Novel Chaotic Strange Phenomena in Piecewise Linear Negative Resistance/Conductance L-C Oscillators : Feasibility Study and Virtual simulation

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Abstract: - In this paper, the chaotic phenomena are outlined in a class of simple NRO (negative resistance oscillator) or NCO (negative conductance oscillator) equivalently. Compared to most chaotic electronic schemes encountered in nonlinear dynamic systems theory and practice, the proposed class of chaotic circuits consists of simple piecewise linear NRO/NCO, involving a minimum number of building components and dynamic states. The feasibility of relevant chaotic phenomena is proven from the existence of their Van Der Pol dynamic models according to the least square estimation principle. Then, despite the simplicity of the 2nd order NRO/NCO, a relevant parameterization strategy associated with virtual simulation techniques, are used as key exploration means for creating relevant chaotic phenomena. Therefore, new strange chaotic attractors with 2D/3D shapes are created and presented. Moreover, a comparative study with a sample of existing chaotic NRO/NCO schemes, show that the proposed class of chaotic NRO/NCO is optimal since it provides minimum building constituents and lower dynamic order, while offering a new palette of chaotic attractors with great topological strangeness.

Key-Words: - Negative resistance/conductance, nonlinear dynamic systems, oscillators, chaotic phenomena, strange chaotic attractors, topological strangeness, virtual simulation.

1 Introduction

The chaotic phenomena were surprisingly discovered a long time ago in different contexts by a few pioneering researchers, including Henri Pointcaré in 1892 (when solving the interaction problem among planets [1-2]), Lyapunov for his PhD thesis presented in 1892 on the study of initial conditions sensibility of chaotic systems [3], Van Der Pol (when simulating a class of nonlinear electric circuits [4-5]), and Edwar Lorenz in 1963 [6]. However, following the state-of-the art of further researches on chaos theory and relevant applications [7-8], it is known nowadays that the chaos is a strange behaviour of natural and artificial nonlinear dynamic systems, e.g., planet interactions, weather dynamics, electrical machines, animal world, electronic oscillators, communication systems and more. It is important to recall that, independently of their building technology, chaotic dynamic systems are characterized on by two following key properties: a) sensibility to neighbouring initial conditions, e.g., small changes of initial conditions in state or input variable(s), might cause local instability and global bounded trajectories after long runs; b) sensibility to parameter(s), e.g., small changes one or a few key parameters, might also cause local instability and global bounded trajectories. As a consequence, a strange visual attraction object can arise from instability and bounded trajectory phenomena. Since the discover of chaos, abundant research works have been published in that emerging research topic [1-18]. However, in most of these research works, the relevant contributions rely on the creation of new complex chaotic systems from existing chaotic schemes. Even though a significant effort for the reduction of the complexity of electronic chaotic circuits have been done in a few recent research works [19-22], the final chaotic electronic circuit involved still remain notoriously intricate in terms of the total number of building constituents. As a novelty, the major emphases of this paper is on a better trade off strategy between the structural simplicity and the topological strangeness of the proposed class of simplest chaotic *oscillator*s. It consists of a basic piecewise linear NRO (*negative resistance oscillators*) or NCO (*negative conductance oscillators* equivalently), involving the minimum number of building components and high topological strangeness of chaotic attractors.

In the following sections of the paper are organized as follows: A brief recall on building components of NRO/NCO is provided IN Section 2. Then, the feasibility of chaotic phenomena in piecewise linear NRO/NCO is outlined in Section 3 from the existence of its equivalent Van Der Pol versions according to least square estimation principle. Then, in section 4, virtual simulations under *Multisim* and Simulink environment are conducted under key parameterization strategies, in order to create and show a rich palette of strange chaotic attractors. Furthermore, a comparative study is conducted in section 5 between the proposed class of piecewise linear NRO/NCO and a sample of existing chaotic NRO schemes, in order to outline the novelty of this research work. Finally, the paper is concluded in section 6.

2 Piecewise NRC/NCC and related NRO/NCO works

The piecewise NRC (negative resistance circuit) or its dual piecewise NCC (negative conductance circuit), is widely used as a building element in a wide variety of electronic instrumentation systems, including autonomous oscillator, switching or sine wave modulators for communication systems, interfacing driver for power electronics converters, signal processing device for analog-to-digital and digital-to-analog converters. As shown in Fig. 1, the basic NRC/NCC circuits can be analogically implemented using a single operational amplifier.

