	88.50	83.47	51.76
2.0 pF	89.75	89.75	89.69	89.08	83.00
	89.87	89.86	89.81	89.31	84.38
20 pF	89.15	89.15	89.08	88.47	82.40
	89.95	89.95	89.94	89.89	89.39
0.2 nF	83.26	83.26	83.19	82.58	76.51
	89.96	89.96	89.95	89.95	89.90

Table 7. The relative error ε_{rel} [%] between the symbolic expressions and the numerical results

C_{out} \ C_M	20 fF	0.2 pF	2.0 pF	20 pF	0.2 nF
0.2 pF	0.97	1.02	1.50	6.47	37.76
2.0 pF	0.13	0.13	0.14	0.26	1.66
20 pF	0.90	0.90	0.96	1.60	8.48
0.2 nF	8.05	8.05	8.12	8.92	17.50

Analysing the figure 7 and table 6 it can be observed that in order to maintain the phase margin around 90°, the Miller capacitance can be varied in the sense of increasing it and the load capacitance can be varied in the sense of decreasing it (Fig. 8).

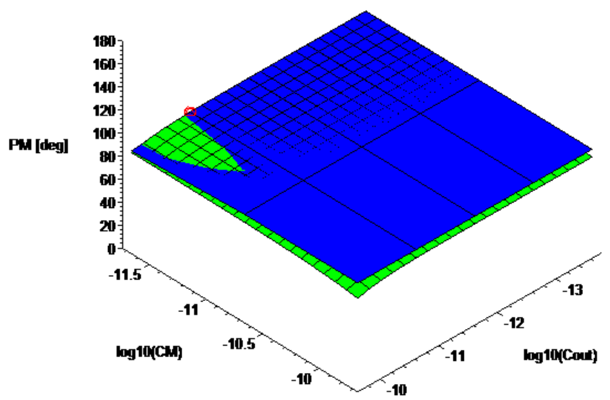

Fig. 8 The validity range of the PM as a function of C_M and the C_{out} capacitances

4 The settling time optimization

Starting from the transfer function computed for the Op-Amp in open loop, the closed loop transfer function is:

$$H_{cl}(s) = \frac{1}{1 + H_{op}(s)} \quad (9)$$

In order to compute the settling time, the amplifier's response to the unit step input voltage is computed as:

$$\frac{H_{cl}(s)}{s} \quad (10)$$

Keeping as symbol only the s variable and expanding in partial fraction the above function, the following expression is obtained:

$$\frac{1}{s} - \frac{1.01}{s + 9.91 \cdot 10^4} + \frac{1.75 \cdot 10^{-2}}{s + 8.09 \cdot 10^6} - \\ - \frac{8.16 \cdot 10^{-4}}{s + 1.44 \cdot 10^8} + \frac{1.73 \cdot 10^{-4}}{s + 6.47 \cdot 10^{12}} \quad (11)$$

The time domain response to the unit step is computed as the inverse Laplace transform:

$$1 - 1.01 e^{-9.91 \cdot 10^4 t} + 1.75 \cdot 10^{-2} e^{-8.09 \cdot 10^6 t} - \\ - 8.16 \cdot 10^{-4} e^{-1.44 \cdot 10^8 t} + 1.73 \cdot 10^{-4} e^{-6.47 \cdot 10^{12} t} \quad (12)$$

The dominant term is $1 - 1.01 e^{-9.91 \cdot 10^4 t}$ which gives the time constant:

$$\tau = \frac{1}{9.91 \cdot 10^4} = 10.01 \mu s \quad (13)$$

The Settling Time (ST) is the time to reach and stay within a specified percentage of the final value and can be computed function of the time constant as:

$ST = 3 \cdot \tau = 30.03 \mu s$ for a 5% error,
$ST = 4 \cdot \tau = 40.04 \mu s$ for a 2% error,
$ST = 5 \cdot \tau = 50.05 \mu s$ for a 1% error.

In order to improve the ST in the sense of decreasing it, C_M and C_{out} are swept around the design point from the validity range of the PM (Fig. 9).

The time constant variation with C_M and C_{out} can be found in the graphic in figure 9 and in table 8.

