



Editors

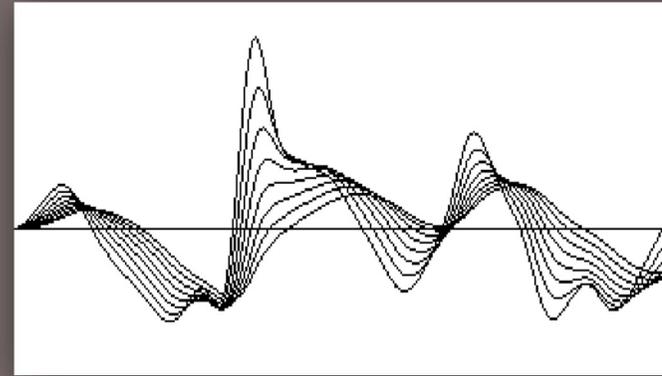
Valeri Mladenov

Nikos Mastorakis



Advanced Topics on Applications of Fractional Calculus on Control Problems, System Stability and Modeling

by Prof. Mihailo Lazarević



ISBN: 978-960-474-348-3

Advanced Topics on Applications of Fractional Calculus on Control Problems, System Stability and Modeling



Advanced Topics on Applications of Fractional Calculus on Control Problems, System Stability and Modeling

Editors

**Prof. Valeri Mladenov
Prof. Nikos Mastorakis**

Authors

**Mihailo Lazarević
Milan R. Rapaic
Tomislav B. Sekara
Sreten B. Stojanovic
Dragutin Lj. Debeljkovic
Zoran Vosika
Goran Lazovic
Jovana Simic-Krstic
Djuro Koruga
Dragan T. Spasic
Andjelka N. Hedrih
Katica R. (Stevanovic) Hedrih**

Advanced Topics on Applications of Fractional Calculus on Control Problems, System Stability and Modeling

Published by WSEAS Press

www.wseas.org

Copyright © 2014, by WSEAS Press

All the copyright of the present book belongs to the World Scientific and Engineering Academy and Society Press. All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise, without the prior written permission of the Editor of World Scientific and Engineering Academy and Society Press.

All papers of the present volume were peer reviewed by two independent reviewers. Acceptance was granted when both reviewers' recommendations were positive.
See also: <http://www.worldses.org/review/index.html>

ISBN: 978-960-474-348-3



World Scientific and Engineering Academy and Society

Preface

In this monograph several aspects of fractional calculus will be presented ranging from its brief history over control applications and stability problems for time delay systems to applications in bio-engineering fields with illustrative examples.

The advantages of fractional calculus have been described and pointed out in the last few decades by many authors. Fractional calculus is based on derivatives and integrals of non integer arbitrary order, fractional differential equations and methods of their solution, approximations and implementation techniques. It has been shown that the fractional order models of real systems are regularly more adequate than usually used integer order models.

The monograph consists of seven chapters and an appendix where related a list of references include in the end of chapters.

The monograph begins in Chapter 1 with a brief historical review of the theory of fractional calculus and its applications. The theory of non-integer order differentiation and integration is almost as old as classical calculus itself, but nevertheless there seems to be an astonishing lack of knowledge of this field in most mathematicians. A look at the historical development can in parts explain the absence of this field in today's standard mathematics textbooks on calculus and in addition give the reader not familiar with this field a good access to the topics addressed in this monograph. In this chapter some well known definitions and properties of fractional order differ integrals are also stated.

Chapter 2 is devoted to the problem of discrete-time (digital) implementation of fractional order systems, i.e. fractional differ integrators, where two novel methods have been closely investigated: direct optimal and indirect. Both methods produce approximations of fractional differ integrators, which are then used to create approximations to more complex fractional order systems. It has been demonstrated by means of a number of numerical examples that both presented methods.

Some of stability problems for time delay systems have been discussed in the two following chapters (Chapters 3, 4). While Lyapunov methods have been developed for stability analysis and control law synthesis of integer linear systems and have been extended to stability of fractional systems, only few studies deal with non-Lyapunov stability of fractional systems. Here, finite-time stability of fractional order time-delay systems is considered in Chapter 3. Sufficient conditions for finite-time stability for (non) linear (non)homogeneous as well as perturbed fractional order time-delay systems are obtained and presented.

