Robust Tracking and Disturbance Rejection for Descriptor-Type Neutral Time-Delay Systems

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Abstract—The problem of designing a controller to achieverobust stability and asymptotic tracking despite certain disturbancesis considered for linear time-invariant descriptortypeneutral time-delay systems. The necessary and sufficientconditions for the solvability of this problem are derived. Inthe case a solution exists, the structure of the controller whichsolves this problem is also presented.

Key Words: Systems, Systems Theory, Control, Time-Delay Systems

1. Introduction

Two general requirements in controller design are stability and performance. One of the most important performance requirements is asymptotic tracking, where a certain output of the system is required to track a given reference signal asymptotically. Furthermore, in many cases, such a tracking must be achieved despite certain disturbances affecting the system. Moreover, since an exact model of any physical system, in general, can not be obtained, both stability and tracking must be robust against perturbations in the system model. The problem of designing a robust controller to achieve stability and asymptotic tracking despite disturbances is usually called as the *robust servomechanism problem*. This problem has extensively been studied for linear time-invariant (LTI) finite-dimensional systems (e.g., see [1]–[11]). Besides finite-dimensional LTI systems, finite-dimensional nonlinear [12]–[14] and discrete-event [15] systems have also been studied.

Many systems, however, may involve time-delays. Since the state of such systems, usually called as *time-delay systems*, can not be represented by a finite-dimensional vector these systems are infinite-dimensional [16]. The dynamics of these systems can, in general, be described by delay-differential equations [17]. For such systems, when the delay-differential equations describing the system dynamics do not involve delayed versions of the highest derivative, the system is said to be *retarded*. Otherwise, it is said to be *neutral*.

However, delay-differential equations alone may not be sufficient to describe the dynamics of some time-delay systems. One may need to use delay-algebraic equations coupled with delay-differential equations to describe the dynamics of some time-delay systems [18]. These systems are inherently neutral [19] and sometimes are called as *descriptor-type* systems [20].

Earlier considerations of the robust servomechanism problem for time-delay systems can be found, e.g., in [21]–[23]. The necessary and sufficient conditions for the solvability of the robust servomechanism problem for LTI time-delay systems, has first appeared in [24]. In [24], however, only retarded time-delay systems were considered. Neutral time-delay systems were later considered in [25]. The consideration in [25], however, was restricted to non-descriptor-type systems. In the present work, we extend the results of [25] to LTI descriptor-type time-delay systems. We first formally state the *strong* and *weak* versions of the robust servomechanism problem for descriptor-type neutral time-delay systems. We then derive the necessary and sufficient conditions for the solvability of both problems. The structure of the controller, which solves these problems (when exists) is also presented.

Throughout the paper, $\det(\cdot)$ and $\mathrm{rank}(\cdot)$ respectively denote the determinant and the rank of the indicated matrix. For positive integers k and l, I_k, 0_k, and $0_{k\times l}$ respectively denote the $k \times k$ dimensional identity matrix, the $k \times k$ dimensional zero matrix, and the $k \times l$ dimensional zero matrix. When the dimensions are apparent, I and 0 are used to denote, respectively, the identity and the zero matrices of appropriate dimensions. $\mathbf{R}^k$ and $\mathbf{R}^{k\times l}$ respectively denote the spaces of k dimensional real vectors and $k \times l$ dimensional real matrices. For $\xi : \mathcal{I} \to \mathbf{R}^l$, where $\mathcal{I}$ is an interval of the real line, $\dot{\xi}$, $\ddot{\xi}$, and $\xi^{(k)}$ respectively denote the first, the second, and the kth derivative of ξ. Finally, $\otimes$ denotes the Kronecker product.

