Developing UFIR Filtering with Consensus on Estimates for Distributed Wireless Sensor Networks

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Abstract: Recent decades have celebrated a growing interest to wireless sensor networks (WSNs), both in theory and applications. Organized to have a large number of nodes, the WSN allows for redundant measurements that makes the distributed optimal estimation an adequate sensor fusion technique. The estimators developed for WSNs should ensure the consensus in the network while respecting restrictions imposed by the battery life, real-time estimation, and low computing burden. In this work, we develop the unbiased finite impulse response (UFIR) filtering technique to operate under consensus on the estimates in the distributed WSN. Properly tuned on optimal horizons, the distributed UFIR filter with consensus on estimates reduces the mean square error (MSE) as compared to the centralized UFIR. It also demonstrates higher robustness against model errors while respecting the restrictions of the WSN.

Key–Words: Optimal estimation, WSN, Robustness

1 Introduction

The interest to wireless sensor networks (WSN) has grown in recent decades, especially in industrial applications [1, 2, 3]. A dramatic progress in the development of WSN has become possible mostly due to technological advances in smart sensors, which allow a more ubiquitous and large scale deployment of WSN in many fields.

The large scale deployment of the WSN nodes allows a desired quantity $\mathcal{Q}$ to be measured by a big number of sensors. However, the measurement process is provided in the presence of noise. Therefore, optimal estimation is required along with adequate sensor fusion techniques [4, 5, 6, 7]. None the less, the restrictions of WSN caused by limited battery life and required processing power put a stress on the development of algorithms to ensure an efficient use of these limited resources.

Distributed filtering has been introduced in order to estimate $\mathcal{Q}$ in real time [8, 9] and improve battery life by eliminating wireless transmissions of the nodes over long distances to a centralized estimator. However, an important issue in distributed filtering remains: it needs to ensure the consensus in the network, which the nodes achieve by averaging the estimates, measurements, or information matrices [10].

Nowadays, Kalman filter (KF) remains most popular among other sensor fusion techniques [11] due to its optimality and low computational cost. In [12], the author has proposed a KF structure that requires each node to locally aggregate its measurement and covariance matrix with those of its neighbors and, in a posterior step, compute the estimate using a KF with a consensus term. Based on this approach, other algorithms were introduced, for example [13, 14], using the Kalman-like approach.

Over the years, many distributed techniques have been developed based on the Kalman filter [15, 16, 17, 18], the optimality nature of KF does not always ensure the robustness, scalability, and fault tolerance required by the WSNs [19, 20]. It has been proven that better robustness can be achieved using filter operating on finite data horizons [21, 22, 23, 24]. Under such an assumption, a moving average estimator has been designed in [25] for weak observability. A consensus finite-horizon H_∞ approach was developed in [26] under missing measurements. In [27], an unbiased finite impulse response (UFIR) filter was developed for consensus on measurements. Although this filter has shown an ability to be more robust and adequate than the KF for WSN, it was designed under the condition that all sensors measure the same state at the same time.

In this work, we make attempts to design a distributed UFIR filter with consensus on estimates. Unlike its predecessor [27], the new filter will be designed to not require the nodes to measure the same state, and having the same variance. That predetermines better performance of the solution that we will show based on simulations of the WSN.

2 Model and problem formulation

Let us suppose that a physical quantity $\mathcal{Q}(t)$ in question is represented for the WSN conditions in discrete-time state-space with the following linear K-state space equations

$$x_k = A_k x_{k-1} + B_k w_k \,, \quad (1)$$
$$y_k = H_k x_k + v_k \,, \quad (2)$$