The problem of stability (simple stability and robust stability) of linear discrete-time fractional order systems is addressed in Chapter 4 where it is shown that some stability criteria for discrete time-delay systems could be applied with small changes to discrete fractional order state-space systems. The approach is based on the idea of constructing novel Lyapunov-Krasovskii functionals combined with free-weighting matrices or algebraic methods.

The next three Chapters (5, 6, 7) are related to applications of fractional calculus in bio-engineering fields. Chapter 5 is dedicated to the mathematical modeling of skin structure applying fractional calculus where it is proposed the skin structure as a more complex system consisting of several layers which describes series of structures via continuous generalizing (distributed order type) the Cole equation. According to this model and experimental data of the skin bioimpedance measurements, one may predict more complex equivalent electrical circuit and define new time parameters which correspond to each reduced Cole element.

In Chapter 6, a thermodynamically consistent rheological modified Zener model of viscoelastic body, i.e. standard fractional linear viscoelastic body is studied and presented. Proposed model comprises both fractional derivatives of stress and strain and the restrictions on the coefficients that follow from Clausius Duhem inequality. In that way, it should be included in both analytical and experimental projects ab initio, particularly in experiments in which newly developed materials are tested.

Finally, Chapter 7 concludes this monograph showing an useful modeling double DNA helix main chains of the free and forced fractional order vibrations applying fractional calculus. Different models are focusing on different aspects of the DNA molecule (biological, physical and chemical processes in which DNA is involved). The aim of this study was to model the DNA dynamics (vibrations of DNA chains) as a biological system in a specific boundary condition that are possible to occur in a life system during regular function of a DNA molecule.

I hope that this monograph will be value to Ph.D. students and fractional systems researchers as well as the other readers will find something in this monograph exciting.

Also, I want to thank very much Mrs. Ranki Gajic for the support in the preparation of the manuscript for English edition.

Belgrade, August, 2012

Dr Mihailo Lazarević
(Михаило Лазаревић)
University of Belgrade
Faculty of Mechanical Engineerig
Belgrade, Serbia

Acknowledgements

Authors gratefully acknowledge the support of Ministry of Education, Science and Technological Development of the Republic of Serbia under the projects: No.35006, No.41006, No.174016, No. ON174001, No.33020 as well as works on this book were partially supported through NATO Collaborative Linkage Grant No 984136.

The Authors

Table of Contents

Preface	iii
Acknowledgements	v
Part I. Introduction to Fractional Calculus	
1 Introduction to Fractional Calculus with Brief Historical Background	3
<i>by Mihailo P. Lazarevic, Milan R. Rapaic, Tomislav B. Sekara</i>	
1.1 Brief History of Fractional Calculus	4
1.2 Basic Definitions of Fractional Order Differintegrals	9
1.3 Basic Properties of Fractional Order Differintegrals	12
<i>References</i>	15
Part II. Control and Stability Issues	
2 Direct and Indirect Method for Discretization of Linear Fractional Systems	19
<i>by Tomislav B. Sekara, Milan R. Rapaic</i>	
2.1 Introduction	19
2.2 Motivation and Formulation of the Problem	21
2.3 The Direct Optimal Method for Discretization of Fractional Integrators	24
2.3.1 The Optimality Criterion	24
2.3.2 Optimal Discrete Approximations of the Fractional Integrators	25
2.3.3 Remarks on the Optimization Procedure	29
2.4 Indirect Method for Discretization of Fractional Integrators	29
2.5 Examples	33
2.6 Conclusions	38
<i>References</i>	39
3 Finite-Time Stability of Fractional Order Time-Delay Systems	43
<i>by Mihailo P. Lazarevic</i>	
3.1 Introduction	43
3.2 Preliminaries on Integer Time-Delay Systems	45
3.2.1 Some Previous Results Related to Integer Time-Delay Systems	47
3.3 Preliminaries on Stability of Fractional Order Systems Including Time-Delays	48
3.3.1 A Review on Stability of Fractional Order Time Delay System	51
3.4 Finite-Time Stability of Fractional Order Time-Delay Systems	54
3.5 Conclusion	62
<i>References</i>	63
4 Stability of Discrete-Time Fractional Order Systems:	67
An Approach based on Stability of Discrete-Time Integer Order Time-Delay Systems	
<i>by Sreten B. Stojanovic, Dragutin Lj. Debeljkovic, Mihailo P. Lazarevic</i>	
4.1 Introduction	67
4.2 Problem Formulation	69
4.2.1 The Discrete-Time Fractional Order System as Discrete-Time Linear Integer Order Time Delay System: Stability Issue	69
4.3 The Problem Solution	71
4.3.1 Stability of Linear Discrete Time Delay Systems	71
4.3.1.1 Delay-Independent Stability	72
4.3.1.2 Delay-Dependent Stability	80
4.4 Conclusion	83
<i>References</i>	84
Part III. Modeling	
5 Modeling of Human Skin using Distributed Order Fractional Derivative	91

