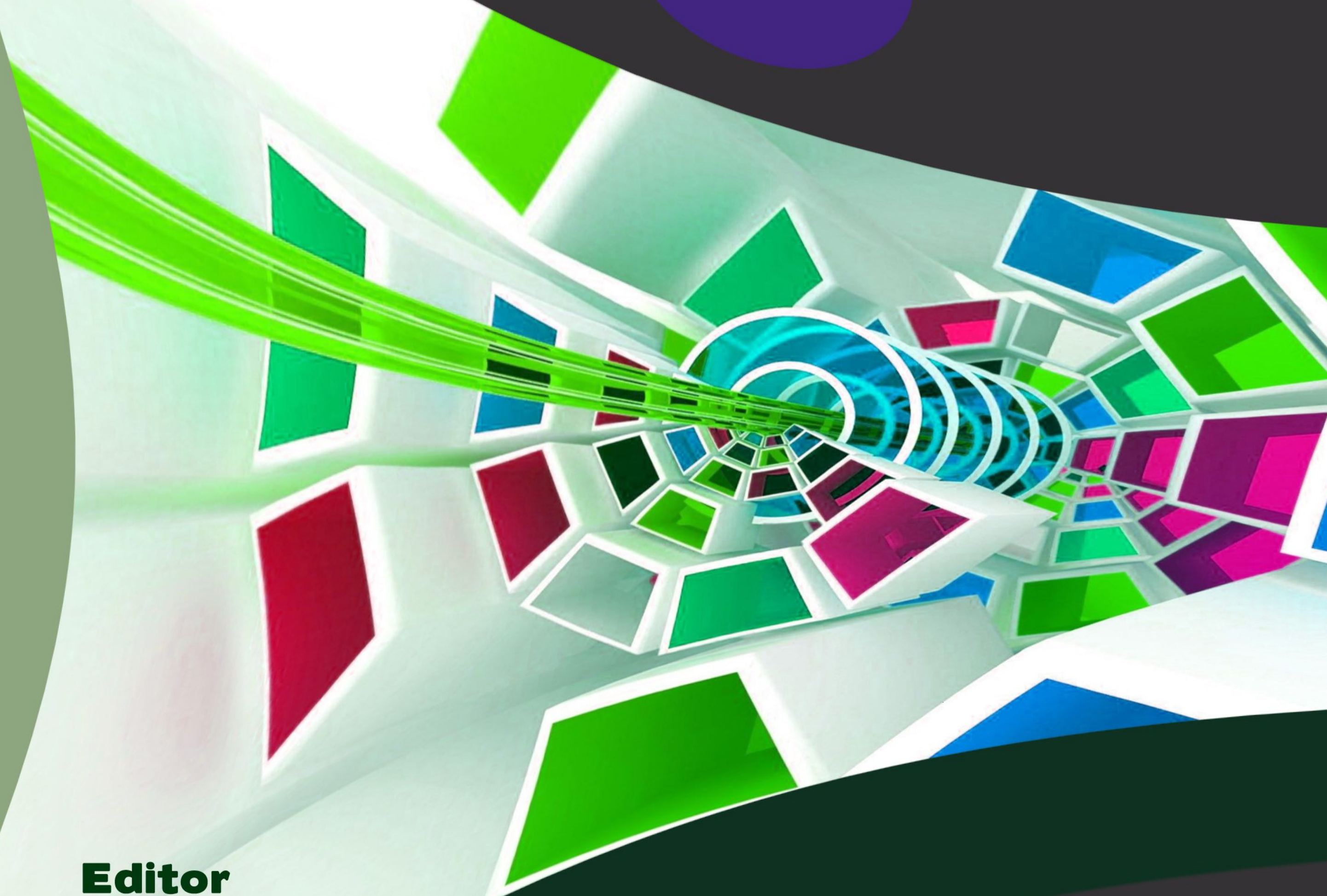
The Minmax Equality and its Technical and Economic Applications



Editor

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Preface

The minmax problem is the most interesting part of the general problem of equilibrium. It is related mainly to the adoption of a prudent strategic behavior and the starting points are the axioms of rational behaviour of decision makers who participate in the decisional process.

The paper proposes a comprehensive approach of the problem starting with the analysis of algebraic and topological conditions which contain solutions, continuing with the presentation of the main algorithms that can be used to solve the problem and ending with solving actual minmax problems encountered in technical economic practice.