where $x_k \in \mathbb{R}^K$ is the state vector in the discrete time index k, $y_k = [\, y_k^{(1)^T} \; \ldots \; y_k^{(n)^T} \,]^T \in \mathbb{R}^{np}$, $p \leqslant K$ is the measurement vector in which $y_k^{(i)} = H_k^{(i)} x_k + v_k^{(i)} \in \mathbb{R}^p$ represents each sensor's individual measurement in an ad hoc network of n inclusive neighbors j. Here, $H_k = [\, H_k^{(1)^T} \; \ldots \; H_k^{(n)^T} \,]^T \in \mathbb{R}^{np\times K}$, $H_k^{(i)} \in \mathbb{R}^{p\times K}$, $A_k \in \mathbb{R}^{K\times K}$, and B_k are known matrices of proper dimensions. The noise vectors, w_k and $v_k = [\, v_k^{(1)^T} \; \ldots \; v_k^{(n)^T} \,]^T \in \mathbb{R}^{np}$, are supposed to be zero mean Gaussian with the covariances Q_k and $R_k = \mathrm{diag}[\, R_k^{(1)^T} \; \ldots \; R_k^{(n)^T} \,]^T \in \mathbb{R}^{np\times np}$, respectively. In what follows, $\hat{x}_{k|r}$ is the estimate of x_k at k via measurements up to and including at time-index k.

The problem can now be discussed as follows. Given individual estimates provided in each WSN node separately, how can we find the consensus for the estimates? With this aim, one KF was used in [12] to obtain optimum estimates for an individual sensor and another one to provide optimum estimation using the individual estimates of the inclusive neighbors. Following this approach, let us form the estimate as

$$\hat{x}_k^{ic} = \tilde{K}_{m,k} Y_{m,k} + \lambda \sum_{j}^{n} (\hat{x}_k^{(j)} - \hat{x}_k^{(i)}) \,, \quad (3)$$

which implies that errors produced by centralized estimation are corrected via the difference between the ith estimate and each individual estimate provided by its neighbors. The estimation is provided in such a way that $\sum_{j=1}^{n} (\hat{x}_k^{(j)} - \hat{x}_k^{(i)}) = 0$ means a perfect consensus between all the inclusive neighbors of the ith node. Otherwise, errors in the centralized estimation will be compensated using the difference between the individual estimates and λ, which is chosen to minimize the root mean squared error (RMSE),

$$\lambda = \arg\min_{\lambda} \{ \mathrm{tr}\, P(\lambda) \} \,, \quad (4)$$

where $P = E\{(x - \hat{x}^{ic})(x - \hat{x}^{ic})^T\}$ is the relevant error covariance. The implementation of (3) requires that each node performs a centralized estimation that will result in high computational burden for a smart sensor. However, in a practical scenario, the computational load can be greatly weakened. In the following section, we will discuss a more suitable version of (3) for a smart sensor.

3 Distributed UFIR Filtering

A simplification of (3) can be obtained in the batch form if to express it as follows,

$$\begin{aligned} \hat{x}_k^{ic} &= \tilde{K}_{m,k} Y_{m,k} && (5a)\\ &\quad + \lambda \sum_{j}^{n} (\bar{K}_{m,k}^{(j)} Y_{m,k}^{(j)} - \bar{K}_{m,k}^{(i)} Y_{m,k}^{(i)}) \\ &= \tilde{K}_{m,k} Y_{m,k} \lambda \tilde{K}_{m,k}^{(j)} Y_{m,k} && (5b)\\ &\quad - n\lambda \bar{K}_{m,k}^{(i)} Y_{m,k}^{(i)} \\ &= (\tilde{K}_{m,k} + \lambda \tilde{K}_{m,k}^{(j)}) Y_{m,k} && (5c)\\ &\quad - n\lambda \bar{K}_{m,k}^{(i)} Y_{m,k}^{(i)} \\ &= \dot{\tilde{K}}_{m,k} Y_{m,k} - n\lambda \bar{K}_{m,k}^{(i)} Y_{m,k}^{(i)} \,, && (5d) \end{aligned}$$

where $\tilde{K}_{m,k}^{(j)} = [\tilde{K}_m^{(i)} \ldots \tilde{K}_k^{(i)} \ldots \tilde{K}_m^{(n)} \ldots \tilde{K}_k^{(n)}]$ and $\dot{\tilde{K}}_{m,k} = [\dot{\tilde{K}}_m^{(i)} \ldots \dot{\tilde{K}}_k^{(i)} \ldots \dot{\tilde{K}}_m^{(n)} \ldots \dot{\tilde{K}}_k^{(n)}]$. In order to determine $\dot{\tilde{K}}_{m,k}$, the unbiasedness condition $E\{\hat{x}_k^{ic}\} = E\{x_k\}$ must be satisfied. Following [28], we thus find the filter gain $\dot{\tilde{K}}_{m,k}$ as

