Fuzzy Internal Model Control of Nonlinear Plants with Time Delay based on Parallel Distributed Compensation

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Abstract: The parallel-distributed compensation (PDC) together with Takagi-Sugeno-Kang (TSK) fuzzy modelling proved to be a modern model-based systematic approach for the design of fuzzy nonlinear controllers from stability and robustness requirements. The usual local linear controllers comprise a state feedback. This requires measurements of the state variables or an observer design. The aim of the present investigation is to develop a fuzzy internal model controller (FIMC) for a nonlinear plant with time delay under uncertainties. The FIMC is PDC-TSK based with dynamic local linear controllers, which are designed on the principle of the internal model controllers to ensure local linear systems stability and robustness. The main contribution of the paper is a method for the design of a FIMC, designed FIMC for the control of the air temperature in a laboratory furnace and MATLAB based simulation investigations of the closed loop system stability and dynamic performance. The FIMC reduces the settling time and the overshoot and improves the system robustness compared to a linear PI controller.

Key-Words: - Fuzzy internal model control, MATLAB, robustness, simulation, stability, temperature, time delay

1 Introduction

The recent advances in fuzzy control are based on dynamic fuzzy Takagi-Sugeno-Kang (TSK) plant and controller models with common premises in the fuzzy rules [1-3]. This approach, known as Parallel Distributed Compensation (PDC), is systematic with emphasis on system stability and robustness with performance restrictions. It takes advantage of the well-developed linear control technique for designing of local controllers and solving the global nonlinear fuzzy system stability. Most industrial processes are inertial complex nonlinear time-varying plants with time delay and model uncertainty [4-6]. Often the nonlinear plant can be represented by a finite number of linear models, each for a given operation sub-domain. In this case the fuzzy PDC-TSK has proven to be a suitable advanced and also simple approach for the control of such plants as it accounts for the plant time delay, nonlinearity, uncertainty and complexity in satisfying the high performance demands to the control system [1-5]. The PDC controllers have only a few rules – one for each plant model. Because of the nonlinear nature both of plant and controller, stability and robustness are essential for the practical feasibility of the designed control system and also difficult to ensure [7-12].

In the PDC structure the fuzzy logic controller design problem is decomposed into local linear controllers design and global fuzzy nonlinear system stability ensuring. The local controllers are developed first from the requirement to guarantee local linear systems stability and robustness. Then the global fuzzy nonlinear system stability is studied, employing Lyapunov stability direct method and Linear Matrix Inequalities (LMIs) numerical technique [1, 2] for solving the Lyapunov stability conditions. The usual local controllers comprise a state feedback. In case the state variables are not measurable, a fuzzy PDC state-variable observer is developed. The design problem becomes complicated. This tendency deepens even more when plant time-delay and plant model uncertainties are to be considered [2-5, 7-12] – the local controllers design becomes difficult and the Lyapunov global system stability problem turns into a computationally hard and even impossible task because of the increased in number and complexity LMIs to be solved.

The design of robust stable linear local control systems for local plants with time delay and uncertainty can be greatly simplified applying the internal model controller (IMC) approach [13, 14]. It often results in equivalent PI/PID controllers, which allows to successfully apply the derived in [15, 16] global system stability conditions.

The aim of the present investigation is to develop a method for the design of a fuzzy internal model controller (FIMC) for a nonlinear plant with time delay under uncertainties. The FIMC is PDC-TSK based with dynamic local controllers, which are derived on the principle of linear internal model controllers to ensure local linear systems stability and robustness.

2 Problem Statement

In [15-17] a simple approach for the design of dynamic PI/PID local controllers for local plants with time delay and model uncertainties is suggested and the corresponding Lyapunov sufficient conditions for the global system stability are derived.

The PDC approach assumes that the nonlinear plant can be described by a number of linear plant models usually in the state space, obtained either by linearisation of a known nonlinear plant model in several operation points (applicable mainly for mechanical systems) or as a result of experiments and identification. The linear plant models are supposed to be observable and controllable. They constitute the consequents in the fuzzy rules of the TSK plant model.

The proposed TSK model derivation for industrial processes with time delay in [15-17] is based on identification. Step responses to plant input changes in different operation points are experimentally recorded and approximated by Ziegler-Nichols models. Then models with close parameters from adjacent step responses are grouped to determine sub-domains of linearisation, represented by average Ziegler-Nichols models with transfer functions $P_i^o(s) = K_i^o.e^{-\tau_i^o s}.(T_i^o s+1)^{-1}$, accepted as nominal local linear plants. Due to the smooth plant nonlinearity the plant model gain K, time constant T and time delay τ vary with the operation point. So, K_i, T_i and τ_i are different in each linear sub-domain. The sub-domains can be recognized by the plant output $y(t)$ or its reference $y_r(t)$. When under closed loop control, the plant output follows the reference y_r and smoothly passes through all sub-domains from the current to the final. So, the nonlinear plant changes its parameters. These parameter variations can be described in a multiplicative uncertainty model for each sub-domain $l_i(s) = \Delta P_i(s)/P_i^o(s)$, where the additive uncertainty $\Delta P_i(s) = P_i^o(s) - P_{wi}(s)$ is determined on the basis of the "worst" perturbed plant $P_{wi}(s)$ for the sub-domain – collective virtual plant with the greatest gain $K_{wi}=K_{imax}$ and time delay $\tau_{wi}=\tau_{imax}$, and the smallest time constant $T_{wi}=T_{imin}$, $i=1\ldots r$, with worst effect on system stability.

