

H_∞ Control Design for a Rigid-Flexible Spacecraft under a Fuel Slosh Influence

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Abstract: The spacecraft Attitude Control System (ACS) performance and robustness depend on the interaction effects between the fuel slosh motion, the panel's flexible motion, and the spacecraft rigid motion, mainly during translational and/or rotational maneuvers. In regards to satellite pointing accuracy flexibility and fuel, slosh is the two most important effects that should be considered in the satellite ACS design since their interactions can damage the ACS performance and robustness. Once, the lowest vibration frequencies, normally of the sloshing mode are about six times less than of the ACS bandwidth. Therefore, there is a strong possibility that this mode can destabilize the ACS pointing accuracy. This phenomenon is called spillover because the control effort spills over outside the control bandwidth. As a result, the designer needs to explore the limits between the conflicting requirements of performance, that is, increase of the bandwidth without introduction noise in the ACS keeping the system robustness to parameters variation. In this paper, one applies the H infinity control method which can deal with these two design requirements (performance and robustness) considering the controller error pointing that may be limited by the minimum time necessary to suppress disturbances that affect the satellite attitude acquisition. The equations of motions are obtained considering the Lagrange method for small flexible deformations and a mechanical model of liquid sloshing which allows modeling and investigating the longitudinal dynamic characteristics of a partially filled liquid tank during a pitch maneuver, satisfying performance and robustness requirements.

Key-Words: Control System, Slosh, ACS, H infinity, Satellite, Flexibility

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1 Introduction

THE name sloshing is given to the free movement of the surface of a liquid that partially fills a compartment. The free movement effected by this liquid layer is a vibratory movement that depends on the shape of the tank, the acceleration of gravity, and/or the axial acceleration of the tank. By oscillating the mass of liquid, the center of mass of the body also oscillates, thus disturbing the rigid-flexible part of the vehicle under study. Like the one in [1] the vibratory movement carried out by the liquid is considered to be a standing wave on the free surface of the liquid.

As for disturbances due to flexible solar panels vibrations control one has developed for accurate attitude control strategy using piezoelectric actuators to actively control the vibrations [2, 3]. On the other hand, a rigid manipulator control application, a feed-

back control law considering the coupling between a rigid manipulator and a flexible appendage has been proposed to actively control the appendage vibrations [4]. Likewise, for large space structure stability, distributed vibration suppression during on-orbit assembly has been studied [5].

In the simultaneous control of the movement of rigid-flexible structures and liquid inside tanks, the modes of vibration that cause a greater disturbance in the system are due to the slosh phenomenon as demonstrated in [6]. The reason is because the liquid movement has a lower frequency of vibration, and it moves a great quantity of mass, thus displacing the center of mass of the liquid. As a result, changing the moment of inertia of the system. Figure 1 shows the correlation between the vibration and the displacement of the center of mass of the liquid.

The sloshing modeling began in the sixties with Abramson's article [7], not having undergone many modifications since then. Due to the complexity of

creating the analytically model for the fluid that moves freely within a closed container, a simplified system is used called a mechanical analog, as will be seen in the following.

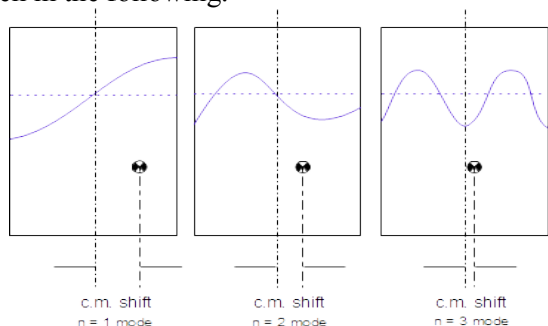


Fig. 1 Correlation between the vibration and the displacement.

In the paper [8] is presented the determination of a satellite-based on the Lagrangian mechanics, and are presented two types of mechanical analogs that can replace the complex dynamics of sloshing. For the model in which sloshing is replaced by a pendulum dynamic, two control laws are designed, one based on Lyapunov and another using the LQR method. These two laws apply to the nonlinear dynamics of the satellite. The results showed that both laws were able to control the movement of the liquid satisfactorily.

An analysis to model propellant slosh for the Europa Clipper mission using a two-pendulum model and compare the results with a computational fluid dynamics (CFD) is presented in [9]. The simulation results were used as real slosh behavior for two propellants at three fill fractions. They have concluded that the two-pendulum model can accurately capture slosh behavior either before or after the first peak.