The book provides a summary of the main theoretical results found in specialized literature, while the elements of novelty and originality of the work are highlighted as follows: presenting new entropic optimality criteria and specifying the conditions of equivalence; extending the results obtained to a criterion less studied in specialized literature – the equalization criterion; the introduction of a new alignment criterion in the decision theory – the maximum probability criterion - and the analysis of its degree of generality; presenting an entropic solution to the problem of distribution of final outcomes (an issue commonly seen in the context of cooperative game theory); the analysis of the stability of coalitions formed by a special technique based on algebraic and entropic calculations.

We also hope that the paper is a useful tool for professionals working in economic and technical fields, because it provides complete solutions for several important applications, such as: determining the moments of equipment failure, the analysis of mining stability in open mining, establishing the market equilibrium interest, solving important capitalization problems in terms of variable interest.

The author wishes to thank the World Scientific and Engineering Academy and Society (WSEAS) for their support and for the special concern shown towards the publishing of this book under impeccable graphics.

May 2011 Ilie Mitran

Introduction

Decision theory is an important area of applied mathematics, the development of which begins during the first decades of the last century. It is based on fundamental results from mathematical analysis, probabilistic and statistical calculations, numerical calculations, optimization theory etc.

In fact, this new mathematical theory tries to model the most complex movement, the social movement (obviously, in terms of known economic and technical restrictions). In this respect, there were introduced rational behaviour concepts, the concept of utility, optimal decisional behaviour criteria in various situations (cautious behavior, risk behavior, total and partially cooperative cases).

The paper aims to highlight and develop the main results of sequential decision theory emphasizing an important category of optimal points - the equilibrium points.

It consists of six chapters; the first three are theoretical and the last chapters are concerned with actual application. Each chapter concludes with a separate paragraph entitled "Bibliographical Notes and Comments" which is a brief summary of the main results obtained by other authors and of the main open problems encountered (if any).

Chapter 1 is an introductory chapter and presents the main concepts and results needed to develop the next chapters. Primarily, it dwells on the problem of entropy, a concept that is really fascinating for a wide range of mathematicians, engineers and economists.

The problem is not completely analyzed, but it highlights the main types of entropy encountered (weighted and unweighted) and entropic concepts used in the system theory (degree of organization, degree of concentration, forecast).

A separate paragraph of this chapter is dedicated to the analysis of some reference decisional processes; this analysis is being performed only at the level of basic concepts and fundamental results.

The second chapter is entitled “The Minmax Inequality and Equality”. It starts from the fact that for a zero-sum game and two decision makers, a saddle point represents, actually, a particular case of equilibrium point. The double inequality that characterizes a saddle point is equivalent (under some algebraic and topological conditions) to a special optimal equation also known as the minmax equation. One can define the existential conditions of the minmax equality and one may characterize analytically the properties of the solutions of this equation. These goals are achieved both in the situation when there is no information exchange between players and in the (more complicated) case when exchanging information between players is allowed. There are some numerical methods for solving the minmax problem (both in simple strategies and in mixed strategies) and the emphasis falls on two constructive methods (actually two combined methods) due to the author.

The third chapter is entitled “Minmax Decisions” and it constitutes a reference chapter of the paper. Basically, it is an extension of some results presented in the specialized literature and also of some issues approached by the author. First, the principles of optimal behavior are considered for a sequential decision problem in all possible stages: the non-cooperative case, the total cooperative and partially cooperative case. The reference optimal principles for the non-cooperative case are analyzed and extended (the maximum probability criterion, the criterion of maximum profit, the principle of stability etc.) and the results of the equalization principle are also developed (a lesser-known optimal principle). A new coalition criterion is being introduced in decision theory and its properties and its degree of generalization can be analyzed. The novelty of the chapter and, thus, of the entire paper, is the study of coalition stability (through an algebraic-probabilistic method and an entropic method), as well as giving an entropic solution of final gains, knowing that the distribution of final gains remains an open problem of cooperative games.

The last chapters of this work have an applicatory characteristic.

The fourth chapter is entitled “Applications of Minmax Equalities and of Equilibrium Points in the context of Analyzing the Operation Safety of a System”.

The problem of determining the moments of a system failure is extremely difficult, especially when taking into account the structure of the system and the possibility of renewal. Basically, it starts from the graph associated with a system made up of several components, a system subject to the operating - failure - renewal requirements. The graph is immediately associated with a finite difference equation system and a system of differential equations, respectively. Applying the Laplace transformation we are led to solving an extremely difficult algebraic system, the equations being transcendental. Using the method of successive approximations, one can approximate the solutions of these equations and hence the unavailability of the system caused by each component. System availability is determined as a solution of a special maxmin problem and the moments of failure can be found immediately as equilibrium points.