2. Problem Statement

We consider a LTI descriptor-type neutral time-delay system with ν discrete time-delays whose dynamics are described as

$$\dot{x}_1(t) + \sum_{i=1}^{\nu} F_i^1 \dot{x}_1(t - h_i) = \sum_{i=0}^{\nu} \left[A_i^{11} x_1(t - h_i) \right.$$
$$\left. + A_i^{12} x_2(t - h_i) + B_i^1 u(t - h_i) + G_i^1 w(t - h_i) \right] \quad (1)$$

$$0 = \sum_{i=0}^{\nu} \left[A_i^{21} x_1(t - h_i) + A_i^{22} x_2(t - h_i) \right.$$
$$\left. + B_i^2 u(t - h_i) + G_i^2 w(t - h_i) \right] \quad (2)$$

where $x_1(t) \in \mathbf{R}^{n_1}$ and $x_2(t) \in \mathbf{R}^{n_2}$ are the state vectors for the delay-differential and delay-algebraic parts, respectively, at time t. Furthermore, $u(t) \in \mathbf{R}^{n_u}$ is the *control input* and

$w(t) \in \mathbf{R}^{n_w}$ is the *disturbance input* at time t. Moreover, $h_i > 0$, $i = 1, \ldots, \nu$, are the time-delays. We use $h_0 := 0$ for notational convenience (i.e., $i = 0$ in (1)–(2) corresponds to the delay-free part of the system). All the matrices shown in (1)–(2) are appropriately dimensioned constant matrices. It is assumed that $\det\left(A_0^{22}\right) \neq 0$. This assumption guarantees existence and uniqueness of solutions to (1)–(2) under appropriate initial conditions [16].

The system (1)–(2) is assumed to have two kinds of outputs:

i) The *output to be regulated*, $z(t) \in \mathbf{R}^{n_z}$, which is given as:

$$z(t) = \sum_{i=0}^{\nu} \left[C_i^{11} x_1(t - h_i) + C_i^{12} x_2(t - h_i) \right. \\ \left. + D_i^1 u(t - h_i) + E_i^1 w(t - h_i) \right] \quad (3)$$

ii) The *measured output*, $y(t) \in \mathbf{R}^{n_y}$, which is given as:

$$y(t) = \sum_{i=0}^{\nu} \left[C_i^{21} x_1(t - h_i) + C_i^{22} x_2(t - h_i) \right. \\ \left. + D_i^2 u(t - h_i) + E_i^2 w(t - h_i) \right] \quad (4)$$

All the matrices shown in (3)–(4) are appropriately dimensioned constant matrices. Here, the measured output (which, hereafter, will be called the *measurement*), $y(t)$, is available to the controller, which is to determine the control input (which, hereafter, will be called the *input*), $u(t)$. The output to be regulated (which, hereafter, will be called the *output*), $z(t)$, which is not directly available to the controller, on the other hand, is required to track a *reference*, $r(t)$, asymptotically. i.e., it is required that

$$\lim_{t \to \infty} e(t) = 0 \quad (5)$$

where

$$e(t) := z(t) - r(t) \quad (6)$$

is the *tracking error* (which, hereafter, will be called the *error*).

The reference, $r(t)$, is assumed to be available on-line but not known in advance. The disturbance input (which, hereafter, will be called the *disturbance*), $w(t)$, on the other hand, is assumed to be neither available, nor measurable, nor known in advance. It is, however, assumed that both $r(t)$ and $w(t)$ satisfy

$$\mathcal{D}r(t) = 0 \quad \text{and} \quad \mathcal{D}w(t) = 0 \, , \quad (7)$$

where

$$\mathcal{D} := \sum_{i=0}^{\nu} \sum_{j=0}^{\mu} a_i^j p^j \delta_{h_i} \quad (8)$$

is a linear delay-differential operator of neutral-type, where μ is the differential degree of $\mathcal{D}$, a_i^j's are constant coefficients, $p^j := \frac{d^j}{dt^j}$ is the differentiation operator of order j, and δ_h is the delay operator by h (i.e., $\delta_h f(t) = f(t - h)$, for any (vector) function $f(\cdot)$). Here, it is assumed that $a_0^\mu \neq 0$. Thus, hereafter, without loss of generality, we will assume $a_0^\mu = 1$.

Remark 1: The time-delays of the system (1)–(4) and of the operator $\mathcal{D}$ are assumed to be the same for notational simplicity. This assumption can be made without loss of generality, since any time-delay of the system, which is not a time-delay of $\mathcal{D}$, can be included in (8) by taking the corresponding coefficients as zero and any time-delay of the operator $\mathcal{D}$, which is not a time-delay of the system, can be included in (1)–(4) by taking the corresponding matrices as zero.