Model-Frequency Domain		
<i>by Zoran Vosika, Mihailo Lazarevic, Goran Lazovic, Jovana Simic-Krstic, Djuro Koruga</i>		
5.1	Introduction	91
5.2	Distributed Order Type Fractional Derivative Model of Impedance	93
5.2.1	Some Basic Results Related to Dielectric Properties of Materials	93
5.2.2	Basic Facts Related to Bio-Impedance of Human Skin	94
5.3	Distributed Order Type Fractional Derivative Model of Impedance	95
5.3.1	Fractional Calculus Preliminaries	95
5.3.1.1	Basic Definitions	95
5.3.1.2	Distributed Caputo Derivatives and Integrals	95
5.3.1.3	Distributed Caputo-Weyl Derivatives and Integrals	96
5.3.2	Cole and Distrubeted Order Cole Element	98
5.3.3	Materials and Methods	100
5.3.4	Results and Discussion	101
5.4	Conclusion	103
	<i>References</i>	104
6	A Thermodynamically Consistent Rheological Model for Engineering Applications	107
<i>by Dragan T. Spasic</i>		
6.1	Introduction	107
6.2	The Modified Zener Model	110
6.3	The Applications	114
6.3.1	The Impact Against a Rigid Wall - An Ideal Case	114
6.3.2	The Forced Vibrations with Fractional Type of Dissipation Pattern	119
6.3.3	A Column-Like Structure under Seismic Load	124
6.3.4	A Geometrically Nonlinear Problem	129
6.3.5	The Impact Problem in the Presence of Dry Friction	134
6.4	Conclusion	141
	<i>References</i>	141
7	Modeling Double DNA Helix Main Chains of the Free and Forced Fractional Order Vibrations	145
<i>by Andjelka N. Hedrih, Katica R. (Stevanovic) Hedrih</i>		
7.1	Introduction - DNA-Structure and Function	145
7.2	Mechanical Properties of DNA Achieved Experimentally	147
7.3	Mechanical Models of the DNA	147
7.4	DNA Models by N. Kovaleva and L. Manevich	148
7.5	Modified DNA Models by N. Kovaleva and L. Manevich for the Forced Regimes	150
7.6	Consideration of the Basic DNA Model - Linearized Kovaleva-Manevich's DNA Model	150
7.6.1	Consideration of the Free Vibrations of a Basic DNA Model - Linearized Kovaleva-Manevich's DNA Model	151
7.6.2	Boundary Conditions of the Double DNA Chain Helix	153
7.6.3	Consideration of the Forced Vibrations of a Basic DNA Model - Linearized Kovaleva-Manevich's DNA Model	154
7.6.4	Consideration of the Forced Vibration Regimes of a Basic DNA Model - Linearized Kovaleva-Manevich's DNA Model-Resonance and Dynamical Absorption	159
7.7	The Double DNA Fractional Order Chain Model on the Basis of the Linearized Kovaleva-Manevich's DNA Models for Free and Forced Vibrations	160
7.7.1	Constitutive Relation of the Standard Light Fractional Order Creep Element	160
7.7.2	The Double DNA Fractional Order Chain Free Vibration Model on the Basis of the Linearized Kovaleva-Manevich's DNA Model	160
7.7.3	Analytical Solutions of the Subsystems of the Main Chains Fractional Order Differential Equations for Free Fractional Order Vibrations	163

7.7.4	Main Coordinates of the Fractional Order Double DNA Helix Chain System and Corresponding Partial Fractional Order Oscillators	168
7.7.5	Visualization of the Main Modes of Fractional Order Double DNA Helix Chain System Free Vibrations and Corresponding Partial Fractional Order Oscillator Modes	171
7.7.6	The Double DNA Fractional Order Chain Forced Vibration Model on the Basis of the Linearized Kovaleva-Manevich's DNA Model	172
7.7.7	Analytical Solutions of the Subsystems of the Main Chains Fractional Order Differential Equations for Forced Regime Oscillations	174
7.7.8	Forced Eigen Modes of the Subsystems of the Main Chains of a Fractional Order Double DNA Helix Chain System Forced Vibrations	176
7.8	Concluding Remarks	180
	<i>References</i>	181
	Appendix	185
	Subject Index	201

