Modelling the Dynamics of Vacuum Aspiration

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Abstract: - Two mathematical models of vacuum aspiration, being a method widely used in various branches of medicine, are considered. In particular, vacuum aspiration is an effective method of prostatitis therapy. The purpose of vacuum aspiration in this urological procedure is to destroy the plug, which clogs up the ill acinus. We analyze plausible physical mechanisms of purification of prostatic acini and ducts on the basis of two different models. A mechanical model is offered to describe the process of plug destruction during vacuum aspiration procedure. The majority of medical practitioners believe that the plug is extracted as a whole during such procedure. However, our theoretical research demonstrates that the sucking of a plug as a whole, previously viewed as the most likely mechanism, is not consistent with the experimental data. The second model implies non-uniform structure of the plug. The problem considered in the framework of the latter assumption belongs to self-organized criticality type of problems. This approach reveals the plug destruction dynamics which corresponds to essential clinical data.

Key-Words: - vacuum aspiration, prostatitis, physical models, lattice of springs, self-organized criticality

1 Introduction

Vacuum aspiration [1,2] is a method extensively used in various branches of medicine. Otorhinolaryngology, urology, general and gynecology surgery are among these fields.

Vacuum aspiration is based on the suction of liquid or solid fraction from a cavity by means of generated negative pressure. The negative pressure can remain constant during a certain time interval. Also, it can oscillate between atmospheric pressure and some negative (reduced) value of pressure. The choice of the value of negative pressure depends on the specific aim of the procedure.

In vascular and heart surgery, the vacuum aspirators are applied to remove the thrombi [3,4]. Also, vacuum aspirators occupy a prominent place among the tissue sample devices (See, e.g., [5]). In pregnancy termination, the vacuum aspiration is the most common method [6]. Obviously, all these tasks require different devices suited for specific procedures.

In fact, medical practitioners have a rather vague idea of how vacuum aspiration devices perform. Yet, advanced medical equipment requires detailed understanding of the physical processes which underlie such therapeutic procedures. Therefore, an adequate model of such physical processes appears to be indispensable. Such model would be also indispensable for the analysis of collected clinical data. As we demonstrate below, the construction of an adequate model of the aspiration procedure is a complicated task, and even the collected clinical data do not lead us to the unambiguous choice of the model.

In the present article, we analyze feasible physical mechanisms of purification of prostatic acini and ducts by means of transurethral vacuum aspiration. Though the most common treatment is with anti-biotics and anti-inflammatory medicaments [7], the preliminary physiotherapy can also be very useful. In particular, vacuum aspiration applied at the first stage of the treatment makes subsequent antibacterial medication much more efficient. In particular, such method allows doctors to shorten treatment period as well as reduce the dosage of anti-bacterial drugs due to a much more efficient transportation of the latter to the acini and ducts previously drained by vacuum aspiration. A detailed explanation of physical processes underlying vacuum aspiration of secretions from plugged acini and ducts is, undoubtedly, valuable for further development of this method – considering the fact that prostatitis affects at least 40% of men aged 35-50 years.

The paper is organized as follows. Section II offers the description of the device used for the treatment of bacterial prostatitis by vacuum aspiration. The method [8,9] was developed by A. R. Guskov and his collaborators at the Central Aviation Scientific-Research Hospital (Moscow, Russia). At present, it is used in a few medical institutions. In Section III we analyze the model of the suction of the plug as a whole. This is how the majority of medical practitioners views the process. Our investigation reveals that this model contradicts the clinical data. In Section IV we analyze the model which takes into account the non-uniform structure of the plug. It describes the gradual erosion of the plug by the oscillating pressure. This model reproduces some important features of plug destruction dynamics which are observed in clinical practice. In our opinion, we deal here with self-organized criticality type of problems [10,11]. The final section summarizes the conclusions concerning the actual physical processes involved in the discussed mechanisms.

