An electrical analogy to compute general scenarios of soil consolidation

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Abstract: - The network method is revealed as a useful numerical technique to solve non-linear problems such as the soil consolidation process, ruled by diffusive equations in which the coefficients of the addends are complex functions of the dependent variable, the effective pressure. The equivalence between the physical and network model is established by introducing a formal analogy between the dependent and independent variables of the problem and the different elements or variable**s** of an electric circuit. Based on the network method, and making use of the powerful computational algorithms implemented in the circuit simulation codes, the numerical models proposed in this paper are solved, performing an analysis on the accuracy of the method, computing times and grid size required depending on the degree of non-linearity of the problem. Finally, from a series of illustrated applications, a comparison is made between the different models proposed, analyzing in depth the most general and precise of them.

Key-Words: - Numerical model, network method, soil consolidation

1 Introduction

Many problems in engineering, particularly most of those that are non-linear, can only be solved by numerical techniques. Non-linearities can arise due to different causes as, among other, the existence of not constant parameters whose relation with the dependent variable is approached by mathematical functions.

The advanced soil consolidation problem belongs to this class of problems since the unitary dependences of the parameters involved, void ratio and hydraulic conductivity, are proportional to the unitary changes in the effective stress, directly related with the excess pore water pressure, the dependent variable [1,2]. This means that these parameters are potential functions of the dependent variable. In addition, the expulsion of water from the porous domain involves the contraction of the soil and, as a consequence, a decrease in the size of the volume element adopted for the domain discretization. This makes the consolidation process to fit within the so-called moving-boundary problems, whose simulation is more complex [3].

In this paper a numerical model for the reliable simulation of non-linear consolidation problems is proposed. Its design, based on the network method [4,5], allows the implementation of a circuit whose mathematical model is equivalent to the finite-difference differential equations that result from the spatial discretization of the partial-differential equations of the consolidation problem; the time remains as a continuous variable in the network model.

The main advantage of using an electric model is that its simulation is carried out by means of the powerful mathematical algorithms implemented in the circuit solution codes such as NgSpice [6], necessary for the type of complex signals and very high frequencies involved in the electronic and communications devices. These codes generally provide the exact solution of the circuit, relegating errors to the mesh size imposed to the domain.

On the other hand, the design of the model requires the application of very few programming rules based on both the constitutive relations of linear electrical devices, such as resistors, capacitors and constant generators, and non-linear devices called controlled voltage or current generators. The latter, versatile enough to implement any type of non-linearity or coupling existing in the governing equations.

Each term or addend of the finite-difference differential equation is implemented in the network of the volume element or cell by a suitable device whose electric current is balanced with the currents of the other devices in a common node, according to the topology of the equation. Cells, in turn, are also coupled together by ideal electrical connections to reproduce the network of the whole domain. Finally, the boundary conditions are added by using the same kind of devices. Once the complete model is ready, the code provides its solution without further mathematical manipulation than the processing of the tabulated data from the simulation.

An analysis of the accuracy of the network model has been carried out, concluding that it is a numerical method that needs very few resources, with mesh grids that do not require a high number of cells and low computation times to reach very precise solutions, with errors below 1%.

A large number of applications have been solved with the aim of identifying and analyzing the main differences between the models proposed in this paper, illustrating in depth the most general and precise of them.

2 Nomenclature

C_i electric capacitor
c_v coefficient of consolidation (m^2/s)
$c_{v,1}$ initial coefficient of consolidation (m^2/s)
e void ratio (dimensionless)
e_o initial void ratio (dimensionless)
G_i controlled current source
H_1 initial thickness (m)
H_2 final thickness (m)
H_i instant thickness (m)
j_C electric current of a capacitor (A)
j_G electric current of a source (A)
k hydraulic conductivity (m/s)
k_1 initial hydraulic conductivity (m/s)
k_2 final hydraulic conductivity (m/s)
m_v coefficient of volumetric compressibility (m^2/N)
$m_{v,1}$ initial coefficient of volumetric compressibility (m^2/N)
q_o surface applied load (N/m^2)
S_∞ final settlement (m)
S_i instant settlement (m)
t time (s)
u excess pore water pressure (N/m^2)
$\bar{U}_s$ average degree of settlement (dimensionless)
V volume (m^3)
V_1 initial volume (m^3)
V_2 final volume (m^3)
z vertical spatial coordinate (m)
γ_k non-linear coefficient of change of permeability with effective pressure (dimensionless)
γ_v non-linear coefficient of compressibility (dimensionless)
γ_w specific weight of water (N/m^3)
σ' effective pressure (N/m^2)
σ'_1 initial effective pressure (N/m^2)
σ'_2 final effective pressure (N/m^2)

3 Governing equations

The physical scheme of the 1D soil consolidation process is shown in Fig.1. Due to the application of loads on the soil surface, the saturated porous of the soil gradually expel the water contained in its interstices reducing its thickness until reaching a stationary situation.