$$\begin{aligned} \dot{\tilde{K}}_{m,k} &= (I + n\lambda)(C_{m,k}^T C_{m,k})^{-1} C_{m,k}^T && (6a)\\ &= (I + n\lambda) G_k^{-1} C_{m,k}^T && (6b) \end{aligned}$$

and notice that (3) has another equivalent form of

$$\hat{x}_k^{ic} = (I + n\lambda)\bar{\tilde{K}}_{m,k} Y_{m,k} - n\lambda \bar{K}_{m,k}^{(i)} Y_{m,k}^{(i)} \,, \quad (7)$$

in which the filter gain $\bar{\tilde{K}}_{m,k}$ is computed by

$$\begin{aligned} \bar{\tilde{K}}_{m,k} &= G_k^{-1} C_{m,k}^T \,, && (8a)\\ &= (C_{m,k}^T C_{m,k})^{-1} C_{m,k}^T && (8b) \end{aligned}$$

and the extended observation vector $Y_{m,k}$ and matrix $C_{m,k}$ are given by

$$Y_{m,k} = \left[y_m^T \; y_{m+1}^T \; \dots \; y_k^T \right]^T , \tag{9}$$

$$C_{m,k} = \begin{bmatrix} H_m (F_k^{m+1})^{-1} \\ H_{m+1} (F_k^{m+2})^{-1} \\ \vdots \\ H_{k-1} A_k^{-1} \\ H_k \end{bmatrix} , \tag{10}$$

where the product of system matrices is assigned as

$$F_k^r = \begin{cases} A_k A_{k-1} \dots A_r, & r < k+1 \\ I & r = k+1 \\ 0 & r > k+1 \end{cases} . \tag{11}$$

For the individual estimates, $\bar{K}_{m,k}^{(i)}$ is given by the product

$$\bar{K}_{m,k}^{(i)} = G_k^{(i)} C_{m,k}^{(i)^T} , \tag{12}$$

where the *generalized noise power gain* (GNPG) matrix $G_k^{(i)}$ [29] is provided by

$$G_k^{(i)} = \bar{K}_{m,k}^{(i)} \bar{K}_{m,k}^{(i)^T} = (C_{m,k}^{(i)^T} C_{m,k}^{(i)})^{-1} . \tag{13}$$

As can be seen, all information required to calculate $\tilde{\bar{K}}_{m,k}$ and $\bar{K}_{m,k}^{(i)^T}$ is provided by the K-states space model. If for a certain application we have $H_k^{(i)} = H_k^{(j)}, \forall j \neq i$, then the transition matrix should be preloaded in the sensors, so only measured data will be required from the nodes. On the other hand, if sensors measure different states, the nodes should transmit such information to neighbors.

3.1 Iterative form

For a large number of nodes, estimate (7) will require a heavy computational load for a smart sensor. In order to reduce this burden, an iterative form of (7) using recursions can be found if derive the algorithm for a sum of a centralized filter and an individual filter as

$$\hat{x}_k^{ic} = (I + n\lambda_k)\hat{x}_k^c - n\lambda_k \hat{x}_k^{(i)} . \tag{14}$$

The iterative forms for $\hat{x}_k^c$ and $\hat{x}_k^{(i)}$ can be derived following [28, 27]. First, represent the centralized filter as

$$G_l = [H_l^T H_l + (A_l G_{l-1} A_l^T)^{-1}]^{-1} , \tag{15}$$

$$\hat{x}_l^c = A_l \hat{x}_{l-1}^c , \tag{16}$$

$$\hat{x}_l^c = \hat{x}_l^{c^-} + G_l H_l^T (y_l - H_l \hat{x}_l^{c^-}) \tag{17}$$

and do the same for the individual estimates as

$$G_l^{(i)} = [H_l^{(i)^T} H_l^{(i)} + (A_l G_{l-1}^{(i)} A_l^T)^{-1}]^{-1} , \tag{18}$$

$$\hat{x}_l^{(i)^-} = A_l \hat{x}_{l-1}^{(i)} , \tag{19}$$

$$\hat{x}_l^{(i)} = \hat{x}_l^{(i)^-} + G_l^{(i)} H_l^{(i)^T} (y_l^{(i)} - H_l^{(i)} \hat{x}_l^{(i)^-}) \tag{20}$$