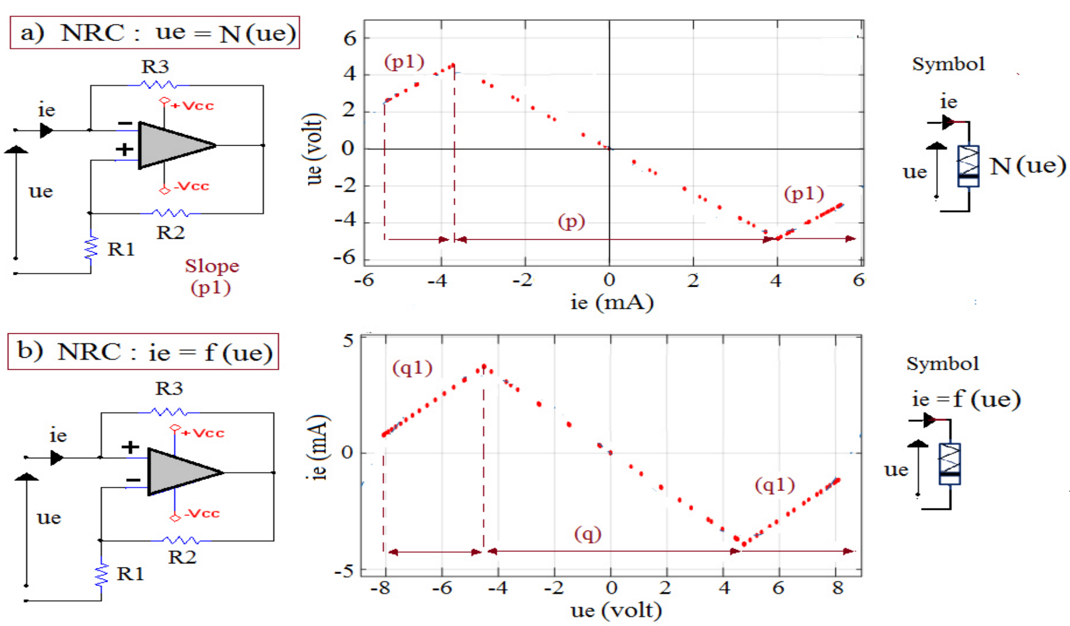


Fig.1 Simple piecewise NRC and NCC

The input-output characteristics $ue \sim ie$ and $ie \sim ue$ of NRC and NCC respectively, are both piecewise linear functions given by:

$$\begin{aligned} ue &= N(ie) \qquad (a) \\ ie &= f(ue) \qquad (b) \end{aligned} \qquad (1)$$

In (1) the slopes parameters p and $p1$ for $N(ie)$ in Fig. 1(a)), and q and $q1$ for $f(ue)$ in Fig. 1(b), are given by:

$$\begin{aligned} p &= -\frac{R1}{R2}R3,\ p1 = R3 \qquad (a) \\ q &= -\frac{R2}{R1\ R3},\ q1 = \frac{1}{R3} \qquad (b) \end{aligned} \qquad (2)$$

The piecewise linear NRC/NCC are used as building components of a class of NRO/NCO as shown in Fig.2. The serial L-C oscillator presented in Fig. 2(a) is built using a NRC, and its dual L-C parallel configuration shown in Fig. 2(b), relies on a NCC core. In each case, the NRO/NCO can be under autonomous or controlled operating conditions from an external voltage source Vs(t).