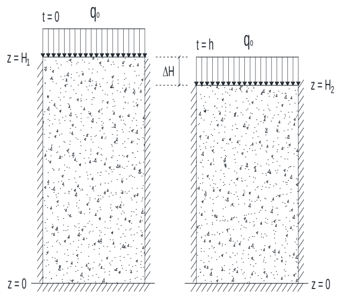

Figure 1. Physical scheme of 1D consolidation

The mathematical model of the non-linear soil consolidation problem, under the hypotheses of incompressibility in water and soil particles, is deduced from the water conservation balance in combination with the Darcy's flow equation

$$\frac{d\sigma'}{dt} = -\frac{1}{\gamma_w m_v}\frac{\partial}{\partial z}\left(k\frac{\partial u}{\partial z}\right) \qquad (1)$$

Assuming, on the one hand, negligible change in thickness and weight of sample water, as well as the hypotheses of the oedometer test

$$\frac{\partial u}{\partial z} = -\frac{\partial \sigma'}{\partial z} \quad \text{and} \quad \frac{\partial u}{\partial t} = -\frac{\partial \sigma'}{\partial t}$$

and, on the other hand, the most common dependences between soil volume, hydraulic conductivity and effective soil pressure, given by Juárez-Badillo [7]

$$\frac{dV}{V} = -\gamma_v \frac{d\sigma'}{\sigma'}$$

$$\frac{dk}{k} = -\gamma_k \frac{d\sigma'}{\sigma'}$$

whose integration provides

$$\frac{V_2}{V_1} = \frac{H_2}{H_1} = \left(\frac{\sigma'_2}{\sigma'_1}\right)^{-\gamma_v}$$

$$\frac{k_2}{k_1} = \left(\frac{\sigma'_2}{\sigma'_1}\right)^{-\gamma_k}$$

and introducing the coefficients and relations

$$\lambda = 1 - \gamma_k$$

$$\frac{k_1 \sigma'_1}{\gamma_w \gamma_v} = \frac{k_1}{\gamma_w m_{v,1}} = c_{v,1}$$

$$c_v = \frac{k}{\gamma_w m_v} = \frac{k_1 \sigma'_1}{\gamma_w \gamma_v}\left(\frac{\sigma'}{\sigma'_1}\right)^{1-\gamma_k} = c_{v,1}\left(\frac{\sigma'}{\sigma'_1}\right)^{\lambda}$$

equation (1) writes in the form

$$\frac{\partial \sigma'}{\partial t} = \sigma' c_{v,1} \frac{\partial}{\partial z}\left[\left(\frac{\sigma'}{\sigma'_1}\right)^{-\gamma_k} \frac{1}{\sigma'_1}\frac{\partial \sigma'}{\partial z}\right] \quad (2)$$

According to Fig.1, and assuming a negligible change in thickness, the following boundary and initial conditions complete the mathematical model:

$$\frac{\partial u}{\partial z}_{(z=0,t)} = 0 \quad \text{(Impervious bottom edge)}$$

$$u_{(z=H_1,t)} = 0 \quad \text{(Upper free drainage)}$$

The model has no analytical solutions except for the case λ=0 [7].

A less restrictive variant that assumes the initial value of the void ratio no negligible leads [8] to

$$\frac{\partial \sigma'}{\partial t} = \frac{(1+e_o)\sigma'_1 k_1}{\gamma_w \gamma_v}\left(\frac{\sigma'}{\sigma'_1}\right)^{1-\gamma_k}\left[-\frac{\gamma_k}{\sigma'}\left(\frac{\partial \sigma'}{\partial z}\right)^2 + \frac{\partial^2 \sigma'}{\partial z^2}\right] \quad (3)$$

an equation nearly the same that (2) except for the factor (1+e_o).

4 The network model

The equivalence between the physical and network model is established by introducing a formal analogy between the dependent and independent variables of the problem and the different elements or variables of an electric circuit: the excess pore pressure (or the effective pressure) is assumed to be the voltage quantity of the model while the flow of water is the electric current.

Developing the equation (2), we have

$$\frac{1}{\sigma'}\frac{\partial \sigma'}{\partial t} = -\frac{c_{v,1}\gamma_k}{(\sigma'_1)^2}\left(\frac{\sigma'}{\sigma'_1}\right)^{-\gamma_k - 1}\left(\frac{\partial \sigma'}{\partial z}\right)^2 + \frac{c_{v,1}}{\sigma'_1}\left(\frac{\sigma'}{\sigma'_1}\right)^{-\gamma_k}\frac{\partial^2 \sigma'}{\partial z^2} \quad (4)$$

an equation that, following the nomenclature of Fig.2, can be expressed in finite-difference form as

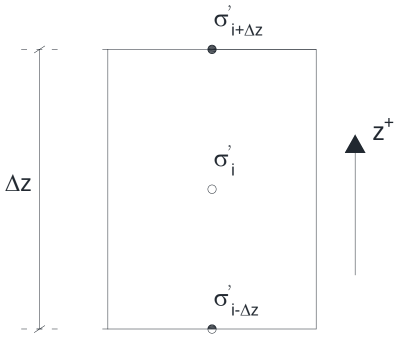

Figure 2. Nomenclature of the 1D element cell

$$\frac{\partial \sigma'}{\partial t} = \left(\frac{\sigma'_i}{\sigma'_1}\right)^{-\gamma_k+1}\left[\frac{\sigma'_{i+\Delta z} - \sigma'_i}{\frac{(\Delta z)^2}{2c_{v,1}}}\right] - \left(\frac{\sigma'_i}{\sigma'_1}\right)^{-\gamma_k+1}\left[\frac{\sigma'_i - \sigma'_{i-\Delta z}}{\frac{(\Delta z)^2}{2c_{v,1}}}\right] - \\ -\frac{c_{v,1}\gamma_k}{\sigma'_1}\left(\frac{\sigma'_i}{\sigma'_1}\right)^{-\gamma_k}\left[\frac{(\sigma'_{i+\Delta z} - \sigma'_{i-\Delta z})^2}{(\Delta z)^2}\right] \quad (5)$$