The local linear controllers for the linear plants in the sub-domains are dynamic PI/PID controllers with transfer functions $C_i(s) = K_{pi}.[1+1/(T_{ii}s)]$ with gain K_{pi} and integral action time T_{ii} as tuning parameters. Each control action is presented as incremental in state space form in the consequents of the corresponding fuzzy rule of the PDC fuzzy controller. The necessary integrator for the incremental control is included as augmentation of the local plant at its input. Thus the fuzzy rules in the TSK models of the plant and the controller respectively for $i=1\ldots r$ are determined as:

R$_i$: IF $y(t)$ is M_{i1} **AND** $e(t)$ is M_{i2} **AND** $\dot{e}(t)$ is M_{i3}

THEN. $$\left| \begin{array}{l} \dot{x}_i(t) = A_{io}x_i(t) + B_{id}\dot{u}_i(t-\tau_i^o) \\ y_i(t) = C_i x_i(t) \end{array} \right. \quad (1)$$

R$_i$: IF $y(t)$ is M_{i1} **AND** $e(t)$ is M_{i2} **AND** $\dot{e}(t)$ is M_{i3}

THEN $\dot{u}_i(t) = -F_i x_i(t) + G_i x_r$ (2)

or $\dot{u}_i(t) = K_{pi}\dot{e}(t) + (K_{pi}/T_{ii})e(t)$,

where: M_{ij} are linguistic values, defined as membership function (MF) of fuzzy sets; $x(t)\in\mathbf{R^n}$ is the state vector; $u(t)\in\mathbf{R^m}$ is the input control vector; $y(t)\in\mathbf{R^q}$ is the output vector; $e_i(t) = y_r - y_i(t)$ is the error in the local closed loop system for constant reference y_r;

$$x_i(t) = \begin{bmatrix} x_{i1}(t) = y(t) \\ x_{i2}(t) = \dot{x}_{i1}(t) \end{bmatrix};\ x_r = \begin{bmatrix} x_{r1} = y_r \\ x_{r2} = 0 \end{bmatrix};$$

$$A_{io} = \begin{bmatrix} 0 & 1 \\ 0 & -1/T_i^o \end{bmatrix};\ B_{id} = \begin{bmatrix} 0 \\ K_i^o/T_i^o \end{bmatrix};\ C_i = [1 \quad 0];$$

$F_i = [K_{pi}/T_{ii}^o \quad K_{pi}]$ and $G_i = [K_{pi}/T_{ii}^o \quad 0]$.

The PDC fuzzy controller has three inputs $y(t)$, $e(t)$ and $\dot{e}(t)$ - $z^T=[y\ e\ \dot{e}]$ (or $y(t)$, y_r and $\dot{y}(t)$), and one output - the control rate $\dot{u}(t)$. Each local controller's parameters are tuned from robust stability or robust performance criterion, considering local nominal plant and multiplicative plant model uncertainty [15-17].

The linear system is robustly stable if for all significant frequencies ω [13]:

$$\left|\Phi^o(j\omega).l(j\omega)\right| < 1,\ \forall\omega \quad (3)$$

where $|\Phi^o(j\omega)|$ is the magnitude of the frequency response of the closed loop system for nominal plant, obtained for $s=j\omega$ from the closed loop system transfer function with respect to reference.

Robust performance is defined as a bounded $\boldsymbol{H}_\infty$-norm of the magnitude of the system error e -

$\sup_{\omega}\left|e(j\omega)\right| < 1$ for all significant frequencies ω. The linear system has a robust performance if:

$$\left|S^{\mathrm{o}}(j\omega).W_{\mathrm{f}}(j\omega)\right| + \left|\Phi^{\mathrm{o}}(j\omega).I(j\omega)\right| < 1, \quad \forall\omega \ge 0 \quad (4)$$

where $\mid S^{\mathrm{o}}(j\omega) \mid$ is the system sensitivity function for nominal plant and $\mid W_{\mathrm{f}}(j\omega) \mid$ - the magnitude frequency response of the shaping filter for the disturbance at the plant output (usually $\left|W_{\mathrm{f}}(j\omega)\right|$=0,3…0,9) [13].

Derived Lyapunov sufficient conditions in [15, 16] allow proving the global closed loop system stability by solving the corresponding LMIs. They state that the closed loop system (1)-(2) is quadratically stable if there exist matrices ***P***>0, and ***Q***>0 such that the following matrix inequalities are satisfied for *i*, *j*=1…*r*, *j*>*i*:

$$\left|\begin{array}{l} PA_{\mathrm{io}} + A_{\mathrm{io}}{}^{\mathrm{T}}P + PB_{\mathrm{id}}F_{\mathrm{i}}Q^{-1}F_{\mathrm{i}}{}^{\mathrm{T}}B_{\mathrm{id}}{}^{\mathrm{T}}P + Q < 0 \\ P.0.5(A_{\mathrm{io}} + A_{\mathrm{jo}}) + [0.5(A_{\mathrm{io}} + A_{\mathrm{jo}})]^{\mathrm{T}}P + \\ + 0.5(B_{\mathrm{id}}F_{\mathrm{j}}Q^{-1}F_{\mathrm{j}}{}^{\mathrm{T}}B_{\mathrm{id}}{}^{\mathrm{T}} + B_{\mathrm{jd}}F_{\mathrm{i}}Q^{-1}F_{\mathrm{i}}{}^{\mathrm{T}}B_{\mathrm{jd}}{}^{\mathrm{T}}) + Q \le 0 \end{array}\right. \quad (5)$$

These results make the basis of the research, described in the present paper.

The aim here is to derive the internal model linear dynamic controllers of the PDC-based FIMC for the local linear plants with time delay and plant model uncertainty in order to ensure local systems robustness [13]. In this way the complexity of the local controllers design in [1-4, 7-12] will be reduced and both their tuning and the global stability problem will be simplified.