To model the slosh to better study the dynamics irregularities that were presented during Saturn S-IVB rotation maneuvers, it was proposed a comparison between slosh, modeled with a mechanical equivalent, with a scale model capable of getting the slosh behavior data [10]. It was concluded that the dynamics of the pendulum represented an excellent performance of the slosh when compared to the experimental model, another important conclusion was that only seven percent of the liquid participates in the rigid body movement of the tank during the rotation excitation.

The main objective of this paper is to simulate the longitudinal control (pitch maneuver) of a space vehicle with a flexible solar array and a partially filled liquid in an internal tank, developed using H infinity, when a transfer orbit is realized. This paper is organized as follow, first the

dynamics of rigid-flexible spacecraft with the slosh dynamics are detailed, secondly the H-infinity control method is presented with its performance requirements, thirdly the simulations and results are discussed. Finally, the conclusion are presented.

2 Mathematical Model

The model adopted was inspired by the European spacecraft ATV - Automated Transfer Vehicle - built by European Space Agency (ESA), whose mission is to transport supplies and fuel to the International Space Station (ISS). This vehicle, shown in Figure 2, can carry seven tones (being almost one tone in liquid supplies) was first launched in [11]. Its Attitude Control System and Orbit (SCAO) has four motors capable of individually generating a thrust of 490 N and other twenty-eight small engines capable of individually generating 220 N. And once attached to the station, its actuators assist the ISS SCAO in the attitude maneuvers [11] of the set.



Fig. 2 Artistic design of ATV. in:
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In a way to simplify the model, for the fluid moving freely within a closed tank, it is used a mechanical analogous [6]. To do this simplification in necessary considering the following criteria for the system:

- Small displacement.
- A rigid tank.
- Non-viscous, incompressible and homogeneous

Under these conditions, the slosh dynamics can be approximate by a mechanical system consisting of a mass-spring or pendulum. For this article, the dynamics of the slosh is approximate by the pendulum dynamic [11].

The parameters of the system in study are: b is the distance from the pendulum attachment point to the

center of mass of the rigid body, a is the length of the pendulum rod and l is the length of the flexible rod of the solar array.

The state variables are: θ is the attitude angle of the satellite (rigid body) in respect to the fixed body reference, ψ is the angle formed between the pendulum rod with the reference axis, representing slosh movement, δ are the elastic nondimensional deformation, and the radial and transversal components of the velocity of the center of the tank are given by v_x and v_z , respectively.

The control variables are: f the transversal force (external torque) applied by the lateral thrusters, M is the pitching moment (internal torque) applied by a reaction wheel and the lower jet impulse F (due to the orbital transfer is being made, this impulse is considered constant).

The mass proprieties: m is the rigid body mass, m_f is the liquid mass, m_p is the mass of the solar array, I is the rigid body inertia and I_f is the moment of inertia of the liquid.

Figure 3 the simplified ATV schematic model is represented.

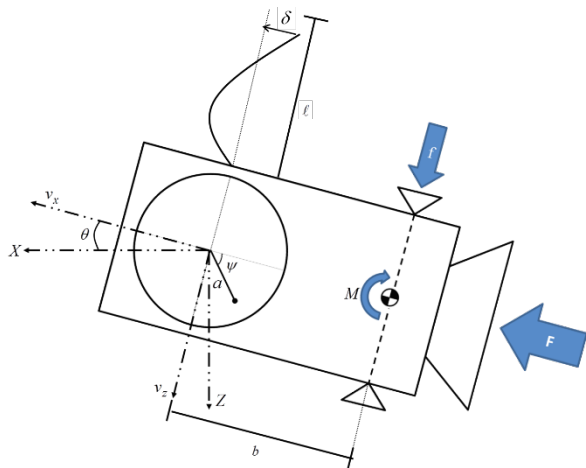


Fig. 3 Mechanical analogous type pendulum.

The dynamics equation is obtained from the energies kinetic and potential, admitting a dissipation of energy expressed by the Rayleigh dissipation function applied to Lagrange's equations of motion, as shown in [1].