The initial values G_{l-1} and $\hat{x}_{l-1}$ are computed as

$$G_s = (C_{m,s}^T C_{m,s})^{-1} , \tag{21}$$

$$\hat{x}_s^c = G_s C_{m,s}^T Y_{m,s} , \tag{22}$$

and for the individual filter as

$$G_s^{(i)} = (C_{m,s}^{(i)^T} C_{m,s}^{(i)})^{-1} , \tag{23}$$

$$\hat{x}_s^{(i)} = G_s^{(i)} C_{m,s}^{(i)^T} Y_{m,s}^{(i)} . \tag{24}$$

Now, the iterative UFIR algorithm can be listed as Algorithm 1.

Algorithm 1: Iterative UFIR Filtering Algorithm with Consensus on Estimates

Data: y_k, N
Result: $\hat{x}_k$
begin
  for $k = N-1 : \infty$ **do**
    $m = k - N + 1, \quad s = m + K - 1$;
    $G_s = (C_{m,s}^T C_{m,s})^{-1}$;
    $G_s^{(i)} = (C_{m,s}^{(i)^T} C_{m,s}^{(i)})^{-1}$;
    $\tilde{x}_s^c = G_s C_{m,s}^T Y_{m,s}$;
    $\tilde{x}_s^{(i)} = G_s^{(i)} C_{m,s}^{(i)^T} Y_{m,s}^{(i)}$;
    for $l = s+1 : k$ **do**
      $G_l = [H_l^T H_l + (A_l G_{l-1} A_l^T)^{-1}]^{-1}$;
      $G_l^{(i)} = [H_l^{(i)^T} H_l^{(i)} + (A_l G_{l-1}^{(i)} A_l^T)^{-1}]^{-1}$;
      $\tilde{x}_l^c = A_l \tilde{x}_{l-1}^c + G_l H_l^T (y_l - H_l A_l \tilde{x}_{l-1}^c)$;
      $\tilde{x}_l^{(i)} = A_l \tilde{x}_{l-1}^{(i)} + G_l^{(i)} H_l^{(i)^T} (y_l^{(i)} - H_l^{(i)} A_l \tilde{x}_{l-1}^{(i)})$;
    end for
    $\hat{x}_k^{ic} = (I + n\lambda)\tilde{x}_k^c - n\lambda \tilde{x}_k^{(i)}$;
  end for
end

4 Application example

In this section, we apply Algorithm 1 to track a moving object on the ground basis trough redundant noisy measurements provided by the nodes of a WSN. It is assumed that the measurements of all the nodes are available for the entire trajectory of the object without delay. The test network consist of four nodes at fixed positions as shown in Fig. 1. Such configuration allows us to examine the behavior of the proposed filter for different number of neighbors, ranging from one to three, as it is assumed that each node can only communicate its first order neighbors.

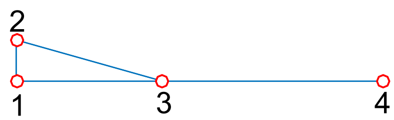

Figure 1: Simulated WSN with four nodes, which track a mobile object over all time.

Each node of the WSN tracks an object traveling along a circular, clockwise trajectory from its initial position $x_0 = 10m$, $y_0 = -10m$. Fig. 2 shows such trajectory whose dynamics are described by the transition matrix

$$A = \begin{bmatrix} a & b \\ -b & a \end{bmatrix},$$

where $a = 0.9996$, $b = 0.03$, $B = I$. At $k = 350$, an object receives an unpredicted impact, which is modeled by substituting a with $a + \delta$ in A during $350 \leq k \leq 370$. The noise components of $w_k = [w_{1k}\ w_{2k}]^T$ have the variance $\sigma_w^2 = 0.01\text{m}^2$ and the covariance $Q = \text{diag}[\,\sigma_w^2\ \sigma_w^2\,]$.