3 Method for Design of PDC-based Fuzzy Internal Model Controller

In Fig.1 is shown the *i*-th local linear system in a PDC-TSK structure of a fuzzy system, where by d_{i} and n_{i} are denoted respectively the disturbance and the noise. It consists of the local plant $P_{\mathrm{i}}(s)$ and a local internal model controller (LIMC) $R_{\mathrm{i}}(s)$. The LIMC is based on a nominal plant model $P_{\mathrm{i}}^{\mathrm{o}}(s)$ and a controller $Q_{\mathrm{i}}(s)=[P_{\mathrm{i}}^{\mathrm{o}}(s)]^{-1}.F_{\mathrm{i}}(s)$, where $[P_{\mathrm{i}}^{\mathrm{o}}(s)]^{-1}$ is the inverse nominal plant model and $F_{\mathrm{i}}(s)$ is a filter, designed to make $Q_{\mathrm{i}}(s)$ proper. The ideal controller transfer function $Q_{\mathrm{i}}^{\mathrm{o}}(s)=[P_{\mathrm{i}}^{\mathrm{o}}(s)]^{-1}$ is derived in case the nominal plant model $P_{\mathrm{i}}^{\mathrm{o}}(s)$ precisely describes the plant and no noise and disturbances take place from the requirement that

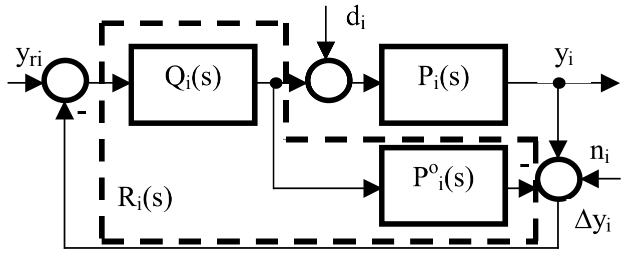


Fig.1. Local internal model control system

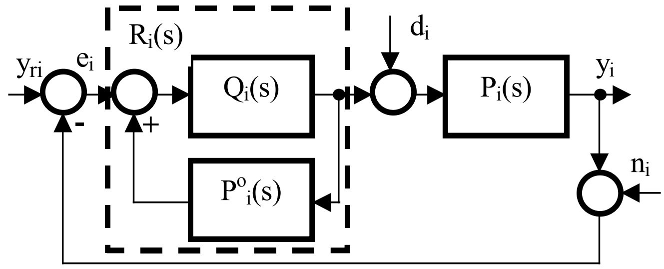


Fig.2. Transformed internal model control system

the plant output $y(t)$ follows the reference $y_{\mathrm{r}}(t)$. However, the controller feasibility requires the filter $F_{\mathrm{i}}(s)$ since the inverse plant model is improper transfer function.

In order to simplify the structure of the LIMC, the system in Fig.1 is equivalently transformed into the system, shown in Fig.2. Then the following transfer function $R_{\mathrm{i}}(s)$ for the LIMC can be derived:

$$R_{\mathrm{i}}(s) = \frac{Q_{\mathrm{i}}(s)}{1 - Q_{\mathrm{i}}(s).P_{\mathrm{i}}^{\mathrm{o}}(s)} = \frac{[P_{\mathrm{i}}^{\mathrm{o}}(s)]^{-1}.F_{\mathrm{i}}(s)}{1 - F_{\mathrm{i}}(s)}. \quad (6)$$

The time delay in the inverse plant model can be neglected according to [13]. For nominal plant model $P_{\mathrm{i}}^{\mathrm{o}}(s) = K_{\mathrm{i}}^{\mathrm{o}}.\mathrm{e}^{-\tau_{\mathrm{i}}^{\mathrm{o}}s}.(T_{\mathrm{i}}^{\mathrm{o}}s+1)^{-1}$ the inverse plant without the time delay becomes:

$$\{[P_{\mathrm{i}}^{\mathrm{o}}(s)]^{-1}\}^{*} = \frac{T_{\mathrm{i}}^{\mathrm{o}}}{K_{\mathrm{i}}^{\mathrm{o}}}s + \frac{1}{K_{\mathrm{i}}^{\mathrm{o}}}. \quad (7)$$

The filter $F_{\mathrm{i}}(s)$ to be added to make this transfer function (7) proper and also to ensure no steady state error in the closed loop system for step input changes, is of the type of a first order time lag - $F_{\mathrm{i}}(s)=1/(\lambda_{\mathrm{i}}s+1)$ [13].

The substituting of the filter into (6) yields:

$$R_{\mathrm{i}}(s) = \frac{[P_{\mathrm{i}}^{\mathrm{o}}(s)]^{-1}.[1/(\lambda_{\mathrm{i}}s+1)]}{1-[1/(\lambda_{\mathrm{i}}s+1)]} = \frac{[P_{\mathrm{i}}^{\mathrm{o}}(s)]^{-1}}{\lambda_{\mathrm{i}}s}. \quad (8)$$

Considering (7) in (8), for $R_{\mathrm{i}}(s)$ it is obtained:

$$R_{\mathrm{i}}(s)=\frac{\frac{T_{\mathrm{i}}^{\mathrm{o}}}{K_{\mathrm{i}}^{\mathrm{o}}}s+\frac{1}{K_{\mathrm{i}}^{\mathrm{o}}}}{\lambda_{\mathrm{i}}s}=\frac{T_{\mathrm{i}}^{\mathrm{o}}}{\lambda_{\mathrm{i}}K_{\mathrm{i}}^{\mathrm{o}}}(1+\frac{1}{T_{\mathrm{i}}^{\mathrm{o}}s})=C_{\mathrm{PIi}}(s)\cdot \quad (9)$$

The LIMC is a PI controller $C_{\mathrm{PIi}}(s)$ with only one tuning parameter λ_{i}, which is selected to satisfy the robustness criterion (3) or (4).