2 Vacuum Aspiration for the Treatment of Prostatitis

In this section, we briefly describe general principles of vacuum aspiration as applied to the treatment of prostatitis. Fig. 1 presents the scheme of the prostate which consists of stroma and glands (acini). Acini drain into urethra by means of prostatic acini. The scheme below shows only several healthy and plugged acini, whereas the total number of acini is about forty and their size is significantly smaller compared to the size of the prostate. Bacterial prostatitis leads to duct obstruction. The plug consists of dense acini epithelium and secretions. It occludes the passage connecting the acinus to the urethra. Vacuum aspiration procedure results in the destruction of the plug. The fact of the destruction is verified by the analysis of the substance obtained during the procedure.

At Central Aviation Scientific-Research Hospital, at the medical center “SANOS” and other institutions which borrowed this method for the treatment of prostatitis, there are used the machines providing the pressure oscillating between 1 atm (10.1 N/cm2) and 0.5 atm (5.05 N/cm2) at frequency 20 Hz. These parameters have been determined as the most efficient ones over the course of many clinical experiments.

The aim of the present research is to clarify the physical mechanism of the plug destruction during the vacuum aspiration procedure. In our opinion, two radically different mechanisms may account for the

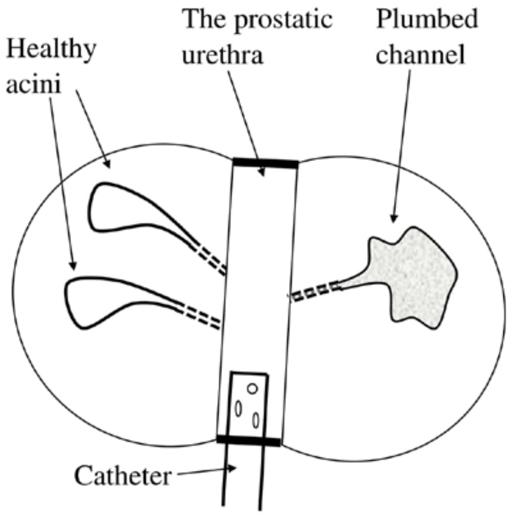

Fig. 1 The scheme of the urethra. Healthy and plugged acini are shown.

effect. The first one accounts for the suction of the plug as a whole, whereas the second - for the progressive fragmentation of the plug by the oscillating pressure. In the latter case, the plug is destroyed gradually, since only the layer facing the urethra is exposed to the destruction. In the following section, we analyze the first model and prove that the hypothesis of the suction of the plug as a whole contradicts experimental data.

3 Analysis of the Model of Suction

We use the following simple model to describe the suction of the plug as a whole. In our model, the duct is represented as a cylindrical tube, and the plug - as a solid cylinder which is held in the tube by the friction forces. The device which is used in the above-mentioned institutions generates the oscillations of pressure at frequency according to the following law:

$$P(t) = p_0(1-\cos(\omega t)) \qquad (1)$$

where p_0 is the amplitude of pressure oscillations, t is the time.

Second Newton’s law determines the motion of the plug:

$$m(t)\ddot{x}(t) = -\pi R_0^2 p_0(1-\cos(\omega t)) + 2\pi k R_0 x(t)\theta(-\dot{x}(t)) - \gamma\dot{x}(t) \qquad (2)$$

where $x(t)$ is the position of the inner side of the plug at time *t*, $m(t)$ - plug mass at time *t*, R_0 - is the radius of the duct, k - the coefficient of friction between the wall of the duct and the side of the plug, γ is the coefficient of viscous friction. Since the applied pressure is never positive, the plug can move only in one direction. Heaviside step function $\theta(\dot{x}(t))$ in the right-hand side of the equation provides for the force of friction only when the plug is moving. The mass of the plug decreases over time since its extruded segment must be easily destroyed by the oscillating pressure. The plug being pressed out into the urethra experiences small viscous friction since the urethra is filled with gaseous medium rather than with a liquid one.