It is assumed that each sensor is able to measure only one of two states of (1) over all time. Sensors 1 and 3 measure the x coordinate, $H^{(1,3)} = [\,1\ 0\,]$, while sensors 2 and 4 observe the y coordinate, $H^{(2,4)} = [\,0\ 1\,]$. Matrix $R_k^{(i)} = E\{v_k^{(i)} v_k^{(i)^T}\} = (\sigma + \phi)^2$ is described for $\sigma = 5$m and $\phi \sim U[-1, 1]$. The filter optimum horizon, $N_{\text{opt}} = 88$, was found at a test stage.

4.1 Effect of λ on RMSE

To implement (7) optimally, λ should be chosen such that the MSE is minimized for the consensus on estimates at each node. To this end, we assume λ is a diagonal matrix, $\lambda = \text{diag}[\lambda_1 \dots \lambda_K]$, where each element corresponds to a compensating factor for each state of the system.

As can be seen in Fig. 3, a correct value of λ ensures the minimum RMSE for the consensus filter. This is more relevant to the first state of node 1 (Fig. 3 a)), where the RMSE of the consensus filter is smaller than that of the centralized and individual filter. For

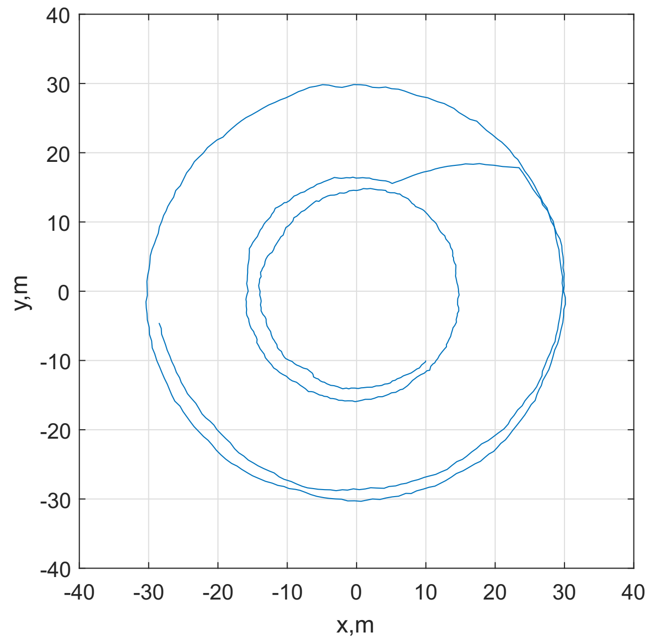

Figure 2: Object with a circular trajectory on a ground space starting with $x_0 = 10m$ and $y_0 = -10m$ at $k = 0$. An external force affects the movement from $k = 350$ to $k = 370$.

the rest of the estimated states of the nodes, the minimum value of RMSE corresponds to either the value of the centralized filter or the individual filter, as an example, for node 2 the minimum RMSE for the first state corresponds to the individual filter and for the second state, to the individual filter, as shown in Fig 3 a) and b) respectively. Regardless of which, an appropriate value of λ will always yield the minimum RMSE.

4.2 Estimation error

Once λ is chosen, we run a new instance to compute the estimation error of both states for every node. Fig. 4 shows that, for the x coordinate of nodes 1 and 2 (Fig 4 a) and b) respectively), the consensus UFIR filter has better robustness in the occurrence of unexpected miss-model errors, as the estimator error is smaller. The error of nodes 3 and 4 for the x coordinate present a similar behavior. However, at some points, the consensus filter produces more errors than the centralized and the individual estimations. This behavior is due to the early definition of λ, which we assume to be diagonal and constant.