Remark 2: The assumption that $r(t)$ and $w(t)$ satisfy the same delay-differential equation is also made for notational simplicity. This assumption can also be made without loss of generality, since if, say, $\mathcal{D}^r r(t) = 0$ and $\mathcal{D}^w w(t) = 0$, for two different linear delay-differential operators, $\mathcal{D}^r$ and $\mathcal{D}^w$, of neutral-type, then they satisfy (7), where $\mathcal{D}$ is the least common multiple of $\mathcal{D}^r$ and $\mathcal{D}^w$.

The formal statement of our problem then is as follows.

Problem 1: Design a controller to determine $u(t)$, using $y(t)$, and $r(t)$, such that the overall closed-loop system is globally asymptotically stable and, for all $r(t)$ and $w(t)$ satisfying (7), for all initial conditions of the system (1)–(2), and for all non-destabilizing (for the closed-loop system) perturbations in the matrices appearing in (1)–(4), (5) is satisfied.

A weaker version of this problem can also be stated:

Problem 2: Design a controller to determine $u(t)$, using $y(t)$, and $r(t)$, such that the overall closed-loop system is globally asymptotically stable and, for all $r(t)$ and $w(t)$ satisfying (7), for all initial conditions of the system (1)–(2), and for all non-destabilizing (for the closed-loop system) perturbations in the matrices appearing in (1)–(2), (5) is satisfied.

Note that the only difference between the two problems is that, in the weaker version (Problem 2) no perturbations in the output and the measurement matrices are allowed.

To present a solution to either problem, let us first note that the system (1)–(4) can be compactly written as

$$\sum_{i=0}^{\nu} F_i \dot{x}(t - h_i) = \sum_{i=0}^{\nu} \left[A_i x(t - h_i) \right. \\ \left. + B_i u(t - h_i) + G_i w(t - h_i) \right] \quad (9)$$

$$z(t) = \sum_{i=0}^{\nu} \left[C_i^1 x(t - h_i) + D_i^1 u(t - h_i) + E_i^1 w(t - h_i) \right] \quad (10)$$

and

$$y(t) = \sum_{i=0}^{\nu} \left[C_i^2 x(t - h_i) + D_i^2 u(t - h_i) + E_i^2 w(t - h_i) \right] \quad (11)$$

where $x(t) := \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \in \mathbf{R}^{n_x}$ ($n_x := n_1 + n_2$) is the overall state vector at time t, and, for $i = 0, \ldots, \nu$,

$$F_i := \begin{bmatrix} F_i^1 & 0 \\ 0 & 0_{n_2} \end{bmatrix} , \quad A_i := \begin{bmatrix} A_i^{11} & A_i^{12} \\ A_i^{21} & A_i^{22} \end{bmatrix} ,$$

where $F_0^1 := I_{n_1}$,

$$B_i := \begin{bmatrix} B_i^1 \\ B_i^2 \end{bmatrix} , \quad G_i := \begin{bmatrix} G_i^1 \\ G_i^2 \end{bmatrix} , \quad C_i^1 := \begin{bmatrix} C_i^{11} & C_i^{12} \end{bmatrix} ,$$

and $C_i^2 := \begin{bmatrix} C_i^{21} & C_i^{22} \end{bmatrix}$.

Next, let us note that, the signals $r(t)$ and $w(t)$ satisfying (7) can be represented as

$$r(t) = C^r q(t) \quad \text{and} \quad w(t) = C^w q(t) , \qquad (12)$$

where C^r and C^w are arbitrary constant matrices and $q(t) \in \mathbf{R}^\mu$ is the solution to the *fictitious system*:

$$\sum_{i=0}^{\nu} \mathcal{F}_i \dot{q}(t - h_i) = \sum_{i=0}^{\nu} \mathcal{A}_i q(t - h_i) , \qquad (13)$$