The transfer function of the local closed loop system for perfect plant model, i.e. $P_{\mathrm{i}}(s)=P_{\mathrm{i}}^{\mathrm{o}}(s)$ and $d_{\mathrm{i}}=0$, $n_{\mathrm{i}}=0$, becomes:

$$\Phi_{\mathrm{i}}(s)=\frac{P_{\mathrm{i}}(s)R_{\mathrm{i}}(s)}{1+P_{\mathrm{i}}(s)R_{\mathrm{i}}(s)}=\frac{P_{\mathrm{i}}^{\mathrm{o}}(s).[P_{\mathrm{i}}^{\mathrm{o}}(s)]^{-1}/\lambda_{\mathrm{i}}s}{1+P_{\mathrm{i}}^{\mathrm{o}}(s).[P_{\mathrm{i}}^{\mathrm{o}}(s)]^{-1}/\lambda_{\mathrm{i}}s}=\frac{1}{\lambda_{\mathrm{i}}s+1}=F_{\mathrm{i}}(s)\cdot$$

This shows that $F_{\mathrm{i}}(s)$ entirely determines the system characteristics – the system behaves as a time lag with time constant λ_{i}. The bigger λ_{i} is - the longer the settling time t_{s} and also the more robust the local system is. On the other hand, the sensitivity of the system $S_{\mathrm{i}}(s)$ with respect to noise and plant model uncertainty, defined as:

$$S_{\mathrm{i}}(s)=1-\Phi_{\mathrm{i}}(s),$$

is $S_{\mathrm{i}}(s)=1-F_{\mathrm{i}}(s)$. The conclusion is that the closer $F_{\mathrm{i}}(s)$ to 1 is, the more precisely the system follows the reference input and the less sensitive it is to noise and plant uncertainties.

The PDC-based FIMC is constructed as incremental accounting for (9) and (8). The fuzzy rules are of the type:

R$_{\mathrm{i}}$: IF $y_{\mathrm{r}}(t)$ is M_{i1} **AND** $e(t)$ is M_{i2} **AND** $\dot{e}(t)$ is M_{i3}

THEN $\dot{u}_{\mathrm{i}}(t)=T_{\mathrm{i}}^{\mathrm{o}}/(\lambda_{\mathrm{i}}K_{\mathrm{i}}^{\mathrm{o}})\dot{e}(t)+1/(\lambda_{\mathrm{i}}K_{\mathrm{i}}^{\mathrm{o}})e(t)$ (10)

or $\dot{u}_{\mathrm{i}}(t)=-F_{\mathrm{i}}x_{\mathrm{i}}(t)+G_{\mathrm{i}}x_{\mathrm{r}}$,

where $F_{\mathrm{i}}=[1/(\lambda_{\mathrm{i}}K_{\mathrm{i}}^{\mathrm{o}})\ \ T_{\mathrm{i}}^{\mathrm{o}}/(\lambda_{\mathrm{i}}K_{\mathrm{i}}^{\mathrm{o}})]$ and $G_{\mathrm{i}}=[1/(\lambda_{\mathrm{i}}K_{\mathrm{i}}^{\mathrm{o}})\ \ 0]\cdot$

By analogy with the PI-PDC controller (2) can be established that

$$1/(\lambda_{\mathrm{i}}K_{\mathrm{i}}^{\mathrm{o}})=K_{\mathrm{pi}}/T_{\mathrm{ii}},$$
$$T_{\mathrm{i}}^{\mathrm{o}}/(\lambda_{\mathrm{i}}K_{\mathrm{i}}^{\mathrm{o}})=K_{\mathrm{pi}}\cdot$$

The FIMC-PDC has three inputs $y_{\mathrm{r}}(t)$, $e(t)$ and $\dot{e}(t)$ - $z^{\mathrm{T}}=[y_{\mathrm{r}}\ e\ \dot{e}]$ and one output - the control rate $\dot{u}(t)$. The fuzzy rules are designed to estimate the degree of belonging of the operating point, defined by the measurement of y_{r} to the different linear sub-domains. The global FIMC-PDC output after a Centre-Of-Gravity defuzzification is obtained as:

$$\dot{u}(t)=-\frac{\sum_{i=1}^{r}w_i(z(t))F_i x(t)}{\sum_{i=1}^{r}w_i(z(t))}=-\sum_{i=1}^{r}h_i(z(t))F_i x(t)\text{,}$$

where $w_i(z(t))=\prod_{j=1}^{p}M_{ij}(z_j(t)),\left|\begin{array}{l}\sum_{i=1}^{r}w_i(z(t))>0\\ w_i(z(t))\geq 0\end{array}\right.$ is the degree of fulfillment of the compound fuzzy condition in the premise, $\mathbf{M}_{\mathrm{ij}}(z_{\mathrm{j}}(t))$ is the degree of match of $z_{\mathrm{j}}(t)$ to $\mathbf{M}_{\mathrm{ij}}$,

$h_i(z(t))=\frac{w_i(z(t))}{\sum_{i=1}^{r}w_i(z(t))},\left|\begin{array}{l}\sum_{i=1}^{r}h_i(z(t))=1\\ h_i(z(t))\geq 0\end{array}\right.$ is the strength of firing of the rule and is normalized, which implies that the MFs comprise an orthogonal system.

An integrator ($1/s$) is used for computing the final control action $u(t)$ from its rate $\dot{u}(t)$.