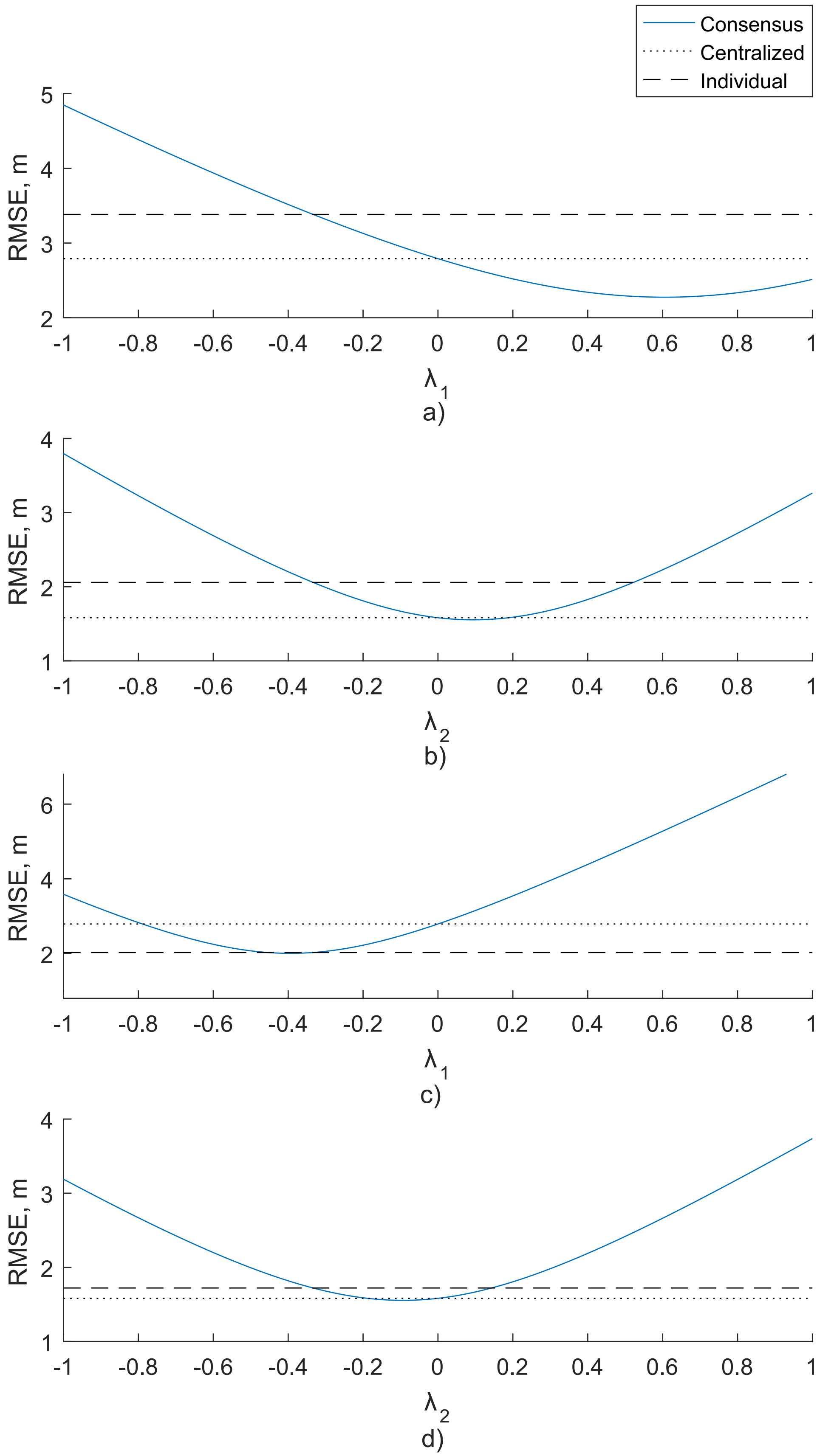

Figure 3: RMSE for x and y coordinate for different values of λ. a),b) corresponds to the x and y coordinate of node 1 respectively. c),d) corresponds to the x and y coordinate of node 2 respectively.