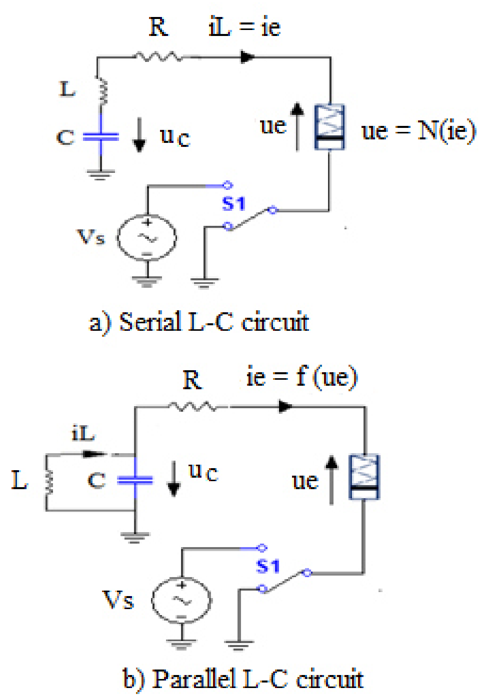


Fig. 2 A Class of NRO and NCO circuits

For the sake of simplicity, let us consider the new notations,

$$\begin{aligned} y &= ue \\ x &= ie \end{aligned} \quad \text{(in Fig. 1(a) and Fig. 2(a))} \qquad (3)$$

and

$$\begin{aligned} y &= ie \\ x &= ue \end{aligned} \quad \text{(in Fig. 1(b) and Fig. 2(b))} \qquad (4)$$

Then, a generic form of (1) could be written as follows:

$$\begin{aligned} y &= N(x) \quad (in\ Fig.\ 1(a),\ Fig.\ 2(a)) \qquad (a) \\ y &= f(x) \quad (in\ Fig.\ 1(b),\ Fig.\ 2(b)) \qquad (b) \end{aligned} \qquad (5)$$

Because of the duality principle between NRO and NCO, the emphasis in this paper without loss of generality, is on the relevant subclass of serial piecewise NRO circuits, which are governed by the following 2nd order nonlinear dynamic model:

$$\begin{aligned} \frac{d^2x(t)}{dt^2} &= -\frac{R}{L}\frac{dx(t)}{dt} - \frac{1}{L\ C}x(t) \\ &\quad - \frac{1}{L}\frac{\mathrm{d\,N}(x(t))}{dt} + \frac{1}{L}\frac{dVs(t)}{dt} \\ &= -\frac{R}{L}\frac{dx(t)}{dt} - \frac{1}{L\ C}x(t) \\ &\quad - \frac{1}{L}\frac{\partial \mathrm{N}(x(t))}{\partial x(t)}\frac{d(x(t))}{dt} \\ &\quad + \frac{1}{L}\frac{dVs(t)}{dt} \end{aligned} \qquad (6)$$

where N(x) is a piecewise linear function outlined earlier in Fig. 1(a).

3 Chaotic Phenomena in Piecewise Linear NRO/NCO

It is important to understand that a class of Van Der Pol oscillators could be derived from piecewise NRO modelled by (5) using LSE (Least Square Estimation) principles. Indeed, let us consider a polynomial estimation model of the resulting piecewise NRC/NCC as presented in Fig. 3, with a cubic shape given as follows:

$$\begin{aligned} y &= a_1 x^3 + a_2 x^2 + a_3 x + a_4 \\ &= \underbrace{\begin{bmatrix} x^3 & x^2 & x & 1 \end{bmatrix}}_{h^{\mathrm{T}}(x)} \underbrace{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}}_{\theta} \end{aligned} \qquad (7)$$

where $\theta = [a1\ \ a2\ \ a3\ \ a4]^T$ is a parameter vector to be determined according to LSE principle from a suitable sample {x(k) , y(k)} of y(k) = N(x(k) with k =1, 2, …, N. In these conditions, the optimal