The global system stability is confirmed by the existence of the matrices $\boldsymbol{P}>0$ and $\boldsymbol{Q}>0$ for $i, j=1\ldots r$, $j>i$, which satisfy the derived in [15, 16] sufficient Lyapunov conditions (5) with the only modification – the new F_{i} and G_{i}:

4 Design of PDC-based FIMC for Air Temperature Control

4.1 Design of PDC-based FIMC

The plant is the air temperature T, $^{\mathrm{o}}$C ($y(t)=T(t)$) in a furnace, shown in Fig.4. It is controlled by electrical heater via the voltage from the controller $u(t)$, which is passed to a Pulse-Width-Modulator (PWM) in order to form the proper duty ratio. During the PWM pulses a Solid State Relay (SSR) connects the heater to net supply voltage. So, during the pulses the heater heats. The duty ratio that corresponds to a given controller output $u(t)$, leads to a corresponding average heat, emitted to the air inside the furnace.

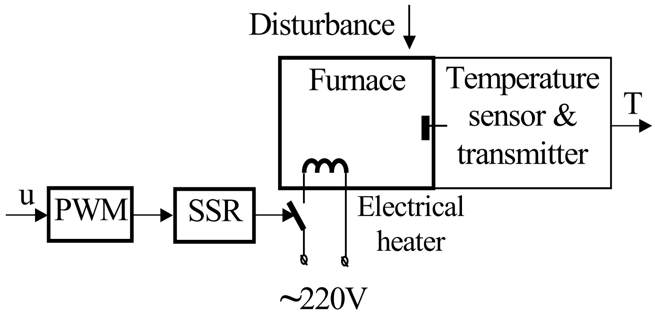


Fig.3. Electrical furnace as a controlled plant

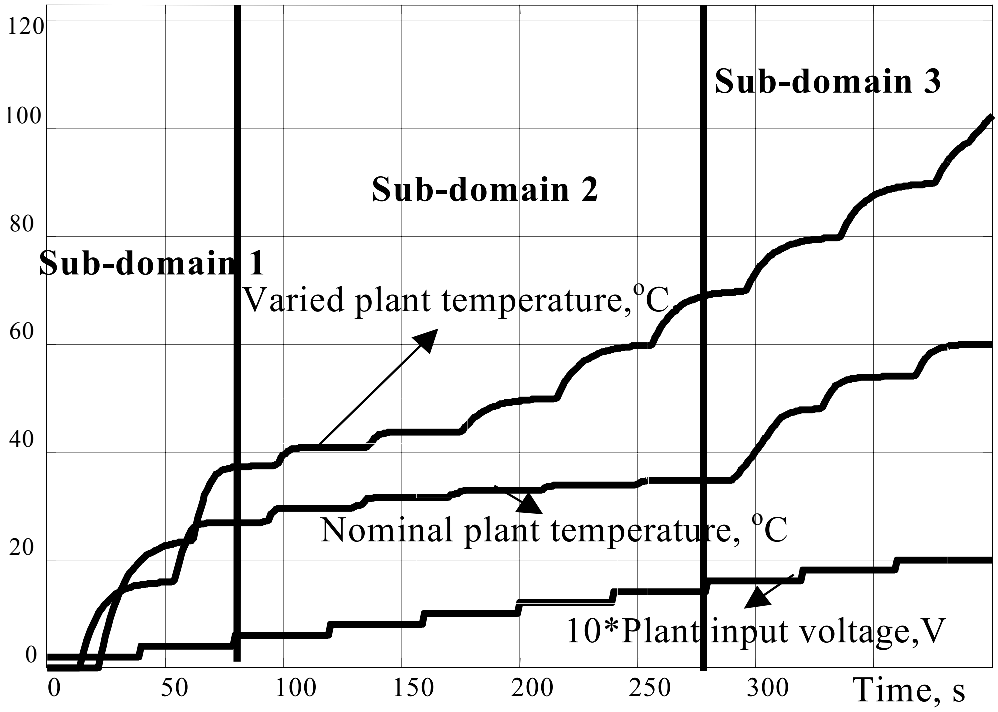


Fig.4. Nominal and varied plant step responses in different operation points

Table 1. Average/worst plant and LIMC parameters

Plant model parameters	K_{iav},°C/V $K_{i\,w}$,°C/V	T_{iav}, min $T_{i\,w}$, min	τ_{iav}, min $\tau_{I\,w}$, min	λ_i
Sub-domain 1	$\boldsymbol{K_{max}}=K_{1av}=66$ $K_{1\,w}=80$	$T_{1av}=8$ $T_{1\,w}=5$	$\boldsymbol{\tau_{max}}=\tau_{1av}=14$ $\tau_{1\,w}=18$	10
Sub-domain 2	$K_{2av}=10$ $K_{2w}=20$	$\boldsymbol{T_{min}}=T_{2av}=6$ $T_{2\,w}=4$	$\tau_{2av}=10$ $\tau_{2\,w}=15$	25
Sub-domain 3	$K_{3av}=50$ $K_{3\,w}=70$	$T_{1av}=9$ $T_{1\,w}=5$	$\tau_{2av}=8$ $\tau_{2\,w}=12$	10

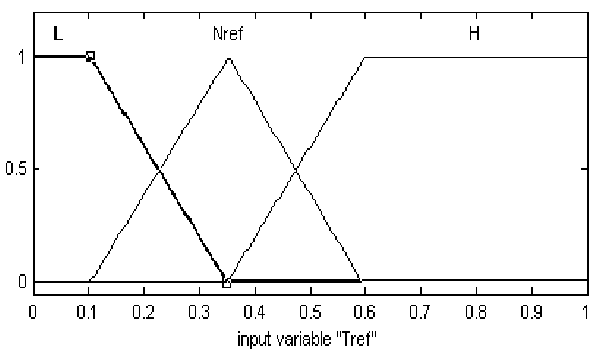


Fig.5. Membership functions for the input – temperature reference $y_r(t)$

The experimentally obtained plant step responses in different operation points for nominal and varied plant are shown in Fig.4. Three linear sub-domains can be distinguished in Fig.4. In Table 1 are given the average and the worst Ziegler-Nichols local plant parameters for each sub-domain.