SUBJECT INDEX

A

Adams-Bashforth Corrector, 31
 Asymptotically Stable, 49, 50, 51, 53, 71-81
 Autonomous, 44, 49, 50, 53, 55, 60, 61, 69,
 70, 151, 152

B

Banach Space, 46
 Bellman-Gronwall Inequality, 43, 44
 Bessel Functions, 23
 BIBO Stability, 44, 49, 53, 54
 Bioimpedance, 92

C

Caputo, 3, 8-13, 44, 48-50, 53, 54, 57, 68,
 95-97, 100
 Cauchy's Integral Formula, 6
 Cauchy Problem, 119, 135
 Caputo-Weyl Derivative, 95, 96
 Characteristic Matrix, 53
 Clausius Duhem Inequality, 107, 110, 119,
 123-125, 133
 Cole-Cole Model, 92, 93
 Cole-Davidson Function, 92
 Collision, 115, 134
 Commensurate, 44, 48, 50
 CRONE, 19

D

Delay Differential Equations (DDEs), 43
 Delay-Independent Stability, 67, 72
 Delay-Dependent Stability, 67
 Degrees of Freedom, 3, 123, 148, 155, 157,
 158, 161, 170, 195-197
 Discretization, 19, 20-24, 29-32, 35, 38, 69,
 114, 136
 Diffusion Equation, 21
 Distributed Order, 92, 93, 95, 98, 99, 103
 Double DNA Helix Chain, 145, 151, 154,
 157-162, 168-172, 176, 179, 180
 Dry Friction, 107, 108, 119, 133, 134, 137, 141

E

Eigenvalues, 50, 51, 53, 80, 81
 Eigenvectors, 81
 Eigen Circular Frequencies, 151, 152, 153,
 159, 169, 170, 178, 180
 Erdely-Kober Definition, 9

F

FIR Filter, 23
 Finite-Time Stability, 43-47, 54, 57, 58, 61, 62
 Fractional Derivative, 3-13, 44, 48, 50, 53, 54,
 57, 68, 91-97, 103, 107, 109, 110, 113, 114,
 118-125, 129, 133, 200
 Fractional Integral, 6-13, 29, 95, 186
 Fractional Order Systems, 3, 13, 14, 19-21, 34,
 38, 43, 44, 48, 51, 54, 57, 59, 60, 62, 67-71,
 75-77, 78, 83
 Fractional Order Controllers, 13, 19
 Forced Vibrations, 119, 120, 145, 150, 151,
 154, 160, 172, 174, 176, 177, 180
 Frequency Analysis, 91

G

Gamma Function, 4, 6, 7, 10, 55, 58, 61, 69,
 95, 110, 185, 186
 Generalized Coordinate, 177, 192
 Grunwald-Letnikov Definition, 7, 9, 12, 20,
 48, 68

H

Havriliak-Negami Function, 92
 Heat Conduction, 21, 22
 Heat Equation, 21, 52
 Heaviside Step Function, 111
 Hereditary Properties, 3, 197, 198
 Hooke Law, 108
 Human Skin, 91-94, 101-103, 189, 190

I

IIR Filter, 23
 Incommensurate, 48
 Initial Value, 49, 52, 56, 199
 Initial Condition, 8, 10-13, 20, 44, 47-60, 95,
 98, 115, 121, 124, 130, 131, 134, 135, 163,
 166-170, 174, 177, 200
 Infinite-Dimensional, 51
 Integro-Differential Inclusions, 107

K

Kelvin-Voigt Viscoelastic Body, 108

L

Lambert Function, 44, 52
 Least-Squares Approximation, 28
 Linear Matrix Inequality (LMI), 44, 51
 Lyapunov-Razumikhin Function, 44, 53

Lyapunov-Krasovskii Functional, 44, 53, 67, 83
 Lyapunov's Second Method, 43

M

Marchaud Fractional Derivative, 7
 Matrix Measure, 43, 46
 Mikhailov Stability Criterion, 44
 Mittag-Leffler Function, 60, 62, 111, 113, 186-188
 Modified Zener Model, 107, 110, 112, 113, 115, 120, 123, 131, 133
 Multi-Pendulum Model, 145, 154, 161