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\omega}} \right) + \omega^\times \left(\frac{\partial L}{\partial \omega} \right) + V^\times \left(\frac{\partial L}{\partial V} \right) = \tau_r \quad (1)$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\psi}} \right) - \left(\frac{\partial L}{\partial \psi} \right) + \left(\frac{\partial R}{\partial \dot{\psi}} \right) = 0 \quad (2)$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{V}} \right) + \omega^\times \frac{\partial L}{\partial V} = \tau_t \quad (3)$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\delta}} \right) - \frac{\partial L}{\partial \delta} + \frac{\partial D}{\partial \dot{\delta}} = 0 \quad (4)$$

where L is the Lagrangian, R and D are energy dissipation, τ_r is an internal torque and τ_t is an external torque. In this formulation $V = [v_x \ 0 \ v_z]^T$, $\omega = [0 \ \dot{\theta} \ 0]^T$, $\tau_t = [F \ 0 \ f]^T$ and $\tau_r = [0 \ (M + fb) \ 0]^T$, the index \times represent an anti-symmetric matrix.

The energy dissipation R for the slosh and D for the flexibility are given by:

$$R = \frac{1}{2} \epsilon \dot{\psi}, D = \frac{1}{2} k d \delta^2 \quad (5)$$

where ϵ is a damping constant and kd is the dissipation constant. It is also allowed an energy elastic potential for the rod, of the form:

$$E_{pot} = \frac{1}{2} k \delta^2 \quad (6)$$

where k is the elastic constant.

The position vector of the satellite mass center with respect to the inertial coordination system (x, y, z) is

$$\vec{r} = (x - b)\hat{t} + z\hat{k} \quad (7)$$

and his velocity, assuming $v_x = \dot{x} + z\dot{\theta}$ and $v_z = \dot{z} + x\dot{\theta}$

$$\dot{\vec{r}} = v_z\hat{t} + (v_x + b\dot{\theta})\hat{k} \quad (8)$$

The position of the center mass of the tank is given by:

$$\vec{r}_f = (x - a \cos(\psi))\hat{t} + (z + a \sin(\psi))\hat{k} \quad (9)$$

and his velocity,

$$\dot{\vec{r}}_f = (v_x + a \sin(\psi)(\dot{\psi} + \dot{\theta}))\hat{t} + (v_z + a \cos(\psi)(\dot{\theta} + \dot{\psi}))\hat{k} \quad (10)$$

For the flexible rod, it is considered that it is subject to two types of motion: a) same angular motion θ that the rigid body, with linear velocity of $l\dot{\theta}$; b) The

deformation δ with respect to the axis Z, with velocity $\dot{\delta}$. Thus, for small deformations,

$$v_p = \dot{\delta} + l\dot{\theta} \quad (11)$$

The Lagrangian ($L = T - E_{pot}$) of all system is given by:

$$L = \frac{1}{2}(m\dot{r}^2 + I\dot{\theta}^2) + \frac{1}{2}(m_f\dot{r}_f^2 + I_f(\dot{\theta} + \dot{\psi})^2) + \frac{1}{2}m_p(\dot{\delta} + l\dot{\theta})^2 - \frac{1}{2}k\delta^2 \quad (12)$$

Substituting Equations 8 and 9 into eq. 12, using the relations given by eq. 1, eq. 2, eq. 3 and performing the derivations, one obtains the satellite equations of motion given by:

$$(m + m_f)a_x + mb\dot{\theta}^2 + m_f a(\ddot{\psi} + \ddot{\theta})\text{sen}(\psi) + m_f a(\dot{\theta} + \dot{\psi})^2 \text{cos}(\psi) = F \quad (13)$$

$$(m + m_f)a_z + m_f a(\ddot{\psi} + \ddot{\theta})\text{cos}(\psi) - m_f a(\dot{\theta} + \dot{\psi})^2 \text{sen}(\psi) + mb\ddot{\theta} = f \quad (14)$$

$$(mb^2 + I_f + m_p l^2)\ddot{\theta} + \delta m_p l + mba_z - \varepsilon \dot{\psi} = M + fb \quad (15)$$

$$(m_f a^2 + I_f)(\ddot{\psi} + \ddot{\theta}) + m_f a(\text{sen}(\psi)a_x + \text{cos}(\psi)a_z) + \varepsilon \dot{\psi} = 0 \quad (16)$$

$$\delta m_p + \ddot{\theta} m_p l + \delta k_d + \delta k = 0 \quad (17)$$

Substituting the acceleration $a_x = \dot{v}_x + v_z \dot{\theta}$ and $a_z = \dot{v}_z - v_x \dot{\theta}$ into eq. 13 and eq. 14,