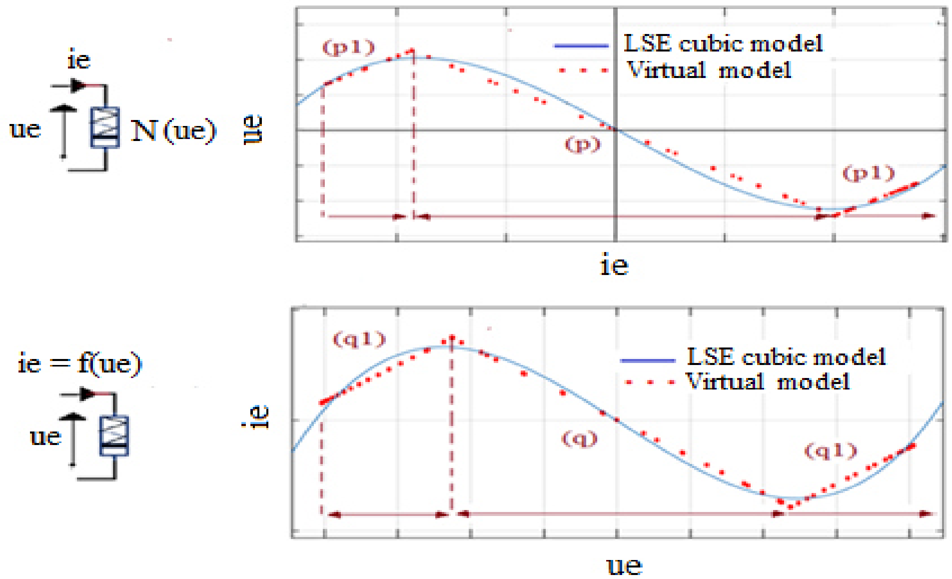


Fig. 3 NRC/NCC and nature of LSE polynomial models

value θ* of θ according to the LSE principle is given as follows [23-24]:

$$\theta^*(N) = \Bigg(\underbrace{\sum_{k=1}^{N} h(x(k))\, h^T(x(k))}_{\text{Sum of N matrixes (4 x 4}} \Bigg)^{-1} * \Bigg(\underbrace{\sum_{k=1}^{N} h(x(k))\, y(k)}_{\text{Sum of N vectors (4 x 1)}} \Bigg) \quad (8)$$

In Matlab-based graphical simulation, it is easy to compute (7) within the *cftool* design framework, i.e., the nature of results to be obtained when computing graphically the LSE solution θ*= [a1* a2* a3* a4*]^T, is illustrated in Fig. 3. Therefore, the suboptimal class of Van Der Pol oscillators obtained from the original piecewise NRO could be modelled from (6) given θ* as follows:

Given that :

$$N^*(x(t)) = a_1^* x^3(t) + a_2^* x^2(t) + a_3^* x(t) + a_4^*$$

$$\frac{\partial N^*(x(t))}{\partial x(t)} = 3 a_1^* x^2(t) + 2 a_2^* x(t) + a_3^* \quad (10)$$

Then (8) becomes:

$$\begin{aligned} \frac{d^2x(t)}{dt^2} &= -\frac{1}{L}\left(R + \frac{\partial N^*(x(t))}{\partial x(t)}\right)\frac{d(x(t))}{dt} - \frac{1}{LC}x(t) + \frac{1}{L}\frac{dVs(t)}{dt} \\ &= -\frac{1}{L}(R + 3a_1^* x^2(t) + 2a_2^* x(t) + a_3^*)\frac{d(x(t))}{dt} - \frac{1}{LC}x(t) + \frac{1}{L}\frac{dVs(t)}{dt} \\ &= -\frac{(a_3^* + R)}{L}\left(1 + \frac{(3a_1^* x^2(t) + 2a_2^* x(t) + a_3^*)}{a_3^* + R}\right) * \frac{d(x(t))}{dt} - \frac{1}{LC}x(t) + \frac{1}{L}\frac{dVs(t)}{dt} \end{aligned} \quad (11)$$

Using the new notations :

$$A = \frac{(a_3^* + R)}{L},\ B = \frac{3a_1^*}{a_3^* + R},\ C = \frac{2a_2^*}{a_3^* + R},\ \omega^2 = \frac{1}{LC}, \quad (12)$$

Then, (11) becomes:

$$\frac{d^2x(t)}{dt^2} + A\left(1 + B x^2(t) + C a_2^* x(t)\right)\frac{d(x(t))}{dt} + \omega^2 x(t) = \frac{1}{L}\frac{dVs(t)}{dt} \quad (13)$$

As a first relevant finding, if the external excitation Vs(t) is a constant, then Equation (12) visually becomes the dynamic model of a family of Van Der Pol NRO. In addition, given the set of parameters:

{R1 = R2 = R3 = 1.71 kΩ] in (2), {a*1 = 4.043e+07, a*2 ≈ 0, a3* = -1710, a4* ≈ 0} in (7), {R= 1 kΩ,, L = 100 mH, C = 22 nF} in Fig. 2(a), {A = 27100, B = 4.4756e+04, ω^2 = 4.549e+08}, the second relevant finding relies on a basic strange attractor obtained associated with (12).

That basis strange attractor is displayed in the x-y plane as shown in Fig. 4. Because of both relevant findings, the feasibility of chaotic behaviour within the original class of piecewise linear NRO is quite established.