The FIMC is designed using MATLAB™ [18]. It is described by three fuzzy rules according to (10) – one for each linear sub-domain. As the conclusion in each rule is a different deterministic function of the inputs e and $\dot{e}$, the input $y_r(t)$ is used to distinguish the sub-domains. Its MFs are shown in Fig.5, where "H" is high, N_{ref} is normal and "H" is low.

The temperature range is [0÷80], °C and the maximal expected error $|e_{max}|= 10$,°C. The control action is bounded in the range [0÷2], V. The fuzzy unit inputs are normalized - the error and the derivative of error in the range [-1÷1], °C, and the reference temperature - in the range [0÷1], °C. The

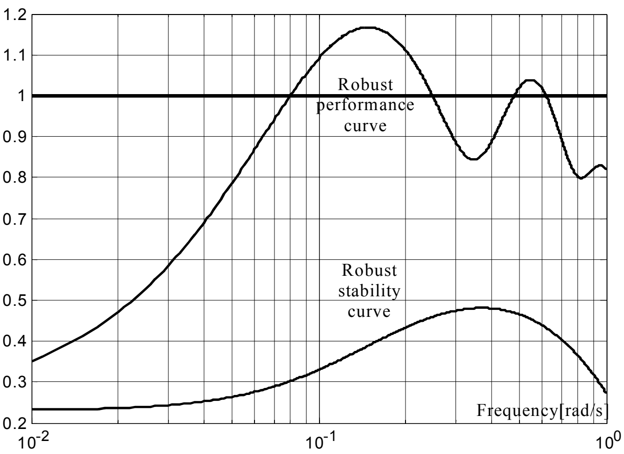


Fig.6. Results from tuning of λ_i of the first local IMC – the robustness curves are mainly below 1

derivative of error is obtained at the output of a noise resistive first order differentiator $s/(s+1)$.

The parameters λ_i of the LIMCs are tuned to satisfy (3) and (4) for the local nominal and worst plant parameters. The robustness curves – the left-hand functions in (3) and (4), which should be below 1 for all significant frequencies $\omega_i=(0.1\div10)*2\pi/T^o_{iav}$, are shown for $LIMC_1$ in Fig.6. The computed values for λ_i are given in Table 1.

The denormalisation factor at the output of the fuzzy unit, which serves also as an integrator gain, is tuned empirically to $K_a=3$.

4.2 System simulation and performance assessment

The designed FIMC-PDC system is studied by simulation in MATLAB™ [18]. The system Simulink model is shown in Fig.7. Its performance is assessed in comparison to a control system with ordinary linear PI controller with PID block instead of the FIMC- PDC.

The PI controller is tuned for the worst plant from the three average plants to ensure a good tradeoff between minimal overshoot and minimal settling time using empiric-tuning method [6, 16]:

$$K_{po}=0.6T_{min}/(K_{max}\,\tau_{max})=0.0036$$
$$T_{io}=0.9T_{min}=5.4 \text{ min.}$$

The plant model is the TSK model, given in Fig.8, which is built to reproduce the step responses of the nominal plant in Fig.4. It consists of a delay block of three units with the different sub-domains time delays to provide the necessary inputs $u(t-\tau_{mi})$, $i=1\div3$, to the fuzzy unit. The fuzzy unit has five not normalized inputs - $u(t-\tau^o_{mi})$, $y_r(t)$ and $y(t)$, one output $\dot{y}(t)$ and the following rules:

R$_i$: IF $u_1(t-\tau^o_{m1})$ is L_{i1} **AND** $u_2(t-\tau^o_{m2})$ is L_{i2} **AND** $u_3(t-\tau^o_{m3})$ is L_{i3} **AND** $y_r(t)$ is L_{i4} **AND** $y(t)$ is L_{i5} **THEN** $\dot{y}_i(t)=(K^o_{mi}/T^o_{mi})u_i(t-\tau_{mi})-(1/T^o_{mi}).y_i(t)$, (11)

where L_{ik}, $k=1\div5$ are the linguistic values for the inputs, defined as MFs of fuzzy sets.

The local models parameters are:

$$\tau^o_m=[14\ 10\ 8], \text{ min},$$
$$K^o_m/T^o_m=[10, 4, 6] \text{ and}$$
$$1/T^o_m=[-0.125, -0.17, -0.5].$$

An integrator after the fuzzy unit computes the final plant output $y(t)$.

Then a TSK perturbed plant model is also developed that differs from the nominal only in the parameters:

$$\tau^v_m=[22\ 18\ 16], \text{ min},$$
$$K^v_m/T^v_m=[12, 6, 5] \text{ and}$$
$$1/T^v_m=[-0.1, -0.05, -0.1].$$

The step responses of the FIMC-PDC system and the PI control system are shown in Fig.9.

The simulation is carried out with nominal and perturbed plant in order to assess robustness. The reference changes ensure operation in different operation points, where the plant has different parameters. The main performance indices – settling time t_s, min, overshoot σ, % and maximal deviation between systems outputs of systems with nominal and perturbed plants $|\Delta y_{max}|$, ^oC, as a measure for robustness, are given in Table 2.

The comparison shows that the FIMC-PDC system shows better robustness, as the responses with the perturbed and with the nominal plant are close. The FIMC-PDC system has also shorter settling time for the low references and no or smaller (for perturbed plant) overshoots in the whole range of operation. The PI control system becomes critically stable with perturbed plant for low references.