In the network model the four terms of the above equation define the following electric currents

$$\left.\begin{array}{l} j_C = \frac{\partial \sigma'}{\partial t}, \quad j_{G+\Delta z} = \left(\frac{\sigma'_i}{\sigma'_1}\right)^{1-\gamma_k}\left[\frac{\sigma'_{i+\Delta z} - \sigma'_i}{\frac{(\Delta z)^2}{2c_{v,1}}}\right] \\ j_{G-\Delta z} = \left(\frac{\sigma'_i}{\sigma'_1}\right)^{1-\gamma_k}\left[\frac{\sigma'_i - \sigma'_{i-\Delta z}}{\frac{(\Delta z)^2}{2c_{v,1}}}\right] \\ j_G = \frac{c_{v,1}\gamma_k}{\sigma'_1}\left(\frac{\sigma'_i}{\sigma'_1}\right)^{-\gamma_k}\left[\frac{(\sigma'_{i+\Delta z} - \sigma'_{i-\Delta z})^2}{(\Delta z)^2}\right] \end{array}\right\} \quad (6)$$

that are balanced in a common node since $j_C = j_{G+\Delta z} - j_{G-\Delta z} + j_G$.

The only linear term of (5) is implemented by a capacitor of capacitance unity (C_i) while the rest terms are by current-sources controlled by voltage. The outputs of these sources are given by (6) and the values of dependent variables in each cell are read at the common nodes of the cell, Fig.2. Thus, the network model (Fig.3) has three sources, G_i, $G_{i-\Delta}$ and $G_{i+\Delta}$, related to the currents j_G, $j_{G-\Delta z}$ y $j_{G+\Delta z}$, respectively. The control nodes of each source are indicated in Fig.3 next to the sources. The resistors in parallel with each source do not play any roll but their implementation is generally required to check the behavior of the model under steady condition.

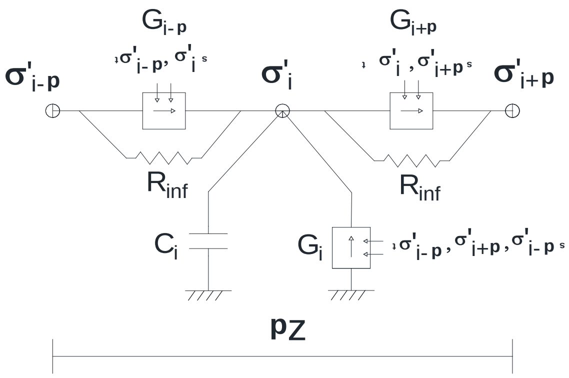

Figure 3. Network model of the volume element

The network model for the equation (3) is nearly the same except for the specification of the output sources G_i, $G_{i-\Delta}$ and $G_{i+\Delta}$; since these currents are implemented by software, the only change is easily made. The currents needed to reproduce to equation (3) are

$$\left.\begin{aligned} j_C = \frac{\partial \sigma'}{\partial t}, \quad j_{G+\Delta z} &= \left(\frac{\sigma'_i}{\sigma'_1}\right)^{1-\gamma_k}\left[\frac{\sigma'_{i+\Delta z}-\sigma'_i}{\left(\frac{(\Delta z)^2}{2c_{v,1}(1+e_o)}\right)}\right] \\ j_{G-\Delta z} &= \left(\frac{\sigma'_i}{\sigma'_1}\right)^{1-\gamma_k}\left[\frac{\sigma'_i-\sigma'_{i-\Delta z}}{\left(\frac{(\Delta z)^2}{2c_{v,1}(1+e_o)}\right)}\right] \\ j_G &= \frac{c_{v,1}\gamma_k}{\sigma'_1}\left(\frac{\sigma'_i}{\sigma'_1}\right)^{-\gamma_k}\left[\frac{\left(\sigma'_{i+\Delta z}-\sigma'_{i-\Delta z}\right)^2}{\frac{(\Delta z)^2}{(1+e_o)}}\right] \end{aligned}\right\} \quad (7)$$

whereas the scheme of the network model coincides with that of the model of equation (2), Fig.3.

5 Network model precision. Mesh grid and calculation time

The network model is the format given to the mathematical model so that it can be used as an input (file) in an electrical circuit resolution program such as NgSpice [6], which solves the network equations and provides the numerical solution of the mathematical model. Obviously, in order to design and program the circuit to be solved, one must have elementary (not necessarily deep) knowledge of circuit theory, but once these fundamentals are mastered, the use of NgSpice is relatively simple.