$$a_x = \frac{F - mb\dot{\theta}^2 - m_f a(\ddot{\psi} + \ddot{\theta})\text{sen}(\psi) - m_f a(\dot{\theta} + \dot{\psi})^2 \text{cos}(\psi)}{m + m_f} \quad (18)$$

$$a_z = \frac{f - m_f a(\ddot{\psi} + \ddot{\theta})\text{cos}(\psi) + m_f a(\dot{\theta} + \dot{\psi})^2 \text{sen}(\psi) - mb\ddot{\theta}}{m + m_f} \quad (19)$$

For the control propose is necessary to linearize the equations of motions, for it is assumed that the system makes small movements around the point of

equilibrium, it can be considered as being very close to zero values.

Substituting the eq. 18 and 19 into eq. 15 and 16 assuming the linearization conditions (small displacements, around the operation point), one has the motion equation [1]:

$$\ddot{\theta} = \frac{(I + m_p l^2 + m^*(b^2 - ba))}{(M + b^*f - m_p l \dot{\delta} + m^* a b \ddot{\psi} + \varepsilon \dot{\psi})} \quad (20)$$

$$\ddot{\psi} = \ddot{\theta} \left(\frac{m^* b a}{I_f + m^* a^2} - 1 \right) - \dot{\psi} \left(\frac{\varepsilon}{I_f + m^* a^2} \right) - \psi \left(\frac{a^* F}{I_f + m^* a^2} \right) - \frac{a^* f}{I_f + m^* a^2} \quad (21)$$

$$\ddot{\delta} = \ddot{\theta} l - \dot{\delta} \frac{k_d}{m_p} - \delta \frac{k}{m_p} \quad (22)$$

where, $b^* = \frac{b m_f}{m + m_f}$, $a^* = \frac{a m_f}{m + m_f}$, $m^* = \frac{m m_f}{m + m_f}$ and $m_f^* = \frac{m_f}{m + m_f}$.

The Equations 20 into 22 shows that the flexible movement, generate by the flexible solar array, are coupled with the slosh and rigid body movement.

3 H Infinity Control Method

The purpose of a H_∞ controller is to shape the response of the given system to a reference and to obtain (or maintain) the closed-loop system stable with the desired performance. A robust controller, in turn, should be able to maintain desired performance and stability, through the presence of plant uncertainties and/or perturbations.

In general, the H_∞ problem consists in: Given the design requirements, set up a system, with the proper filters (weights functions), to adapt the system to the conditions of performance and robustness, then create a problem of minimization of the transfer function matrix, in a closed-loop, using the infinite norm.

In Figures 4 and 5, the P (red box) is the called generalized plant, K is the controller and G is the system plant, with the exogenous inputs w, outputs z, u is the control signal, and is the plant output signal. , and are weight functions.

The weight functions aim to evaluate the performance, robustness, and energy consumed by the system, acting on the sensitivity function (S), complementary

sensitivity function T , and the relationship between the controller gain K and the sensitivity function S .

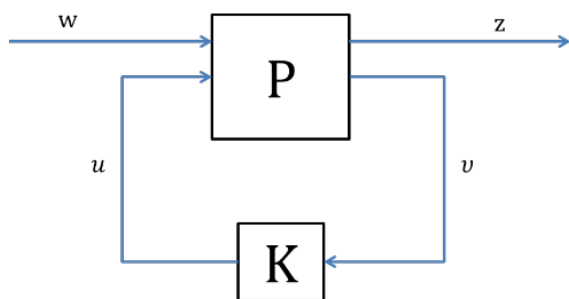


Fig. 4 Generalized plant.

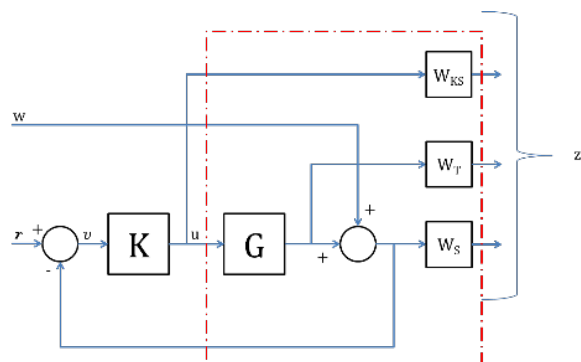


Fig. 5 Expanded generalized plant.