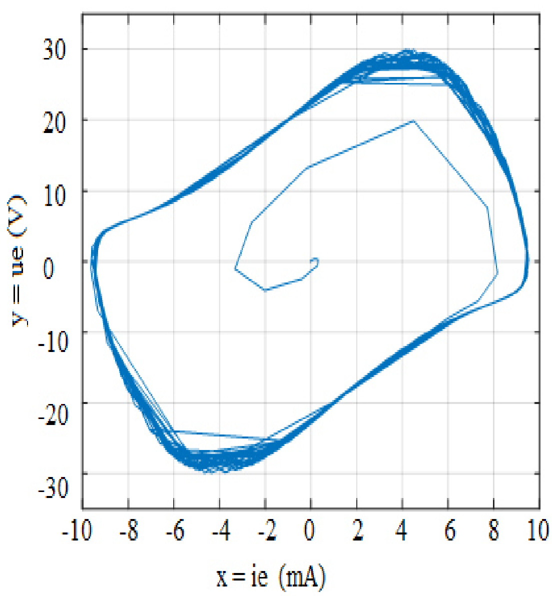


Fig. 4: Basic attractor obtained in the x-y plane

4 Chaotic strange attractors in Piecewise Linear NRO/NCO

The main goal of this section is to show that, more attractive chaotic strange attractors exist within the class of piecewise linear NRO. This is possible from suitable parameterization strategies in autonomous as well as in controlled operating conditions. Table 1 presents the set of data used to explore the expected sample of strange chaotic phenomena as presented in Table 2. In the first colon of Table 1, the notations Vs = 0 and Vs # 0 stand for autonomous and controlled operating conditions respectively (Se Fig. 2 for better clarity), whereas ABCj in Table 1 is related to line *j* = 1, 2, …, 7 in Table 2 where a rich palette of novel 2D/3D chaotic strange attractors are presented.

Given Table 1 and following Table 2, it is important to observe that the search strategy adopted in this work for creating chaotic strange attractors with arbitrary 2D/3D shapes, relies on the identification and adjustment of key parameters of the same 2[nd] order LC oscillators, e.g., the negative slop *p* of the NRC and the external excitation Vs(t). Hence, the minimum dimension (2[nd] order) of the chaotic dynamic space is unchanged and independent of the topological strangeness of the chaotic attractor. These findings are straightforward merits and challenge compared to most existing chaotic electronic schemes in which the topological strangeness of attractors grows according to the increasing complexity of the whole systems in terms of volume, dynamic order, building costs.

Table -1 Set of Data used for chaotic

Mode	ABCj		Vs (V)	fs (Hz)	Negative Slop *p*	α1	R1 (kΩ)	R2 (kΩ)	R3 (kΩ)	R (Ω)	L (mH)	C (nF)
Vs = 0	ABC1		0	-	-1.55e-04	9.63e-01	0.25	1.71	45	10	100	47
	ABC2		0	-	-6.66e-03	5e-01	0.15	5	5	10	10	100
Vs # 0	ABC3		3	10	-9.74e-03	5.66e-02	1.71	5	0.3	10	100	47
	ABC4		3	10	-1,66e-02	2.857e-01	150	5	2	10	10	100
	ABC5		3	10	-1.35e-03	1.68e-01	150	1.71	8.44	10	10	100
Vs # 0	ABC6		8	10	-5.316e-03	9.90e-02	1.71	5	0.55	10	100	47
	ABC7		10	10	- 6.33e-02	9.52e-02	0.15	1.71	0.18	10	10	100

5 Comparative Study With A Sample of Other Chaotic NRO/NCO Schemes

As shown in Table 3, the optimality of simple piecewise linear NRO/NRC presented in this paper, relies on a comparative study with a sample of existing chaotic NRO schemes en countered in the literature.

The comparison criteria retained for OPAM (operational amplifier) implementation technology, consists of the circuit architecture, the hardware complexity (in terms of number of : DC sources, operational amplifiers, diodes), the dynamic order and the topological strangeness of attractor(s).

It is a great challenge to observe that, the proposed class of piecewise NRO is optimal for almost all comparative criteria.

These main relevant findings and further work improvement lead to the conclusion of the paper.