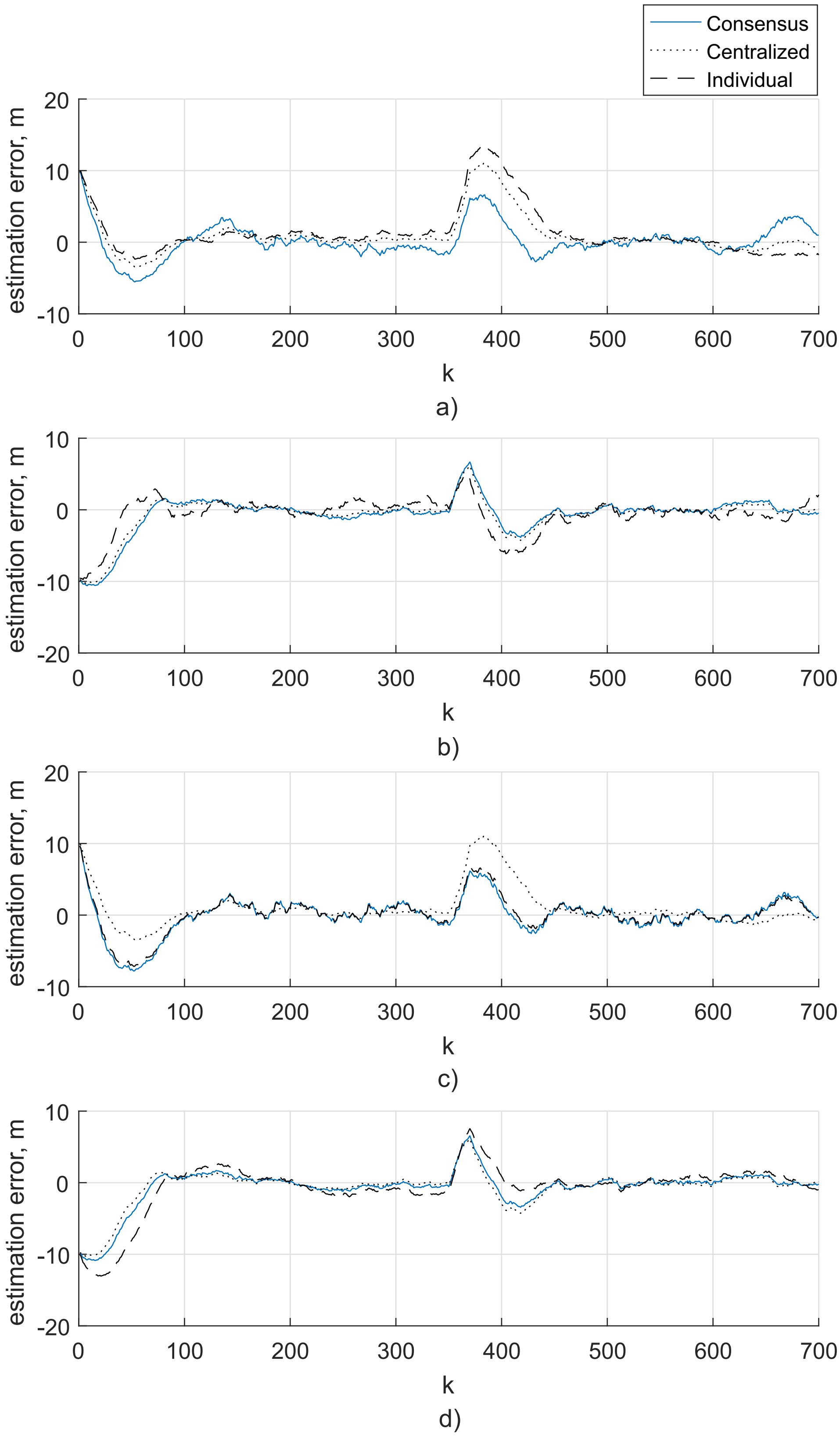

Figure 4: Estimation error for x and y coordinate for different values of λ. a),b) corresponds to the x and y coordinate of node 1 respectively. c),d) corresponds to the x and y coordinate of node 2 respectively.

5 Conclusion

The filter shows better robustness in the sense of the estimation error than the centralized and individual UFIR filters, and its iterative version is appropriate for its implementation in a smart sensor also, depending of the application, the transition matrix $H^{(j)}$ could be preloaded on the sensors, so the nodes should only transmit their measurements, which will have a positive impact on the battery life of the nodes.

A natural improvement for this filter will be to compute the optimum value of λ for every time index k as $\lambda_k = \arg\min_{\lambda_k} \{\operatorname{tr} P_k\}$, where $P_k = E\{(x_k - \hat{x_k}^{ic})(x_k - \hat{x_k}^{ic})^T\}$; this improvement is already being studied and will be presented in a near future, as well as the implementation on a network with a larger number of nodes.

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