with arbitrary initial condition $q(\theta)$, $\theta \in [-h_{max}, 0]$, where $h_{max} := \max\{h_i , \ i = 1, \ldots, \nu \mid a_i^j \neq 0 , \text{ for at least one } j \in \{0, \ldots, \mu\}\}$ (the actual initial condition and the actual C^r and C^w define the actual signals $r(t)$ and $w(t)$; however, since these signals are assumed to be unknown, both the initial condition $q(\theta)$, $\theta \in [-h_{max}, 0]$, and the matrices C^r and C^w are arbitrary), where $\mathcal{F}_0 := I_\mu$ (this follows from the assumption that $a_0^\mu = 1$, which guarantees existence and uniqueness of solutions to (13)),

$$\mathcal{A}_0 := \begin{bmatrix} 0 & 0 & \cdots & 0 & -a_0^0 \\ 1 & 0 & & & -a_0^1 \\ 0 & 1 & & & -a_0^2 \\ \vdots & & \ddots & & \vdots \\ 0 & & & 1 & -a_0^{\mu-1} \end{bmatrix} ,$$

and, for $i = 1, \ldots, \nu$, $\mathcal{F}_i := \begin{bmatrix} 0_{\mu-1} & 0 \\ 0 & a_i^\mu \end{bmatrix}$, and

$$\mathcal{A}_i := \begin{bmatrix} 0 & 0 & \cdots & 0 & -a_i^0 \\ 0 & 0 & \cdots & 0 & -a_i^1 \\ 0 & 0 & \cdots & 0 & -a_i^2 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & -a_i^{\mu-1} \end{bmatrix} .$$

Finally, let us state the following assumptions:

Assumption 1: The state $q(t)$ of the fictitious system (13) is observable through the output

$$\begin{bmatrix} r(t) \\ w(t) \end{bmatrix} = \begin{bmatrix} C^r \\ C^w \end{bmatrix} q(t) . \qquad (14)$$

Assumption 2: $\mathrm{rank}(\mathcal{G}) = n_w$, where $\mathcal{G} := \begin{bmatrix} \mathcal{G}_0 \\ \vdots \\ \mathcal{G}_\nu \end{bmatrix}$ and $\mathcal{G}_i := \begin{bmatrix} G_i \\ E_i^1 \\ E_i^2 \end{bmatrix}$, $i = 0, 1, \ldots, \nu$.

Assumption 3: For any solution $q(t)$ of (13), $\lim_{t \to \infty} q(t) = 0$, only if $q(\theta) = 0$, $\forall \theta \in [-h_{max}, 0]$.

Assumption 4: $\mathrm{rank}(\mathcal{B}) = n_u$, where $\mathcal{B} := \begin{bmatrix} B_0 \\ \vdots \\ B_\nu \end{bmatrix}$.

Assumption 5: $\mathrm{rank}(\mathcal{C}) = n_z$, where $\mathcal{C} := \begin{bmatrix} C_0^1 & \cdots & C_\nu^1 \end{bmatrix}$.

Remark 3: Assumptions 1–5 are made to avoid triviality. All these assumptions can be made without loss of generality. Assumption 1 means that what is produced by the fictitious system determines either the reference or the disturbance. If any part of this system is not observable through both $r(t)$ and $w(t)$, then this part can be removed. If Assumption 2 does not hold, then the linearly dependent columns of $\mathcal{G}$ can be removed together with the corresponding elements of w. Assumption 3 means that the fictitious system is totally unstable. Since our problem is concerned with asymptotic tracking, if the fictitious system has any stable part, then this part can be removed. If Assumption 4 does not hold, then the linearly dependent columns of $\mathcal{B}$ can be removed together with the corresponding elements of u. Finally, if Assumption 5 does not hold, then the linearly dependent rows of $\mathcal{C}$ can be removed together with the corresponding elements of z.