Table 2 Sample of novel chaotic phenomena created in the proposed piecewise linear NRO/NCO

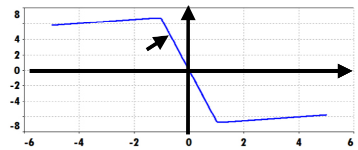
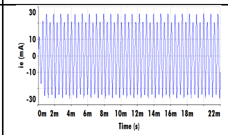
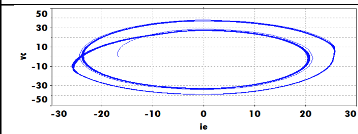
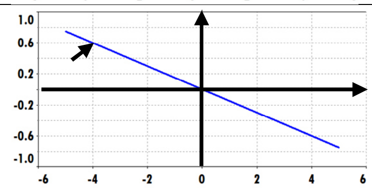
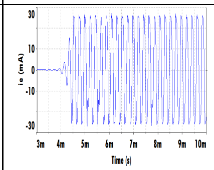
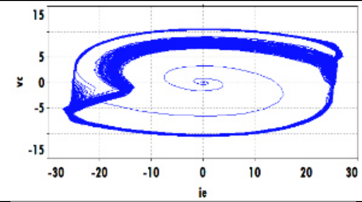
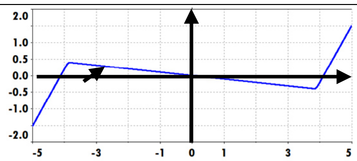
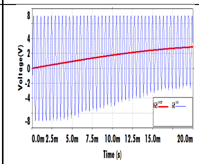
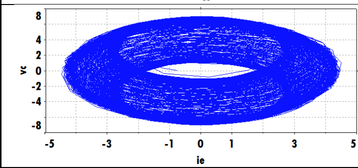
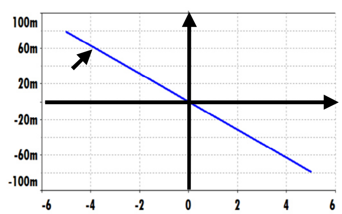
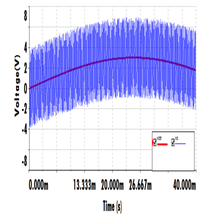
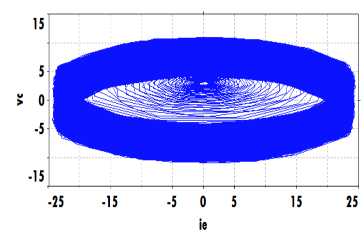
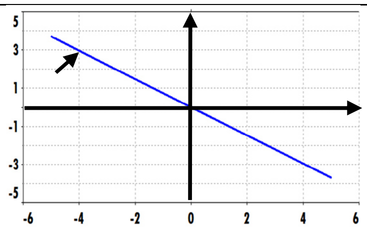
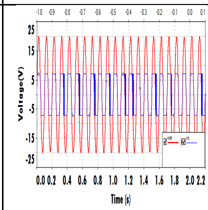
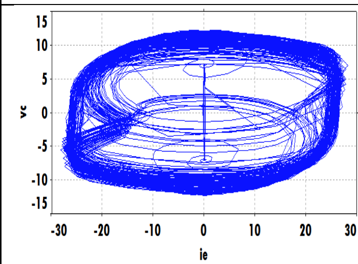
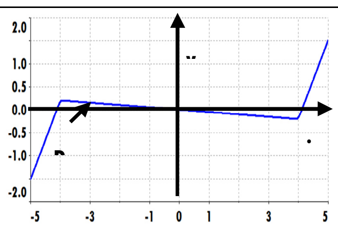
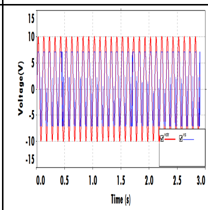
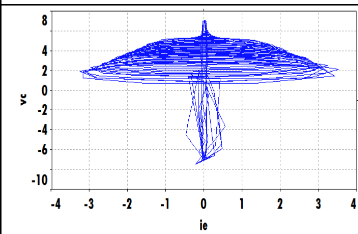
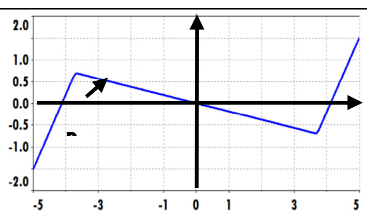
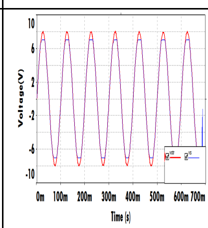
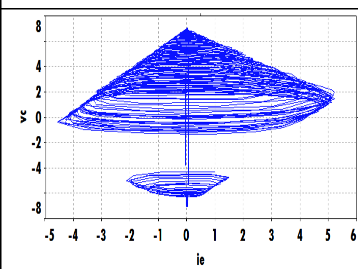
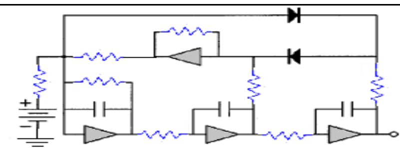
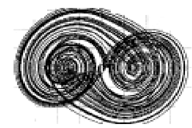
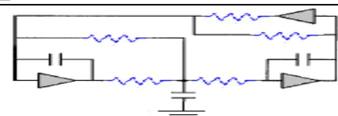
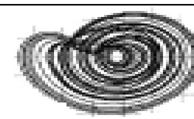
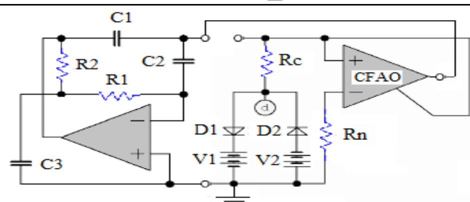
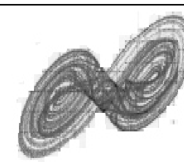
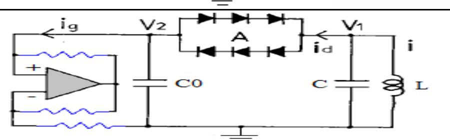
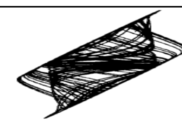
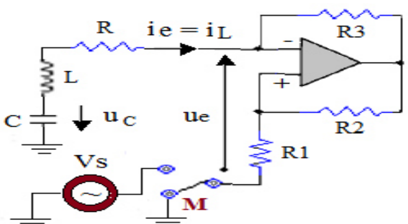
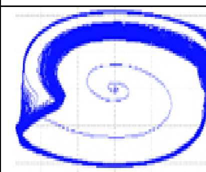
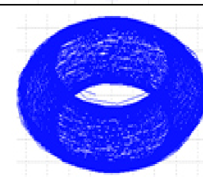
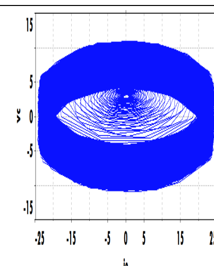
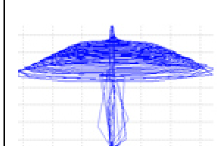
	A	B	C
1			
2			
3			
4			
5			
6			
7			