NgSpice has a series of algorithms that guarantee the convergence of the circuit, as long as it is well designed, ensuring that the numerical response as a result of the simulation is the exact solution of the circuit or model, reducing errors to the grid size of the circuit and the time step chosen for the output of results. With regard to the latter, its influence is reduced only to the bigger or smaller error of interpolation that NgSpice makes when calculating the numerical value of the solution of the problem for a given time of which there is no computed solution. Since NgSpice performs a linear type interpolation, the greater the number of steps used in the computation – which implies a smaller difference of time between two consecutive points of the solution – the smaller the error committed in the interpolation. After numerous simulations and checks, it is estimated that, whatever the duration of the transient consolidation process, it is enough to divide the temporal domain in about 150-500 steps. Below this range, obviously, errors are made in the interpolation (in addition that the convergence of the solution can not be reached), while unnecessary calculations are being carried out above (being also possible to experience difficulties with the convergence of the problem).

On the other hand, the size of the mesh grid (or the number of cells) is a much more influential parameter in reaching a high precision in the solution of the model. In general, the network method is a numerical technique that achieves both the convergence and the accuracy of the solution with a relatively low amount of computing resources, compared to other methods. Thus, in analogous linear problems [8] to that presented in this paper (consolidation in 1-D coordinates), convergence is achieved with a very little number of cells (10 is enough), low computation times – below 5 seconds on any computer with an Intel Core i3 processor or higher – and a high precision in the solution, with relative errors below 10% for the local degree of consolidation (percentage of consolidation achieved in a given position of the domain) and below 2% for the average degree of settlement (percentage of settlement reached on the soil surface). For this type of problems, it has been verified [4, 8] that grids above 50 cells provide very precise solutions whatever the variable in study, in which the maximum relative error does not exceed 0.1%, in any case. Likewise, this can be extended for 2-D and 3-D models, where solutions with negligible errors are guaranteed provided that the number of cells in each spatial direction is above 50.

Non-linear models, on the other hand, require more demanding grids, also needing higher computation times. In this type of problems, the degree of non-linearity is a very important factor, so that as the non-linearity is greater both the number of cells to be used and the computation time increase. However, numerous tests and simulations performed both for this paper and for other research [8] have proved that the need to use smaller grids is only to achieve the convergence of the problem, since, once it has been reached - in the network method -, the accuracy of the solution is guaranteed, with negligible relative errors.

Tables 1-3 present a series of consolidation scenarios, conveniently grouped, which have been simulated using the network method (in the variant of the JB model, equations (3) and (7), the most general and accurate of the models presented here). The objective of the simulations is to study the influence of the different parameters that characterize the process and on which the greater or lesser non-linearity of the problem depends. i) the ratio final/initial effective pressure, σ'_2/σ'_1, ii) the non-linear coefficient of compressibility, γ_v, and iii) the non-linear coefficient of change of permeability with effective pressure, γ_k. In view of the constitutive equations given by Juarez-Badillo [7], the consolidation problem defined in equation (3) presents a more pronounced non-linearity as the three previous parameters increase. In order not to incur unnecessary results or conclusions, or devoid of physical meaning, the study has focused on those consolidation scenarios that can be found in practice; for cohesive soils with consolidation phenomena, the ranges of variation of the values of the three analyzed parameters are: i) σ'_2/σ'_1 between 1 (limit value at which the surface applied load is zero) and 8, ii) γ_v between 0.02 and 0.3, and iii) γ_k between 0.1 and 3. Moreover, for a better comparison between results, the remaining parameters of the problem – on which the dependence is linear – have been kept constant: H_1 = 1 m, e_o = 1, σ'_1 = 30.000 N/m^2 and k_1 = 0.02 m/s.

Each of the tables studies the influence of one of these three parameters separately, keeping the values of the other two constant. All of them show both the number of cells in which it has been necessary to divide the domain to reach the convergence of the problem and the time spent in the calculation of the network model (including the representation of results). The same number of time steps (300) has been chosen for all the simulations. The calculations have been carried out with a personal computer with processor Intel Core i7-4770 CPU 3.40 GHz.

Table 1 reflects the influence of the ratio σ'_2/σ'_1 for a range of values between 2 and 32 (the latter a possible value but far above those that are given in practice) and for the usual fixed values of γ_v = 0.1 and γ_k = 1.

γ_v	γ_k	σ'_2/σ'_1	minimum cells number	time (s)
0.1	1	2	10	1
0.1	1	4	80	6
0.1	1	6	110	9
0.1	1	8	120	10
0.1	1	12	130	10
0.1	1	16	140	11
0.1	1	24	150	12
0.1	1	32	200	14

Table 1. Minimum number of cells and calculation time. Influence of the ratio σ'_2/σ'_1

In view of the simulations performed, it is observed that as the ratio σ'_2/σ'_1 increases, it is necessary to use a smaller grid size, increasing the calculation time. Indeed, and in relation to what has been commented above, this happens as the non-linearity of the problem increases. However, the power and versatility of the network method is reflected in the fact that to solve a consolidation scenario with an effective pressure increase of 32 times with respect to its initial value it is only necessary to use a grid with 200 cells and a computation time of 14 seconds (Table 1), for the usual values γ_v = 0.1 and γ_k = 1.

The influence of γ_v is shown in Table 2. For this, the ratio σ'_2/σ'_1 has been set to 4 (very common in practice), whereas for the coefficient γ_k it has been chosen the usual value of 1. The values for γ_v range from 0.01-0.5. In view of the results obtained, similar conclusions to those mentioned for the influence of σ'_2/σ'_1 can be made in this case.