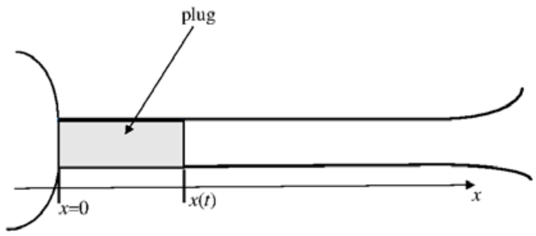

Fig. 2 Scheme of the affected acinus with the plug.

We express the plug mass through its density ρ, so that $m(t) = \rho \pi R_0^2 x(t)$, and introduce dimensionless variables according to the formulae:

$$\varphi = \omega t, \quad P_0 = \frac{p_0}{\rho \omega^2 R_0^2}, \quad K = \frac{2k}{\omega^2 \rho R_0^2}, \quad \Gamma = \frac{\gamma}{\pi \omega \rho R_0^3} \qquad (3)$$

The initial Eq. (2) then transforms into the equation for the new variable $z(\varphi) \equiv x(\varphi / \omega) / R_0$:

$$z(\varphi) z_{\varphi\varphi}(\varphi) = -P_0(1 - \cos\varphi) + K\, z(\varphi) \times \theta(-z_\varphi(\varphi)) - \Gamma z_\varphi(\varphi) \qquad (4)$$

The choice of parameters can be made on the basis of the available experimental data. Some parameters are known, namely, $p_0 = 0.5$ atm = 50662.5 Pa, the frequency $\omega = 20$ Hz. The average value of the duct radius is $R_0 = 5 \times 10^{-5}$ m and the average length of the duct and, correspondingly, the initial (at $t = 0$) length of the plug is $x(0) = L = 1$ mm. The density of the duct can be estimated as $\rho \sim 3000 - 4000$ kg/m^3.

Using these values, we obtain the following expressions for the parameters of Eq. (4):

$$P_0 = 1.70 \times 10^7, \quad K \sim 667.0\, k, \quad \Gamma \sim 4.24 \times 10^7 \gamma \qquad (5)$$

The viscous friction is negligibly small compared to the other forces that our model takes into account. So, we neglect this term. Now there is one unknown parameter - friction coefficient k. We need to find the value of k which would provide for the procedure time.

We use modified Euler method [12] to get the numerical solution of Eq.(4). Clinical trials show that the total time of the aspiration procedure is about 90 min. It takes several sessions, each session taking approximately 15 min. This time cannot be reduced by moderate varying of the device parameters. In the dimensionless parameters (3), this time value corresponds to $\Phi_0 = 6.5 \times 10^6$.

The plug starts moving at some moment $\varphi = \varphi_0$ when the power produced by the device exceeds friction. Therefore, the condition $K z_0 < P_0$, where $z_0 = L / R_0$, is the initial condition for the solution $z(\varphi)$. The second initial condition is zero velocity $z_\varphi(\varphi_0) = 0$. The initial value of time, φ_0, is determined by the condition

$$P_0(1 - \cos\varphi_0) = K\, z_0 \qquad (6)$$

Now we estimate the values of the parameter *K*. We search for the value of *K* which results in the total procedure time equaling Φ_0. The friction coefficient γ for gas is so low that the corresponding term can be neglected. It is clear that the time of the suction of the plug increases as the difference $P_0 - K z_0$ approaches zero remaining positive. Numerical calculations show that for $K = 0.99(P_0 / z_0)$ the total time of suction is as small as $\Phi = 0.05$, which is much smaller than the period of the oscillations. Correspondingly, the time of suction is less than one second. The plug starts to move at $\varphi_0 = \arccos\left(1 - K z_0 / P_0\right) \approx 1.5607961$. Fig. 3 displays $z(\varphi)$ for these parameters (solid line). Visible nonlinear character of the function $z(\varphi)$ is caused by the decrease of mass as the plug moves outward from the acinus channel, as well as by the decrease of the force of friction.