Table 3 Comparative study between existing chaotic NRONCO and the proposed chaotic NR

References	Electronic circuit architecture		Structural complexity				Topological strangeness of attractor
			Number of OPAMs	Number of DC source	Number of Diodes	Dynamic Order	
[20]			4	1	2	3	
[21]			3	0	0	3	
[22]			2	2	2	3	
[23]			1	0	6	3	
Proposed class of simple chaotic piecewise linear NRO/NCO	 **NOVELTY** The discovery of a wide variety of novel chaotic phenomena, relies in this case on a better parameterization strategy within the optimal class of piecewise linear RNO/NCO, with minimum number of building parts, minimum state space dimension (equal to 2), and high topological strangeness of a wide variety of chaotic attractors involved.	Autonomous	**1**	**0**	**0**	**2**	
		Voltage controlled	**1**	**0**	**0**	**2**	
			1	**0**	**0**	**2**	
			1	**0**	**0**	**2**	

6 Conclusion

This research paper can be thought off as a key user guide for better trade off strategies, between structural simplicity and topological strangeness of relevant chaotic phenomena when creating chaotic NRO/NCO. A sample of novel 2D/3D strange chaotic attractors obtained from parameterization exploration and virtual simulations, have shown the merits of a simple methodology for effectively building virtual NRO/NCO circuits and systems.

However, the research work as presented in this paper, is limited to the feasibility analysis associated with well tested chaotic NRO schemes in the virtual world. Therefore, testing and characterizing in a real word novel chaotic strange attractors created and presented here, would be an additional relevant contribution in future research works.