5 Conclusion

The main contributions of this paper are the following.

- A general, computationally simple and easy to implement method for the design of fuzzy PDC-based internal model controllers is suggested. It ensures system stability and robustness in controlling nonlinear plants with time delay and model uncertainties avoiding the complexity of the existing methods.

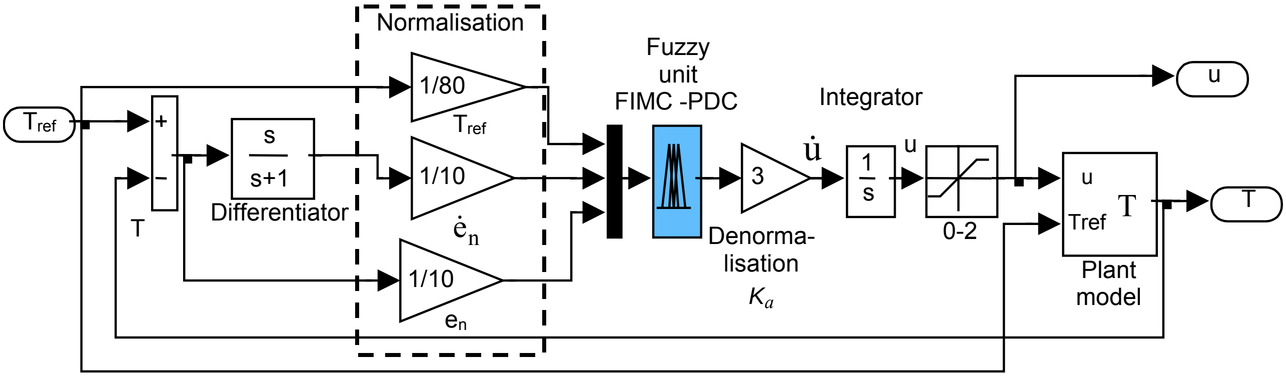


Fig.7. Simulink model of a system with PDC-based FIMC

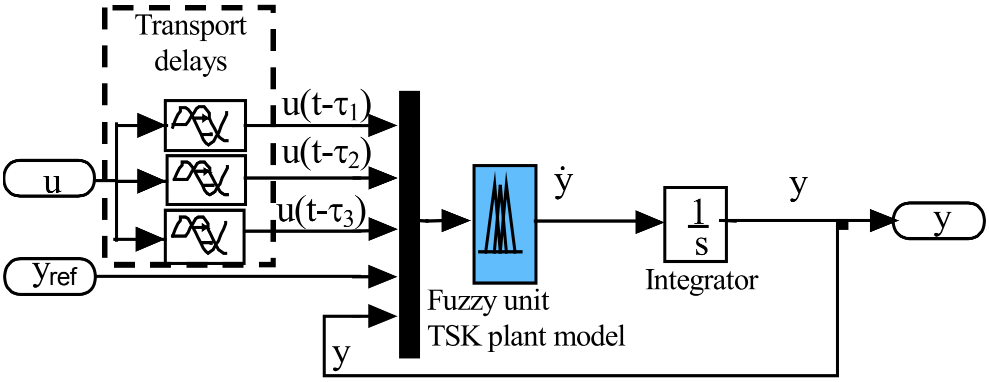
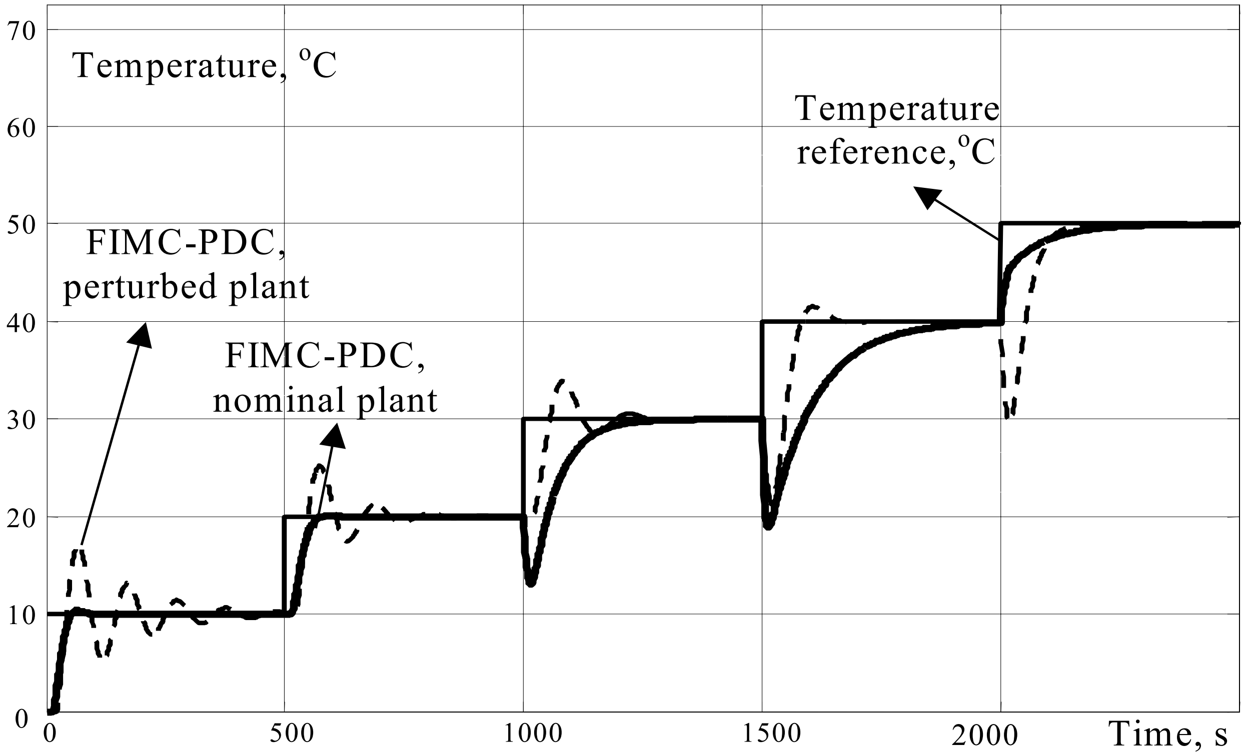