So, we obtained $\Phi << \Phi_0$ for reasonable parameters. To get the experimental total time Φ_0 in this model, we must use the parameters which obey the inequality $0 < \left(P_0 - (Kz_0)\right) / P_0 < 0.01$. Moreover, even small (within one percent) increase of the pressure amplitude leads to a radical decrease of the time of the suction of the plug. Meanwhile, clinical trials show that a more significant variation of the pressure does not lead to a visible decrease of the total time of aspiration procedure. This conclusion is correct for various modifications of the presented model. For instance, if we apply significant value of viscous friction term or assume that the mass of the plug did not decrease, the time of the aspiration is still as small as several orders

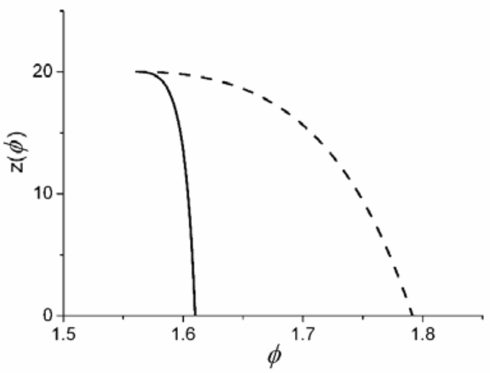

Fig. 3 The dependence of $z(\varphi)$ for the parameters of the existing devices: $P_0 = 1.689 \times 10^7$, $K = 0.99 \times P_0 / z_0$, $z_0 = 20.0$, $\Gamma = 7000.0$, solid line; the same parameters, excluding $\Gamma = 70000.0$, the dashed line.

comparing to Φ_0. Dashed line at Fig. 3 illustrates this conclusion for the model with high viscous friction, $\Gamma = 70000.0$. In principle, it is possible to adjust the parameters K and Γ for which the time of the suction will be as large as experimental value $\Phi_0 = 6.5 \times 10^6$. However, the change of the pressure even by almost negligible quantity (less than 0.01 %) would result in drastic decrease of suction time. But, the vacuum aspirators which are used in urological procedures cannot keep the pressure with such accuracy. If this model were correct, the real procedure would last several seconds rather than around 90 min., as it happens in practice.

We note that possible modifications of the model do not lead to the plug suction dynamics that would agree with the clinical data. For instance, it seems reasonable to take into account the oscillations of friction between the wall of the duct and the side of the plug. To take this into account, we should replace the coefficient of friction (Eq. (2)) by the oscillating term of the following form

$$k \quad \rightarrow \quad k\beta\left(1 - \frac{\beta - 1}{\beta}\cos\omega t\right), \quad 0 < \beta < 1 \qquad (7)$$

Therefore, the friction coefficient harmonically changes between βk and k. This modification of the model results in moderate decrease of the procedure time.

4 Analysis of 'Spring Lattice' Model

In this section, we discuss the model which provides the gradual erosion of the plug by the oscillating pressure. We have noted that the plug consists of heterogeneous material. Let us speculate that the plug consists of dense solid fragments which are relatively weakly connected with each other and with the walls of the acinus. We model such plug by the lattice of the masses and the springs (Fig. 4). The oscillating force, acting on each site of such lattice, decreases exponentially with the distance from the surface of the plug. This exponential decrease of the pressure takes place for dense medium [13]. Let us assume that a spring breaks when its length exceeds some critical value. Also, we assume random distribution of the masses, spring lengths and spring constants. In contrast with the model considered in the preceding section, we cannot fit the clinical data into the parameters of the spring lattice. The laborious investigation of plugs structure is necessary to make possible the evaluation of the model's parameters. Nevertheless, we study general properties of the model, viz. the dependence of time of destruction of the lattice on the amplitude and frequency of oscillations of the external force.