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References

[1] H. Point Carré, *Les méthodes nouvelles de la mécanique celeste*, 1892, Gauthiers-Villard, Paris.

[2] B. Van Der Pol, A theory of the amplitude of free and forced triode vibrations, *Radio Review*, 1920, pp. 701– 710.

[3] Lypunov, Mesure de divergence entre deux orbites issue des conditions initiales voisines, Moscou University, 1892**.**

[4] B. van Der Pol, On Relaxation Oscillations, *I, Phil. Mag*., 1926, pp. 978–992.

[5] B. van Der Pol, The nonlinear theory of electric oscillations, Proc. IRE, **1934**, pp. 1051–1089.

[6] Lorenz, ‘‘Deterministic nonperiodic flow,’’ *Journal Atmos. Sci*., 1963, 20, pp. 130–141.

[7] S. *Wiggins, Introduction to applied nonlinear dynamic systems and chaos,* New York, 1990.

[8] J.C. Sprott and S. J. Linz, Algebraically simple chaotic flows, *International Journal of Chaos Theory and Applications*, 2000, 5(2), pp. 1 – 20.

[9] O. E. Rössler, Continuous chaos, Four prototype equations, Annales N. Y. *Academic Science*, 1976, 316, pp. 376-392.

[10] R. S. Barbosa, J.A. Tenreiro Machado, B. M. Vinagre and A. J. Calderon, Analysis of the van der Pol oscillator containing derivatives of fractional order. *Journal Vibration.Control*, 2007, 13(9–10), pp. 1291–1301.

[11] V. Vaithianathan and J. Veijun, Coexistence of four different attractors in a fundamental power system model. *IEEE Transactions on Circuits Systems I*, 1999, 46(3), pp. 405–409.

[12] G. Chen, *Controlling Chaos and Bifurcations in Engineering Systems,* CRC Press, Boca Raton, 1999, USA.

[13] T/ L/ Vincent and J. Yu, Control of a chaotic system», *J. of Dynamics and Control,* 1991, 1, pp. 35-52.

[14] A. Arneodo, P. Coullet, and C. Tresser, Possible new strange attractors with spiral structure, *Commun. Math. Phys*. 1981, 79, pp. 573–579.

[15] A. Tamasˇevicˇius, G. Mykolaitis, and A. Namaju¯nas, Double scroll in a simple ‘2D’ chaotic oscillator, *Electronic Letters*., 1996, 32, pp. 1250–1251.

[16] B.B. Ferreira, M.A. Savi, A.S. de Paula, Chaos control applied to cardiac rhythms represented by ECG signals, Phys. Scr., 2014, 89(10):105203,.

[17] A. Zerroug, L. Terrissa, A. Faure, Chaotic dynamical behavior of recurrent neural network, *Annu. Rev. Chaos Theory Bifurc. Dyn. Syst*., 2013, 4:55–66.

[18] X. Luo, C. Wang, Z. Wan, Grid multi-wing butterfly chaotic attractors generated from a new 3-D quadratic autonomous system, *Nonlinear Anal. Model. Control*, **2014**, 9(2), pp. 272–285.

[19] J. C. Sprott, A new class of chaotic circuit, Physics Letters A266, 2000, *Elsevier Electronics Letters*, pp. 19-23.

[20] E. N. J. Petržela, Z. Kolka and S. Hanus, Simple chaotic oscillator : From mathematical model practical experiments, *Radioengineering*, April 2000, 15(1), pp. 6-12.

[21] N. INABA, T. Saito, and S. Mori, Chaotic phenomena in a circuit with negative resistance and an ideal switch of diodes, Transactions of the IEICE, August 1987, 70(8), pp. 744-754.

[22] J. C. Sprott, A new class of chaotic circuit, Physics Letters A266, 2000, pp. 19-23.

[23] A. Maddi, A. Guessoum, D. Berkani and O. Belkina, Etude de la méthode des moindres carrée recursive et application au signal de parole, *3rd International Conference: SETIT (Sciences of Electronic, Technologies of Information and Telecommunications*), 2005, March 17-21, Tunisia.

[24] J. CI. Trigeassou, *Recherche des modèles expérimentaux assistés par ordinateur*, Technique et documentation- Lavoisier, 1988, Paris.