γ_v	γ_k	σ'_2/σ'_1	minimum cells number	time (s)
0.01	1	4	30	2
0.02	1	4	40	4
0.05	1	4	60	5
0.10	1	4	80	6
0.20	1	4	120	10
0.35	1	4	150	12
0.50	1	4	180	13

Table 2. Minimum number of cells and calculation time. Influence of the coefficient γ_v

Finally, Table 3 shows how the variation of γ_k affects the convergence of the model. This parameter, despite having a very similar appearance to γ_v (coefficient that governs a potential type dependence with the effective pressure), presents a much greater range of variation for its usual values (0.1-3 for γ_k, 0.02-0.3 for γ_v). For this reason, consolidation scenarios that require smaller grids and higher computation times have been found, as the value of γ_k rises (increasing, in a marked way, the non-linearity), Table 3. However, even for the less favorable case (γ_k = 3), requiring 5,000 cells to reach the solution of the problem, the total calculation time does not exceed 5 minutes (including the representation of results).

γ_v	γ_k	σ'_2/σ'_1	minimum cells number	time (s)
0.1	0.3	4	10	1
0.1	0.5	4	10	1
0.1	0.7	4	40	4
0.1	1	4	80	6
0.1	1.5	4	170	13
0.1	2	4	400	29
0.1	2.5	4	1,500	85
0.1	3	4	5,000	290

Table 3. Minimum number of cells and calculation time. Influence of the coefficient γ_k

With all the above, it can be concluded that the network method is presented as a powerful numerical technique, with which it is possible to solve the non-linear consolidation problem in a relatively low time and with practically negligible errors. Thus, the most unfavorable consolidation scenario that could be found in practice would be solved in a time of the order of 20 minutes, using a grid of approximately 10,000-15,000 cells and ensuring maximum relative errors below 0.1%.

6 Applications

6.1 Analysis of proposed models

In this section, two consolidation scenarios are illustrated, with the aim of comparing and analyzing the results obtained with the different network models designed. For each of the two examples, different values of the parameters have been given to the model: γ_v, γ_k, e_o, H_1 (m), σ'_1 (N/m^2), σ'_2 (N/m^2) and k_1 (m/year) for the non-linear models, equations (2) and (3), JB model and JB variant model respectively, and $c_{v,1}$ (m^2/year) for the well-known linear case [9], Table 4.

set	γ_v	γ_k	e_o	H_1	σ'_1	σ'_2	k_1	$c_{v,1}$
1	0.1	0.5	1	1	30000	60000	0.02	0.61
2	0.1	0.5	3	2	30000	35000	0.04	1.22

Table 4. Physical and geometrical soil parameters. Sets 1 and 2

Fig.4 and Fig.5 show the evolution of the average degree of settlement ($\bar{U}_s$) over time. This important function represents the percentage of settlement achieved in a given time, as the ratio between the instant settlement and the final settlement reached at the end of the consolidation process:

$$\bar{U}_s = \frac{S_i}{S_\infty} = \frac{H_i - H_1}{H_2 - H_1} \quad (8)$$

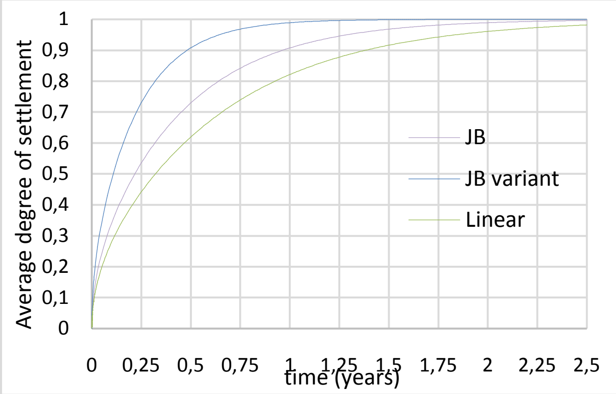

Figure 4. $\bar{U}_s$ as a function of time. Set 1

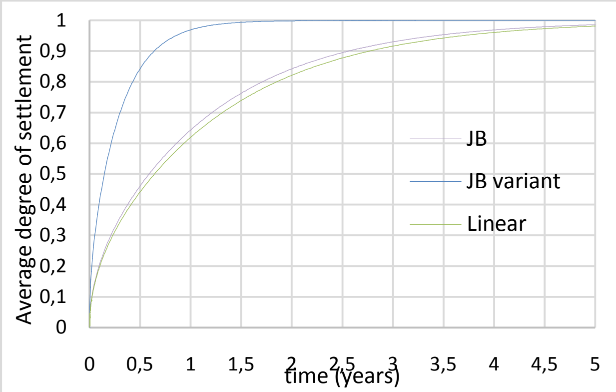

Figure 5. $\bar{U}_s$ as a function of time. Set 2

In both figures it can be observed that the non-linear models report smaller consolidation times in comparison with the solution of the linear model. On the other hand, the JB variant model, which takes into account the value of the initial void ratio (e_o), reports smaller times than the JB model, the smaller the higher e_o, Fig.4 and Fig.5. This fact becomes even more evident in the set 2 (Fig.5), in which when a small loading step is applied, the JB model and the linear one give almost the same solution, whereas the JB variant model does not. This is because, by its formulation, the JB model tends to be a linear model as the load step is smaller.

All this means, therefore, that the JB variant model is much more precise in its solution, giving a value, sometimes, much lower of the consolidation time, which can imply important advantages and cost savings in the execution of an engineering work.