Various versions of the lattice of springs have been exploited in the study of self-organized criticality [10,11,14]. The most well-known model of lattice of springs type is the model of the earthquakes by R. Burridge and L. Knopoff [15]. The models of the lattices of springs are widely used in the investigation of the mechanical properties of heterogeneous media [16,17].

The dynamics of destruction of the lattice of springs is interesting in itself, independently of the vacuum aspiration problem. This type of problem can hardly be investigated analytically. In the present article, we perform numerical analysis of the break of two-dimensional square lattice of springs.

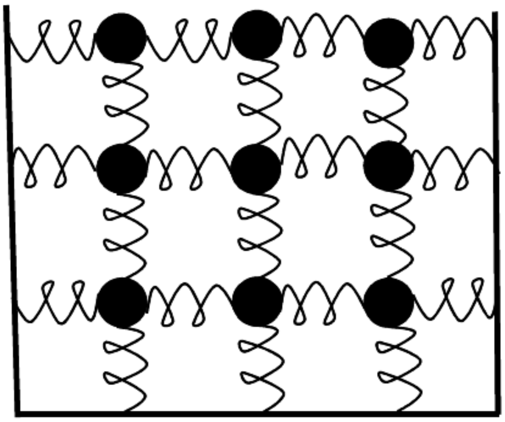

Fig. 4. Two-dimensional lattice of masses and springs which is modelling the fragmented structure of the plug.

The motion of the masses is determined by the spring tension according to Hooke's law and the oscillating pressure which exponentially decreases with the distance from the outer surface of the plug. We choose the break of one quarter of the number of springs as the criteria for the destruction. The calculations are performed for small-size rectangular lattice, consisting of four rows and three columns of the masses. We use arbitrary units in our simulation. We keep the same values of the springs' initial length $l_{hor} = 1.0$, $l_{vert} = 1.4$ for horizontal and vertical springs correspondingly, and spring constants, $k = 1$ for all springs, whereas the masses are uniformly distributed random numbers in the interval [0.8, 1.1].

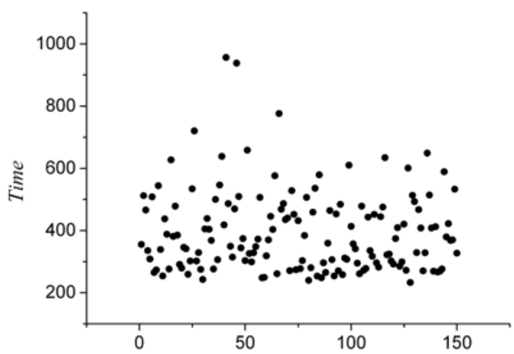

Fig. 5. The dependence of the time of the lattice breaking for different mass distribution. Each point corresponds to some specific distribution of the masses.

Therefore, the masses are the i.i.d. (independent and identically distributed) random variables. The critical length is chosen as $1.5 l_{hor}$ and $1.5 l_{vert}$ for horizontal and vertical springs correspondingly. This random distribution of the masses results in wide spread of the time of the destruction of lattice (Fig. 5). Fig. 6 displays the dependence of the time of the lattice destruction on the magnitude of the oscillating force, $f(x) = f_0 e^{-x/d}(1-\cos\omega t)$, where parameter d determines the exponential decrease of the external oscillating pressure, ω is the oscillation frequency. The oscillating pressure of small amplitude cannot destroy the lattice, hence the time of destruction is infinite. After the pressure exceeds some critical value, the time of destruction decreases drastically to some value, after which the time of destruction shows comparatively weak dependence on f_0. The slightly

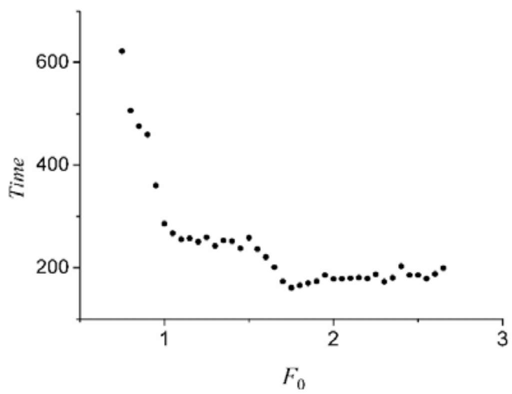

Fig. 6. The dependence of the time of the lattice breaking on the amplitude of the oscillating force. See the parameters of the system in the text.