3. Main Results

In this section we will present the necessary and sufficient conditions for the solvability of Problems 1 and 2. We will also present the structure of the controller when a solution exists. For this, let us first define

$$\hat{\mathcal{F}}_i := \mathcal{F}_i \otimes I_{n_z} , \quad \hat{\mathcal{A}}_i := \mathcal{A}_i \otimes I_{n_z} , \quad i = 0, \ldots, \nu ,$$

and

$$\hat{\mathcal{B}} := \begin{bmatrix} 1 \\ 0_{\mu-1 \times 1} \end{bmatrix} \otimes I_{n_z} .$$

Let us also define the following system

$$\sum_{i=0}^{\nu} \begin{bmatrix} F_i & 0 \\ 0 & \hat{\mathcal{F}}_i \end{bmatrix} \dot{\xi}(t - h_i) = \sum_{i=0}^{\nu} \left(\begin{bmatrix} A_i & 0 \\ \hat{\mathcal{B}} C_i^1 & \hat{\mathcal{A}}_i \end{bmatrix} \xi(t - h_i) \right.$$
$$\left. + \begin{bmatrix} B_i \\ \hat{\mathcal{B}} D_i^1 \end{bmatrix} v(t - h_i) \right) \qquad (15)$$

where $\xi(t) \in \mathbf{R}^{n_x + n_z \mu}$ is the state and $v(t) \in \mathbf{R}^{n_u}$ is the input vector at time t. Our first result gives necessary and sufficient conditions for the solvability of Problem 1.

Theorem 1: Under Assumptions 1–5, there exists a solution to Problem 1 if and only if the following are satisfied:

1) The system (9) is stabilizable through the input u and is detectable through the measurement y, given in (11).
2) The system (15) is stabilizable through the input v.
3) The measurement y, given in (11), *contains* the output z, given in (10), i.e., there exists $T \in \mathbf{R}^{n_z \times n_y}$, such that $z(t) = Ty(t)$, $\forall t \geq 0$.

Proof: We first prove the *only if* part. If Condition 1 is not satisfied, then there esxists no controller which can stabilize the given system. Hence, Condition 1 is necessary.

To show the necessity of Condition 2, let us define

$$\tilde{e}(t) := \begin{bmatrix} e^{\mu-1}(t) \\ \vdots \\ e^1(t) \\ e(t) \end{bmatrix} \qquad (16)$$

where

$$e^1(t) := \sum_{i=0}^{\nu} \left[a_i^\mu \dot{e}(t - h_i) + a_i^{\mu-1} e(t - h_i) \right]$$

$$e^2(t) := \sum_{i=0}^{\nu} \left[a_i^\mu \ddot{e}(t - h_i) + a_i^{\mu-1} \dot{e}(t - h_i) + a_i^{\mu-2} e(t - h_i) \right]$$

$$\vdots$$

$$e^{\mu-1}(t) := \sum_{i=0}^{\nu} \left[a_i^\mu e^{(\mu-1)}(t - h_i) + \ldots + a_i^1 e(t - h_i) \right].$$

Then, we obtain

$$\sum_{i=0}^{\nu} a_i^\mu \dot{e}(t - h_i) = e^1(t) - \sum_{i=0}^{\nu} a_i^{\mu-1} e(t - h_i), \quad (17)$$

$$\dot{e}^j(t) = e^{j+1}(t) - \sum_{i=0}^{\nu} a_i^{\mu-j-1} e(t - h_i), \quad j = 1, \ldots, \mu - 2, \quad (18)$$

and

$$\dot{e}^{\mu-1}(t) = \mathcal{D}e(t) - \sum_{i=0}^{\nu} a_i^0 e(t - h_i). \quad (19)$$

Next, let $\zeta(t) := \begin{bmatrix} \tilde{x}(t) \\ \tilde{e}(t) \end{bmatrix}$, where $\tilde{x}(t) := \mathcal{D}x(t)$. Also let $\tilde{u}(t) := \mathcal{D}u(t)$. Then, using (9), (10), (6), and (7), (17)–(19) gives

$$\sum_{i=0}^{\nu} \begin{bmatrix} F_i & 0 \\ 0 & \hat{\mathcal{F}}_i \end{bmatrix} \dot{\zeta}(t - h_i) = \sum_{i=0}^{\nu} \left(\begin{bmatrix} A_i & 0 \\ \hat{\mathcal{B}}C_i^1 & \hat{\mathcal{A}}_i \end{bmatrix} \zeta(t - h_i) \right.$$
$$\left. + \begin{bmatrix} B_i \\ \hat{\mathcal{B}}D_i^1 \end{bmatrix} \tilde{u}(t - h_i) \right) \quad (20)$$

and

$$e(t) = \begin{bmatrix} 0_{n_z \times (n_x + (\mu-1)n_z)} & I_{n_z} \end{bmatrix} \zeta(t) \quad (21)$$