N

Neutral Type, 44, 53
 Nonhomogenous, 45
 Norm, 46, 55, 59, 67, 71, 72
 Nonautonomous, 47, 48, 50, 58, 60, 61, 154
 Nyquist Frequency, 25, 29, 44

O

Optimality Criterion, 19, 24, 29, 32, 34, 38, 112
 Oustaloup's Rational Approximation (ORA), 20, 29, 31, 32, 38

P

Pade Approximation, 30
 Particle Swarm Optimization (PSO), 19, 29, 112
 Parameter Perturbations, 45
 PID Regulator, 21, 23
 Pnost's Inversion Formula, 114, 116, 117, 119, 121
 Practical Stability, 43, 44, 46, 54, 68

R

Razumikhin Stability Theory, 44, 53
 Rational Approximations, 19, 21, 29, 38
 Rheological Model, 107, 108, 109, 141
 Riemann-Liouville, 3, 6, 12-15, 44, 48-50, 53, 54, 68, 95, 96, 110, 115, 120, 125, 133, 288
 Riesz Potential, 8
 Riemann Sheet, 44, 53
 RNA, 147-150
 Routh-Hurwitz Criterion, 43
 Root-Locus, 44, 48
 Robust Stability, 44, 48, 67, 68, 71, 77, 83
 Rouche's Theorem, 44, 53

S

Seismic Load, 123, 124
 Skin Layer, 91, 92, 190
 Slack Variable, 107, 133, 135, 137
 Solvent, 80-83
 Stress-Strain Relation, 108, 109
 Standard Fractional Linear Viscoelastic Body, 107, 137

T

Time-Delay, 43-62, 67-81
 Time-Delay Systems, 43-47, 51-54, 58, 60, 62, 67-71, 81, 83
 Transcendental Characteristic Equation (TCE), 52
 Transfer Function, 13, 19-22, 48, 52, 53
 Tustin Operator, 20, 28

U

Uncertain Matrix, 70

V

Viscosity, 21
 Volterra Integral Equation, 55, 59

W

Weyl Fractional Integral, 7, 8, 95



Mihailo P. Lazarević is a Professor at the University of Belgrade, Faculty of Mechanical Engineering, Department of Mechanics in Serbia.

Mihailo P. Lazarević was born in Belgrade (Serbia) in 1964. He received four degrees from Belgrade University.

- *B.Sc in Mechanical Engineering (Aerospace),*
- *B.Sc in Electrical Engineering,*
- *M.Sc in Control and System Science Engineering and*
- *Ph. D. in Mechanical Engineering (Robotics).*

He worked at the University of Belgrade at the Faculty of Mechanical Engineering, Department of Mechanics as an Assistant from 1996-2001. He became an Assistant Professor in 2001 and Associate Professor in 2005 and since 2009 he is a full Professor at the same Faculty.

He is a member of the Serbian Society of Mechanics and the Secretary of the Serbian Society of Mechanics since July 2013. During the years of 2009 to 2012 he was the Chief of the Laboratory of Applied Mechanics of the Faculty of Mechanical Engineering in Belgrade University.

He published 4 National Monographs, 4 Chapters in International Monographs, 7 articles in Leading International Journals, 8 articles in International Journals, 12 articles in National Journals, 80 articles in the proceedings of International Meetings, Conferences and Symposiums, 1 Book and 2 Handbooks as collections of solutions and solved problems. Also, he participated in several research projects in Serbia and abroad (EUREKA) related to the field of (bio)robotics and applications of Fractional Calculus.

He is engaged as a reviewer by several International Journals and organizer of many International/National Conferences. According to Scopus, he has over 120 citations and his H index is 7.

His scientific research can be divided into several thematic areas among them the Mathematical Modeling and Control of Rigid Bodies by Electrical Devices (Robotics and Biomechatronics) which has been his first research interest. He has also been working on extension of ideas of Fractional Calculus to Modeling of Mechatronic Systems and Biomechanical Tissues.

In addition, a wide range of topics of his interest are in Different Classes of Time Delay Systems, in Theory of Electroviscoelasticity, in Applications of Fractional PID, in Fractional Iterative Learning Control and in Applications of Fractional Wavelet Transform in Signal and Image Processing.