non-monotonic character of the curve in Fig. 6 is caused by the random dependence of the masses in separate computations. We have performed the averaging of destruction time for 150 calculations - for each value of f_0 - for different random masses. Weak dependence of destruction time on the amplitude of the applied oscillating pressure is the essential feature of experimental data which the model apparently catches.

It is worth noting that the time destruction shows non-monotonic dependence on the frequency ω. The pronounced peaks in the dependence of time destruction on the oscillation frequency are found in certain range of the lattice parameters. Fig. 7

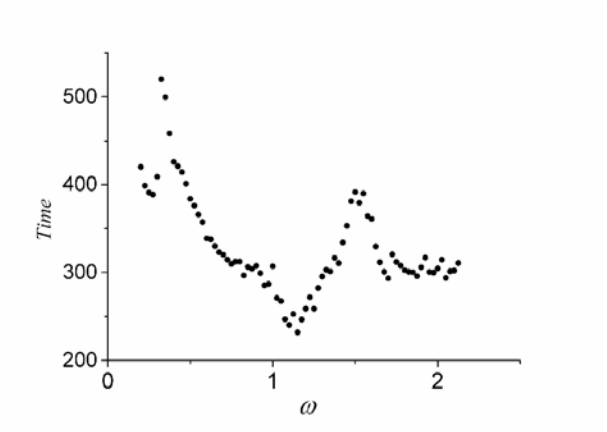

Fig. 7. The dependence of time of destruction of the lattice on the frequency ω.

represents an example of such dependence for $f_0 = 1.0$ and for the same spring constants and the same distribution for masses as used in the previous calculation.

5 Conclusion

We performed the analysis of two different possible mechanisms of the same vacuum aspiration procedure. Our research demonstrates that the sucking of a plug as a whole cannot be the mechanism of draining the clogged acinus during vacuum aspiration procedure. The theoretical investigation of suction of a plug as a whole reveals that a small (about 0.1%) increase of pressure may result in a huge decrease of the time of vacuum aspiration procedure. However, such conclusion contradicts the available clinical data. In our opinion, the fragmentation of a plug by oscillating pressure must be the most probable mechanism of the recovery of inflamed acinus. In that case, the change (increase) of the oscillating pressure magnitude does not lead to the speeding up of the procedure. In order to reproduce this feature of the procedure, we present the model that describes the gradual erosion of the plug by its fragmentation. The choice of adequate parameters for this model is so far an open problem, as we have noted. Still, we demonstrate that the proposed model reproduces the weak dependence of the destruction time on the amplitude of the applied oscillating pressure. It is also worth mentioning that the proposed model is of mathematical interest, since it refers to self-organized criticality phenomenon.

We should underscore, that we investigate the physical mechanism of draining of a prostatic acinus by means of vacuum aspiration rather than analyze medical aspects of the treatment or investigate the mathematical aspects of self-organized criticality exhibiting by the model we proposed in Sec. IV.

As was noted in the Introduction, vacuum aspiration is widely used in different branches of medicine. The related devices are distinguished by various parameters, e.g. by the amplitude of the negative pressure and the frequency (for the devices producing the oscillating pressure). To our knowledge, the present research is the first one where the problem of adequate model of vacuum aspiration in medical procedure is analyzed. There is no doubt that some other mechanisms of the plug destruction can be investigated. Our research, as we believe, draws attention to the fact that in different cases the physical mechanism of suction varies depending on its specific physiological nature.

Acknowledgements

M. Ye. Zhuravlev thanks G. A. Chernykh for the valuable discussion.

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