6.2 Analysis of the JB variant model

In view of the results obtained in the previous section, we proceed to analyze the dependences and variations of the consolidation problem according to the different soil parameters, for the JB variant model, the most general and precise consolidation model presented here. To illustrate the problem conveniently, a total of 16 new consolidation scenarios are shown, grouped conveniently to facilitate understanding and discussion.

The first group focuses on the analysis of the linear part of the consolidation model. It is composed of 5 simulations: set 1, from the previous section, and sets 3-6, Table 5 and Fig.6. In them, the values of e_o, H_1 and k_1 are varied, verifying that the dependence of the model on these parameters is linear. Indeed, if we take as a reference the set 1 and duplicate the initial hydraulic conductivity k_1, keeping the rest of parameters constant (set 3), the consolidation process accelerates, requiring half the time to reach the same average degree of settlement ($\bar{U}_s$). In this way, it follows that the process velocity is inversely proportional to the value of k_1. When H_1 is duplicated (set 4) the process slows down, requiring quadruple of the time to reach the same $\bar{U}_s$ (in this case, the duration of the process is directly proportional to the square of H_1). As for the initial void ratio, e_o, there is also an inversely proportional dependence, this time with respect to the factor ($1+e_o$), set 5. Finally, set 6 is presented to illustrate that the consolidation process remains invariant in time when the changes in these three factors are compensated. With this group of simulations it is highlighted the importance of this group of parameters, higher – in the opinion of the authors – than that of the usually used coefficient of consolidation (c_v), since the interpretation and information they give is much more direct.

set	γ_v	γ_k	e_o	H_1	σ_1'	σ_2'	k_1	$c_{v,1}$
1	0.1	0.5	1.00	1.0	30000	60000	0.02	0.61
3	0.1	0.5	1.00	1.0	30000	60000	0.04	1.22
4	0.1	0.5	1.00	2.0	30000	60000	0.02	0.61
5	0.1	0.5	0.33	1.0	30000	60000	0.02	0.61
6	0.1	0.5	2.00	1.5	30000	60000	0.03	0.92

Table 5. Physical and geometrical soil parameters. Set 1 and sets 3-6

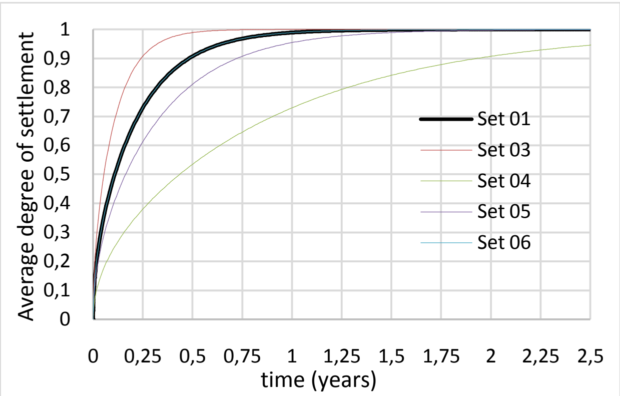

Figure 6. $\bar{U}_s$ as a function of time. JB variant model. Set 1 and sets 3-6

The second group, formed by set 1 and sets 7 and 8, Table 6 and Fig.7, aims to illustrate the influence of the coefficient γ_v in the consolidation process, keeping constant the rest of parameters. It is important to note, at this point, that the comparison has been made for the values of γ_k = 0.5 and σ'_2/σ'_1 = 2. With the results obtained (Fig.7), it follows that the consolidation process is slowed down as the value of the coefficient γ_v rises (increasing soil compressibility), for the values of γ_k and σ'_2/σ'_1 indicated.

set	γ_v	γ_k	e_o	H_1	σ_1'	σ_2'	k_1	$c_{v,1}$
1	0.1	0.5	1	1	30000	60000	0.02	0.61
7	0.2	0.5	1	1	30000	60000	0.02	0.31
8	0.4	0.5	1	1	30000	60000	0.02	0.15

Table 6. Physical and geometrical soil parameters. Set 1 and sets 7-8

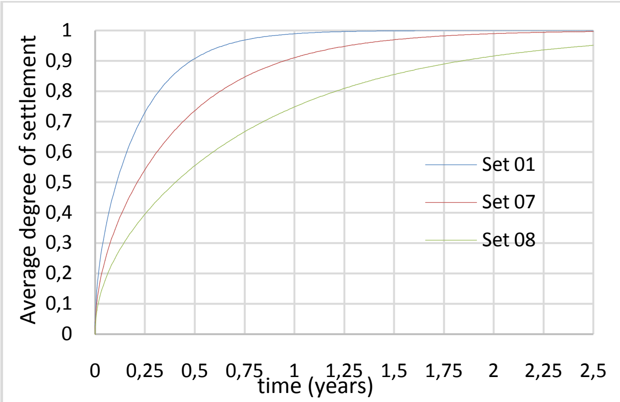

Figure 7. $\bar{U}_s$ as a function of time. JB variant model. Set 1 and sets 7-8