The last part of this chapter analyzes, using the minmax optimization technique, the dependence between the reliability of a system and power consumption. This dependence can be written analytically as a double integral while optimal maxmin and minmax problems can be solved by using Pontriaghin principle of maxmin and some methods specific to the game theory. The results are completed after determining the reliability increasing coefficients and the decrease in consumption of electricity in the case of interventions in moments of equilibrium, as well as the estimating the costs associated to these interventions.

Chapter five is entitled "Applications of Minmax Equality within Problems Regarding Capitalization of Compound Interest" and it provides economic insight for this specific issue.

It starts from the analytical expression of the compound interest capitalization polynomial and it presents the main results of optimal maxmin and minmax problems that occur.

A milestone of this chapter is that it analyzes the situations when the unit interest is variable while the efficiency functions adopted do not allow the use results specific to differential calculus. The results are important and provide interesting economic interpretations.

There are several options to approach the market equilibrium interest calculation; the most rigorous version is when it starts from the analytical expressions of the elasticity coefficients of credit demand and supply.

It is analyzed the situation in which the elasticity coefficients are of polynomial type and unit interest is of equilibrium type; however the market equilibrium interest can only be approximated, not calculated (as the solution of a transcendental equation).

The last chapter of this paper is entitled “Maxmin Optimal Method for Analyzing the Stability of Works in Open Pit Mining”. It dwells on a very important issue from practical point of view, because the exploitation of quarries is often more advantageous (in terms of costs and risk conditions) then the exploitation of underground deposits. On the other hand, the specialized literature does not know a rigorous method of analyzing the stability of mining pits. Basically, in the case of sliding on plane surfaces the mining work is considered to be stable if the stability coefficient is a proper fraction. Since the actual stability coefficient can only be approximated (mainly because of approximate quantification of the working conditions), one cannot say precisely what happens if this coefficient is approximately 1 or even greater. For this reason, Forster's idea seemed interesting: the breaking curve (in section) is not an arc but a section of a normal distribution for which a precise method for calculating the average and the dispersion value is given. Therefore, this chapter further develops this method and it also presents a new method of approximate calculation which determines the center and the radius of the circle according to which the slope is sliding (in fact it is a sequential method).

Acknowledgement

Games theory is the mathematical theory which deals with the substantiation of the decisional methods and strategies used within modeling conflictual situations on global markets. One of the most apparent phenomena in global economy nowadays is the emergence of multinational companies against the globalization context. Many of the multinationals operate within an oligopoly market and the game theory is an indispensable tool for studying and forecasting their behaviour. Researches conducted in Chapter III - “Minmax Decisions”, are part of the research activity carried out by the author during the international project: COST 281/ 2009 Brussels, Action IS0905 - The Emergence of Southern Multinationals and their Impact on Europe. The problem of studying and forecasting the strategic behaviour of multinationals within the globalization context shall be approached in a future paper work in which the emphasis is laid on modeling competitive situations by applying methods and concepts from the game theory.

The Author

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SUBJECT INDEX

Ilie Mitran is Associated Professor of Applied Mathematics at the University of Petrosani, Faculty of Sciences, Department of Mathematics, Romania.

He graduated from the University of Timisoara (Romania) in 1975, receiving the 5 years Diploma in Mathematics, specialization Operations Research. In 1980, he has graduated from the Economic Studies Academy, receiving the 5 years Diploma in Economics, specialization in Cybernetics and Economic Planning. In 1988, he received the Ph.D. in Mathematics, based upon the doctoral thesis "Mathematical Contributions regarding Cybernetic Modelling of the Decisional Process". In 1998, he received the second Ph. D. in Engineering with the doctoral thesis "Contributions regarding the Systems Renewal Modelling".

He published 12 books and monographs, more than 100 papers in various Scientific Journals and International Conference Proceedings.

In June 2009, he was invited to deliver a Plenary Lecture on Criteria of Strategic Behaviour and Optimum Trajectories for the Problem of Sequential Decision, at the 2nd WSEAS International Conference on Finite Elements, Finite Volumes, Boundary Elements (F-and-B'09).

His general research interests are:

Applied Mathematics, Decision Modelling, Econometrics and Operations Research. He is a member of American Mathematical Society (AMS - SUA), Society for Industrial and Applied Mathematics (SIAM - SUA), Mathematical Association of America
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