Thus, under Assumptions 1–5, in order to be able to find an input $\tilde{u}$, which drives $e(t)$ to zero (i.e., which satisfies (5)) under all initial conditions, the observable part (through $e(t)$) of the system (20)–(21) must be stabilizable. Since the part $\tilde{x}$ of the overall state ζ is already stabilizable by Condition 1 and the part $\tilde{e}$ is observable because of relations (17)–(19), this condition, however, is equivalent to the condition that the system (20) must be stabilizable through $\tilde{u}$. The system (20), however, is equivalent to system (15), which means that Condition 2 is necessary.

To show the necessity of Condition 3, note that, for robust tracking, error must be fed back. The error, however, can not be obtained unless Condition 3 holds. This completes the proof of the *only if* part.

We will give a constructive proof for the *if* part. By Condition 3, $z(t)$ can be obtained as $z(t) = Ty(t)$. Thus, since, both $r(t)$ and $y(t)$ are available to the controller, $e(t)$ can be obtained as

$$e(t) = Ty(t) - r(t) \quad (22)$$

Then, as a part of the controller, the following system, called the *servocompensator*, can be build:

$$\sum_{i=0}^{\nu} \hat{\mathcal{F}}_i \dot{s}(t - h_i) = \sum_{i=0}^{\nu} \hat{\mathcal{A}}_i s(t - h_i) + \hat{\mathcal{B}}e(t) \quad (23)$$

where $s(t) \in \mathbf{R}^{\mu n_z}$ is the state vector at time t. Then, the system (9) augmented by (23) is described as

$$\sum_{i=0}^{\nu} \begin{bmatrix} F_i & 0 \\ 0 & \hat{\mathcal{F}}_i \end{bmatrix} \dot{\eta}(t - h_i) = \sum_{i=0}^{\nu} \left(\begin{bmatrix} A_i & 0 \\ \hat{\mathcal{B}}C_i^1 & \hat{\mathcal{A}}_i \end{bmatrix} \eta(t - h_i) \right.$$
$$\left. + \begin{bmatrix} B_i \\ \hat{\mathcal{B}}D_i^1 \end{bmatrix} u(t - h_i) + \begin{bmatrix} G_i \\ \hat{\mathcal{B}}E_i^1 \end{bmatrix} w(t - h_i) \right)$$
$$- \begin{bmatrix} 0 \\ \hat{\mathcal{B}} \end{bmatrix} r(t) \quad (24)$$

where $\eta(t) := \begin{bmatrix} x(t) \\ s(t) \end{bmatrix} \in \mathbf{R}^{n_x + \mu n_z}$ is the overall state vector at time t. Recall that $x(t)$ is detectable through $y(t)$ by Condition 1. Furthermore, $s(t)$ is directly measurable, since (23) is a part of the controller. Thus, the state $\eta(t)$ is detectable to the controller, which can use both $y(t)$ and $s(t)$. Furthermore, by Condition 2, the augmented system (24) is stabilizable through the input $u(t)$, since (24) is equivalent to (15), apart from the exogenous signals $w(t)$ and $r(t)$. Therefore, there exists a controller which globally asymptotically stabilizes (24). Such a controller, augmented by the servocompensator (23), then globally asymptotically stabilizes the original system (9) (equivalently, (1)–(2)). Now, it remains to be shown that such a controller also satisfies (5) under all conditions stated in Problem 1. For this, let us define $\tilde{\eta}(t) := \mathcal{D}\eta(t)$. Then, using (7), (24) becomes:

$$\sum_{i=0}^{\nu} \begin{bmatrix} F_i & 0 \\ 0 & \hat{\mathcal{F}}_i \end{bmatrix} \dot{\tilde{\eta}}(t - h_i) = \sum_{i=0}^{\nu} \left(\begin{bmatrix} A_i & 0 \\ \hat{\mathcal{B}}C_i^1 & \hat{\mathcal{A}}_i \end{bmatrix} \tilde{\eta}(t - h_i) \right.$$
$$\left. + \begin{bmatrix} B_i \\ \hat{\mathcal{B}}D_i^1 \end{bmatrix} \tilde{u}(t - h_i) \right) \quad (25)$$

where $\tilde{u}(t) := \mathcal{D}u(t)$. Now, consider a feedback controller, which uses $y(t)$ and $s(t)$ to produce $u(t)$, that globally asymptotically stabilizes (24). Let us apply the same controller by using $\tilde{y}(t) := \mathcal{D}y(t)$ and $\tilde{s}(t) := \mathcal{D}s(t)$ to produce $\tilde{u}(t)$. Then, this controller will globally asymptotically stabilize (25). Since (25) does not have any exogenous inputs, however, this would imply $\lim_{t \to \infty} \tilde{\eta}(t) = 0$. Since, (25) is equivalent to (20), however, this implies that $\lim_{t \to \infty} \zeta(t) = 0$. Then, (21) implies (5). This completes the proof. $\square$

Our next result gives necessary and sufficient conditions for the solvability of Problem 2.

Theorem 2: Under Assumptions 1–5, there exists a solution to Problem 2 if and only if the following are satisfied:

1) The system (9) is stabilizable through the input u and is detectable through the measurement y, given in (11).
2) The system (15) is stabilizable through the input v.
3) There exists $T \in \mathbf{R}^{n_z \times n_y}$, such that $C_i^1 = TC_i^2$ and $E_i^1 = TE_i^2$, $i = 0, \ldots, \nu$.

Proof: Under the assumption that no perturbations are allowed in the matrices appearing in (3) and (4), Condition 3 implies that the output can be constructed from the measurement and the input as follows:

$$z(t) = Ty(t) + \sum_{i=0}^{\nu} \left(D_i^1 - T D_i^2 \right) u(t - h_i) \qquad (26)$$

Then, the result follows from Theorem 1. $\qquad \square$

Remark 4: Note that Assumptions 1–5 are needed only for the *only if* parts of Theorems 1 and 2. Even if any one of these assumptions fail, the *if* parts of both theorems continue to hold.

From the proof of the *if* part of Theorem 1, it can be deduced that the solution (when exists) to both problems involves two parts:

i) A *servocompensator* whose dynamics is defined by (23) and its input is obtained as in (22) for Problem 1 and as in (26) for Problem 2 (of course, if the stronger Condition 3 given in Theorem 1 holds, the input to the servocompesator can also be obtained as in (22) for Problem 2). This part of the controller is used to suppress the disturbance and to achieve asymptotic tracking. Note that, the dynamics of the servocompensator, in fact, mimics the dynamics of the fictitious system which produce the reference and the disturbance.

ii) A *stabilizing compensator* whose inputs are $y(t)$ and $s(t)$ and whose output is $u(t)$, to be applied to the given system (1)–(2). This controller is designed to globally asymptotically stabilize the augmented system (24). Any stabilizing controller design method developed for descriptor-type neutral time-delay systems (e.g., [26]—[38]) can be used to design this part.

4. Conclusions

Both the strong and weak versions (respectively Problems 1 and 2) of the robust servomechanism problem for LTI descriptor-type neutral time-delay systems have been considered. Necessary and sufficient conditions for the solvability of both problems have been derived and the structure of the controller, which solves these problems (when exists) has been presented. The necessary and sufficient conditions for both problems are generalizations of the conditions given in [5] for finite-dimensional LTI systems and the conditions given in [24] for LTI retarded time-delay systems to the present case. Furthermore, as in the case of finite-dimensional LTI systems [5] and LTI retarded time-delay systems [24], the general structure of the controller which solves the problem (when exists) involves two parts: (i) a *servocompensator*, which is used to suppress the disturbance and to achieve asymptotic tracking; and (ii) a *stabilizing compensator* which is designed to stabilize the given system augmented by the servocompensator.

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