A similar procedure can be used also for the operational amplifiers having a complex-conjugate pair of poles [14].

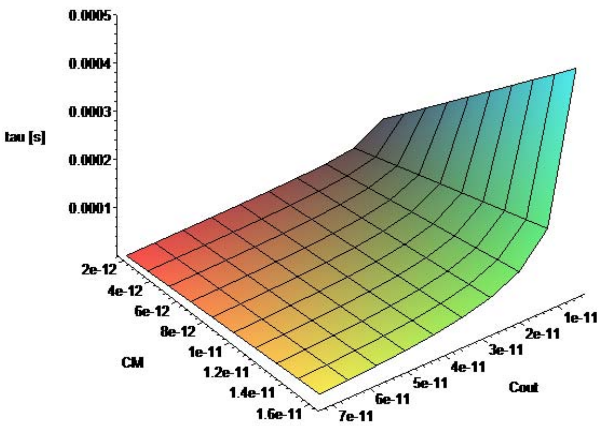

Fig. 9 Time constant as a function of Miller and the load capacitance

Figure 10 presents the time step response of the Op-Amp in the design point (in red) and the improved one (in green).

Table 8. Time constant (in seconds) for various C_M and C_{out}

C_M / C_{out}	1.80p	5.04p	8.28p	11.5p	14.7p
4.00p	51.2u	143u	235u	327u	420u
19.2p	10.5u	29.7u	49.0u	68.0u	87.7u
34.4p	5.71u	16.4u	27.1u	37.8u	48.5u
49.6p	3.75u	11.2u	18.6u	26.0u	33.5u
64.8p	2.65u	8.40u	14.0u	19.7u	25.4u

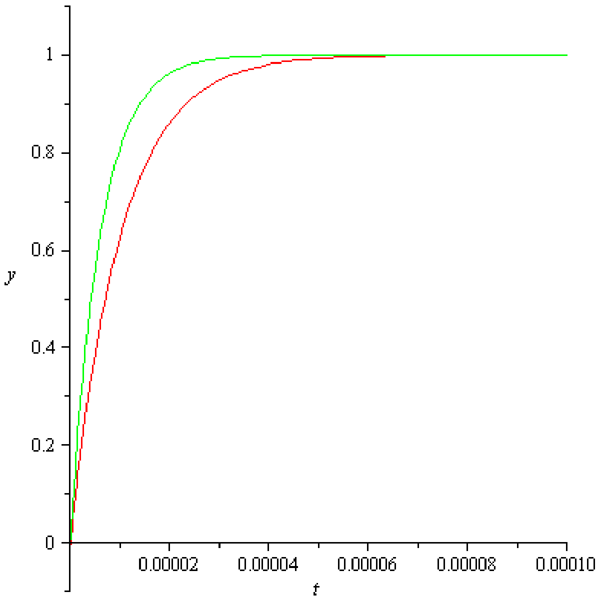

Fig. 10 Settling Time for two sets of Miller and the load capacitance

5 Conclusions

A procedure implemented in MAPLE is used to compute both numerical and symbolical poles/zeros, the transfer functions, the phase margin and the settling time expressions. This procedure can be used to verify a circuit design and to improve it if needed.

First, the symbolic expression of the phase margin is computed as a function of Miller and of the load capacitances for an operational amplifier in open loop designed in 180nm technology. Sweeping both capacitances, the validity range of the desired phase margin can be easily obtained.

Second, the symbolic expression of the time constant which is used to evaluate the settling time is computed function of Miller and the load capacitance for the operational amplifier in closed loop. Varying both capacitances, the optimum time constant can be easily obtained.

Of course these symbolic expression can be obtained as a function of other symbols if desired.

Having the poles, zeros, the phase margin and the settling time expressions, is interesting to see what part of the circuit gives each of these expressions and gives the designers the possibility to modify the project, adjusting only a small part of the circuit (e.g. a transistor, a capacitor or a resistor) in order to correct or to improve the design.

The numerical results have been compared with those obtained using SPECTRE RF. In this case, even though the frequency characteristic exhibit a pole in the range of 10^{12} Hz, the PZ analysis from SPECTRE RF cannot compute it even though the accuracy is increased.

6 Acknowledgment

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