The expressions of the sensitivity function ($S(s)$) and complementary sensitivity function ($T(s)$) are given by:

$$\begin{aligned} S(s) &= (I + G(s)K(s))^{-1} \\ T(s) &= (G(s)K(s))(I + G(s)K(s))^{-1} \\ &= I - S(s) \end{aligned} \quad (23)$$

the term I are an identity matrix in eq. 23. The expressions of output z in Fig. 5, are

$$\begin{aligned} z_1 &= W_{KS}u \\ z_2 &= W_TGu \\ z_3 &= W_S G\omega + W_S Gu \\ v &= -\omega - Gu \end{aligned} \quad (24)$$

then with these relations (eq. 24), is possible write the expression the of the generalized plant P

$$P = \begin{bmatrix} 0 & W_{KS} \\ 0 & W_T \\ W_S I & W_S G \\ -I & -G \end{bmatrix} \quad (25)$$

where I is an identity matrix.

The H_∞ control method consist in calculate a gain K that minimizing the H_∞ norm of the closed loop transfer function,

$$\begin{aligned} \|F_l(P, K)\|_\infty &= \max_\omega \bar{\sigma}(F_l(P, K)(j\omega)) \\ \|F_l(P, K)\|_\infty &< \gamma \end{aligned} \quad (26)$$

where $F_l(P, K)$ is the linear fractional transformation (LFT) of P and K

$$F_l(P, K)\omega = P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21} \quad (27)$$

The γ factor is obtained numerically from a state-space realization as the smallest value of γ such that the Hamiltonian matrix H has no eigenvalues on the imaginary axis,

$$\begin{aligned} H &= \begin{bmatrix} A + BR^{-1}D^T C & BR^{-1}B^T \\ -C^T(I + DR^{-1}D^T)C & -(A + BR^{-1}D^T C)^T \end{bmatrix} \\ & \quad (28) \end{aligned}$$

where A , B , C and D are the space state matrices and $R = \gamma^2 I - D^T D$.

3.1 Selecting weight functions

Then, for the selection of weight functions, it's usual to take the structure (eq. 29) and by a put and try methodology find the best function for the problem:

$$\begin{aligned} W_S &= \left(\frac{s}{\frac{k}{\sqrt{M}} + \omega_b} + \omega_b \right)^k \\ W_T &= \sim \left(\frac{s + \frac{\omega_{bc}}{k\sqrt{M}}}{\omega_{bc} + s^k \sqrt{A}} \right)^k \text{ and } W_{KS} = cte \end{aligned} \quad (29)$$

where the constants are: k is the roll off adjust, ω_b is the bandwidth of the plant, ω_{bc} is a proportional value of ω_b , A steady-error restriction and M are an overshoot restriction.

To guarantee a good performance, the maximum singular value of the sensitivity function ($\bar{\sigma}S(j\omega)$) and the complementary sensitivity function ($\bar{\sigma}T(j\omega)$) must be smaller than the inverse of the W_S and W_T weights function, as shown by the equations below:

$$\bar{\sigma}(S(j\omega)) < \frac{1}{|W_S(j\omega)|}, \quad \bar{\sigma}(T(j\omega)) < \quad (30)$$

$$\frac{1}{|W_T(j\omega)|}$$

Figure 6 represents the eq. 30 graphically.

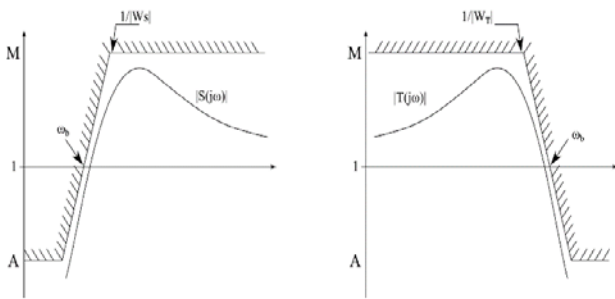


Fig. 6 Performance ratio

3.2 Control design requirements

For the controller design H_∞ the challenge lies in finding a set of weights functions that satisfy the performance and robustness conditions necessary to control the interaction of flexibility with a slosh.

If the control law is poorly designed, can occur the phenomenon known as spillover [12], whose characteristic is to destabilize the system by exciting the modes of vibration causing a much greater control effort than expected, bringing undesirable consequences to the mission.