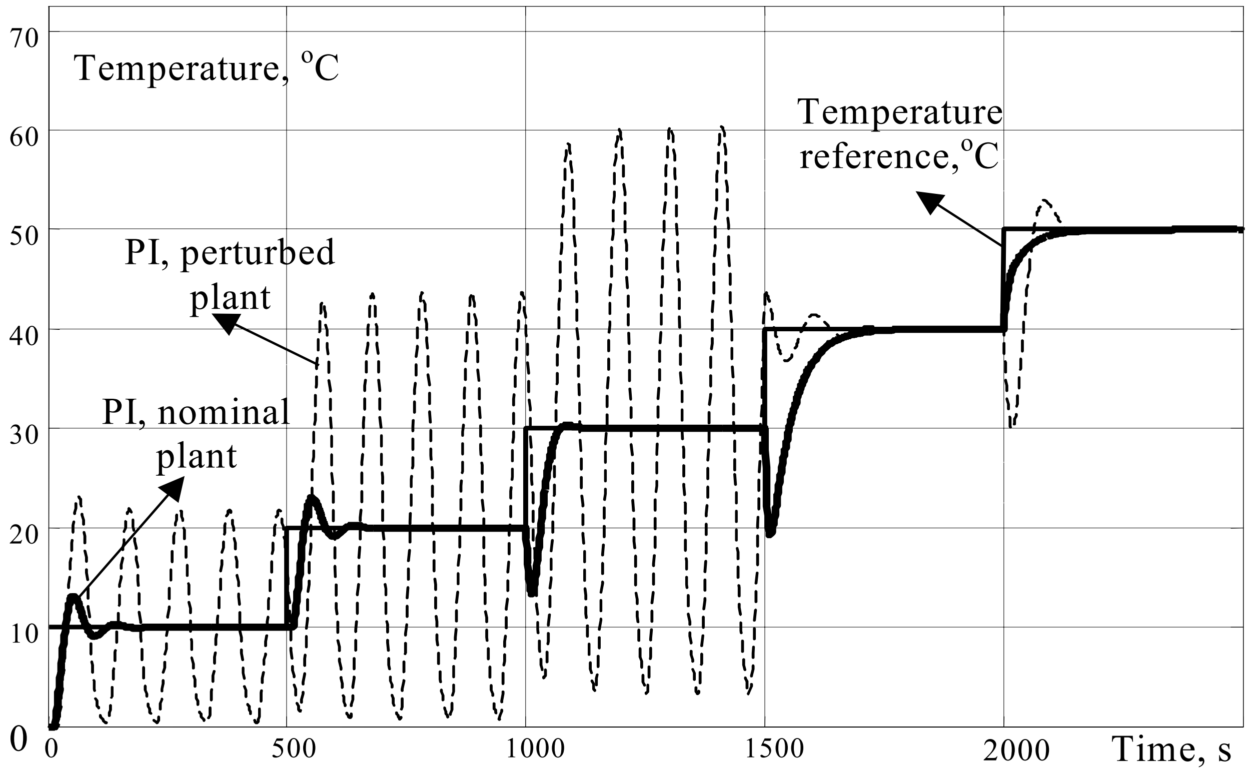
Fig.8. Simulink plant model

Table 2. Comparison of performance indices of investigated systems

Systems	FIMC-PDC Nominal plant		FIMC-PDC Perturbed plant		Robust-ness estimate	PI Nominal plant		PI Perturbed plant		Robust-ness estimate
Reference °C	t_s min	σ %	t_s min	σ %	$\|\Delta y_{max}\|$ °C	t_s min	σ %	t_s min	σ %	$\|\Delta y_{max}\|$ °C
10	50	0	350	70	7	120	30	NA	NA	∞
20	65	0	230	45	6	100	25	NA	NA	∞
30	200	0	200	30	8	80	0	NA	NA	∞
40	350	0	120	10	10	200	0	130	25	22
50	100	0	110	0	12	100	0	110	20	15



a) FIMC-PDC control system



b) PI control system

Fig.9. Step responses of systems with: a) FIMC-PDC and b) PI controller

The local controllers structure and parameters are determined from the nominal plant model. The only tuning parameter is computed to take the minimal possible value that satisfies the robust stability or robust performance criterion. These criteria are defined in the frequency domain, so are convenient to apply for plants with time delay. The minimal possible value ensures also fast transient response.

- The application of the method is demonstrated for the design of FIMC-PDC of the air temperature in a laboratory furnace. The process is nonlinear with time delay and can be approximately described by a TSK model for three linearisation sub-domains.
- The performance of the designed system is assessed via simulation in comparison with the performance of a well-tuned PI control system.
- The FIMC-PDC system retains stability and remains robust at plant perturbations and also has reduced overshoot and settling time. The control action is smooth and economic.
- Future work is envisaged in the real time application of the suggested method using MATLAB™ facilities and industrial programmable logic controllers [19-24]. Comparison will be carried out with the design, tuning and implementation of PI/PID local controllers in PDC structure using the results in [15, 16].

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