Table 7 and Fig.8 show a third group of simulations (sets 1, 9 and 10) in which the influence of the coefficient γ_k is studied. It is important to highlight an important aspect of the physical meaning of this parameter (in view of the constitutive relations proposed by Juarez-Badillo [7]) and that justifies the values assigned to γ_k (0.5, 1 and 1.5) in the simulations carried out in this group. For values of γ_k < 1, the reduction of the hydraulic conductivity (in relation to its initial value) is smaller than the increase of effective pressure (in relation to its initial value) carried out along the consolidation process; that is to say, $k_2/k_1 > \sigma'_1/\sigma'_2$. For γ_k = 1 is fulfilled $k_2/k_1 = \sigma'_1/\sigma'_2$, whereas for values γ_k > 1 is verified $k_2/k_1 < \sigma'_1/\sigma'_2$. For the latter case, the reduction on the hydraulic conductivity turns much more pronounced, compared to the effective pressure increase. For all this, it is obvious that the consolidation process slows down as γ_k increases, for a same value of the ratio σ'_2/σ'_1 (Fig.8).

set	γ_v	γ_k	e_o	H_1	σ'_1	σ'_2	k_1	$c_{v,1}$
1	0.1	0.5	1	1	30000	60000	0.02	0.61
9	0.1	1.0	1	1	30000	60000	0.02	0.61
10	0.1	1.5	1	1	30000	60000	0.02	0.61

Table 7. Physical and geometrical soil parameters. Set 1 and sets 9-10

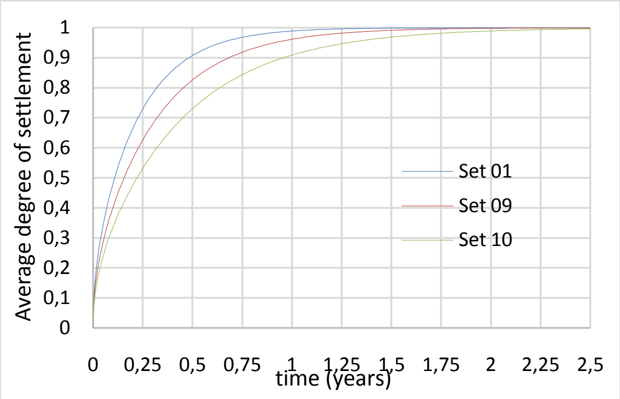

Figure 8. $\bar{U}_s$ as a function of time. JB variant model. Set 1 and sets 9-10

In the fourth group, formed by sets 10-12, Table 8, we re-analyze the influence of the coefficient γ_v, as already done in the second group, but this time for $\gamma_k = 1.5$. The ratio σ'_2/σ'_1 takes again a value of 2. Once the simulations are analyzed, Fig.9, the same conclusions as for the case $\gamma_k = 0.5$ are obtained: the consolidation process takes longer as γ_v increases.

set	γ_v	γ_k	e_o	H_1	σ'_1	σ'_2	k_1	$c_{v,1}$
10	0.1	1.5	1	1	30000	60000	0.02	0.61
11	0.2	1.5	1	1	30000	60000	0.02	0.31
12	0.4	1.5	1	1	30000	60000	0.02	0.15

Table 8. Physical and geometrical soil parameters. Sets 10-12

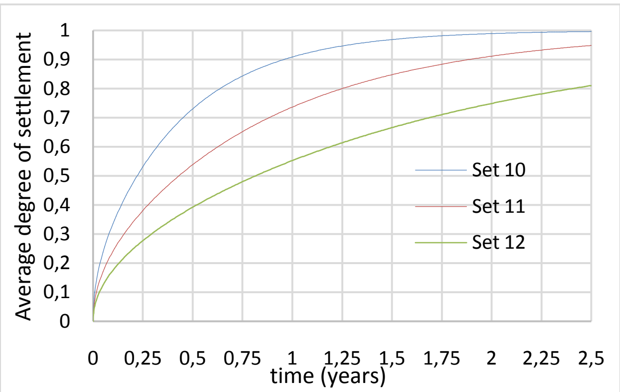

Figure 9. $\bar{U}_s$ as a function of time. JB variant model. Sets 10-12

Finally, to carry out the study of the influence of the factor σ'_2/σ'_1, three groups of simulations are presented, each corresponding to a value of γ_k (0.5, 1 and 1.5, respectively). In all of them, γ_v has been set at 0.1, while the ratio σ'_2/σ'_1 takes the values of 2, 4 and 8. Thus, for the case $\gamma_k < 1$ (fifth group of this section, Table 9) it is observed that the consolidation process accelerates as the value of σ'_2/σ'_1 increases, Fig.10. When γ_k takes the value of 1 (Table 10), the consolidation process keeps its duration constant, regardless of the value of σ'_2/σ'_1, Fig.11. Finally, for values $\gamma_k > 1$, Table 11, the consolidation process is delayed in time as we increase σ'_2/σ'_1, Fig.12.

set	γ_v	γ_k	e_o	H_1	σ'_1	σ'_2	k_1	$c_{v,1}$
1	0.1	0.5	1	1	30000	60000	0.02	0.61
13	0.1	0.5	1	1	30000	120000	0.02	0.61
14	0.1	0.5	1	1	30000	240000	0.02	0.61

Table 9. Physical and geometrical soil parameters. Set 1 and sets 13-14

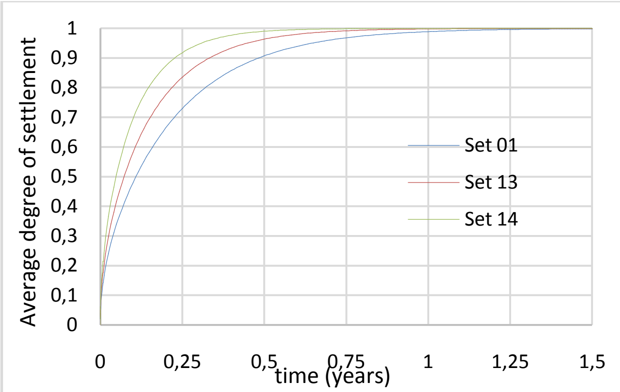

Figure 10. $\bar{U}_s$ as a function of time. JB variant model. Set 1 and sets 13-14

set	γ_v	γ_k	e_o	H_1	σ'_1	σ'_2	k_1	$c_{v,1}$
9	0.1	1	1	1	30000	60000	0.02	0.61
15	0.1	1	1	1	30000	120000	0.02	0.61
16	0.1	1	1	1	30000	240000	0.02	0.61

Table 10. Physical and geometrical soil parameters. Set 9 and sets 15-16

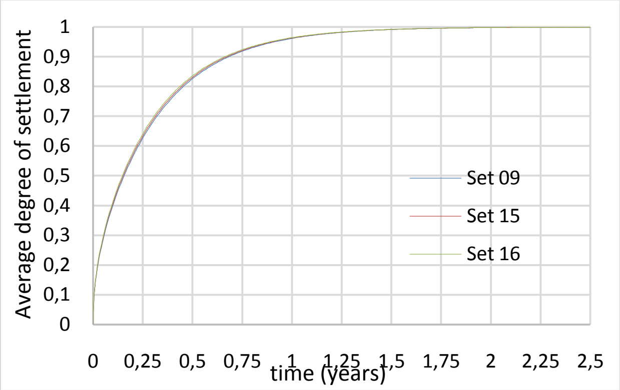

Figure 11. $\bar{U}_s$ as a function of time. JB variant model. Set 9 and sets 15-16

set	γ_v	γ_k	e_o	H_1	σ_1'	σ_2'	k_1	$c_{v,1}$
10	0.1	1.5	1	1	30000	60000	0.02	0.61
17	0.1	1.5	1	1	30000	120000	0.02	0.61
18	0.1	1.5	1	1	30000	240000	0.02	0.61

Table 11. Physical and geometrical soil parameters. Set 10 and sets 17-18

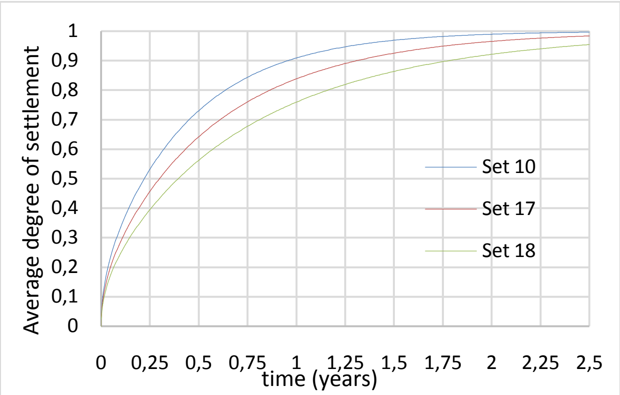

Figure 12. $\bar{U}_s$ as a function of time. JB variant model. Set 10 and sets 17-18

As a summary of all this, we can affirm that the consolidation process ruled by the JB variant model is delayed in time as long as the values of γ_v and γ_k increase. However, in relation to the rise of σ'_2/σ'_1, the behavior of the process (in terms of duration) is influenced by the value of γ_k, so that when $\gamma_k < 1$, the process is accelerated by the increase of σ'_2/σ'_1, whereas for values of $\gamma_k > 1$ the opposite happens.

All the simulations described in this section have been solved with the network method, using in each case a sufficient grid (> 200 cells) to guarantee a high precision in the solution of the consolidation problem, and with 300 time steps for all of them.

7 Conclusion

The network method has proven to be a versatile tool for the design of network models in non-linear problems of soil consolidation. Each term of the equation is assumed to be an electric current that balances with the others in a common node in such a way that the solution – given by a circuit simulation code – provides the currents and voltages that satisfy this condition. The non-linear terms of the equation are implemented, regardless of the type of functions involved within them, by a special kind of component called controlled current source. The output of this source – or the current to which the term is related – is specified by software and may depend on functions whose arguments are the currents at other components or the voltage at other nodes of the circuit. Thus, non-linear or coupled terms may be implement in the model in a direct way by a same type of component.

The complete model extends the volume element between adjacent cells by ideal electrical contacts, to cover the complete geometry of the domain. Finally, the boundary conditions, whatever the functions that define them, are also implemented by controlled sources or other simple components. In short, very few rules are required for the design of the soil consolidation non-linear problem.

The accuracy of the model has been analyzed, confirming that both a higher number of cells and longer computing times are required as the non-linearity of the model increases. However, even in the most unfavorable scenarios, the resolution of the model is achieved with a relatively low amount of computing resources, requiring a maximum of 20 minutes to find reliably the solution of the problem.

A considerable number of real cases have been solved and analyzed with the aim of illustrating the non-linear soil consolidation process. For the resolution of the electric circuit the free version of the NgSpice code simulation has been used.

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