With these the objectives, the controller is limited to stabilizing the angle of attitude (θ) minimizing the effects of slosh and flexibility, acting just in the control of the rigid body.

Therefore, it remains to verify if the design functions satisfy the requirements provided by Equations 30, shown below, respectively;

Figure 7 shows the singular value of the system,

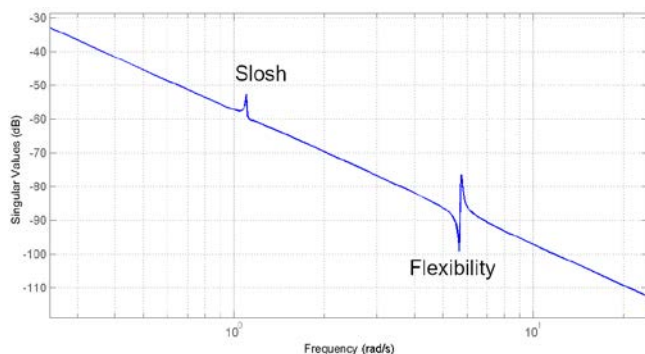


Fig. 7 Singular values of the system.

then the weight function was sectioned with these fallow values for the parameters: $\omega_B = 0.05$ rd/s,

$\omega_{bS} = 15\omega_b$ $A = 10^{-3}$ and $M = 2$. the values of A and M are the same that traditionally one finds in the lectures. These parameters were chosen so that there are no resonance effects on the controller due to the flexibility and slosh modes, and also because they obey the restrictions of the sensitivity and complementary sensitivity functions.

Figure 8 shows that the projected functions respected the conditions of performance and existence of the functions of weight W_S and W_T (see Fig.6). For this particular problem the weighting of the control signal, W_{KS} was considered constant.

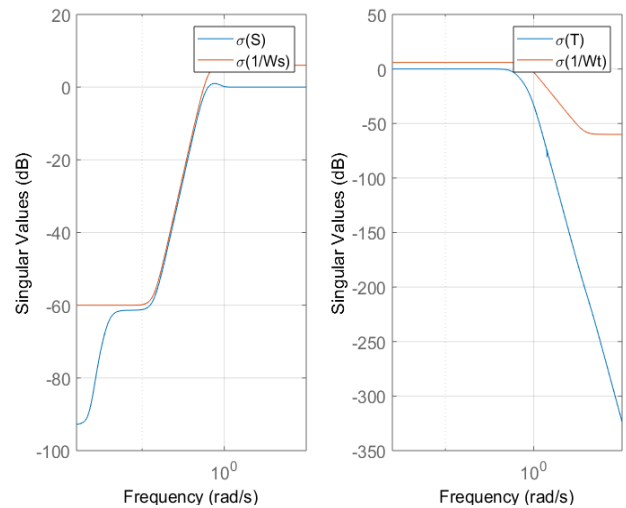


Fig. 8 Singular values of S and T in comparison with $1/|W_S|$ and $1/|W_T|$

4 Simulations and Results

For the simulations, the following parameters were adopted: For the rigid body, $m = 600$ kg, $I = 720$ kgm^2 , $b = 0.25$ m, $F = 500$ N, $\epsilon = 0.19$ kgm^2/s . Being m , I , b , F , the mass of the rigid body without the liquid portion, moment of inertia, distance of the actuators to the center of mass, a constant force and the constant of internal energy dissipation, respectively. For slosh, $m_f = 100$ kg, $a = 0.33$ m, $I_f = 10$ kgm^2 . Since m_f is the mass of liquid in displacement, a size of the stem of the pendulum and I_f moment of inertia of the liquid. For the rod flexible, $m_p = 10$ kg, $\ell = 1.5$ m, $k = 300$ $\text{kg rad}^2 / \text{s}^2$, $kd = 0.48$ $\text{kg rad}^2 / \text{s}$. Being m_p the mass of the rod (flexible appendix), ℓ the size of the panel, k is the constant elastic and kd is the constant of dissipation of energy. The simulations were done for a three-minute (120 s) time interval with an initial condition ($\theta = 1^\circ$).

Figure 9 shows the time evolution of the states: θ rigid body attitude, Ψ slosh compartment given by the pendulum angular displacement and δ the flexible displacement of the flexible rod. For this response of the initial condition these states are stabilized in 60 s.

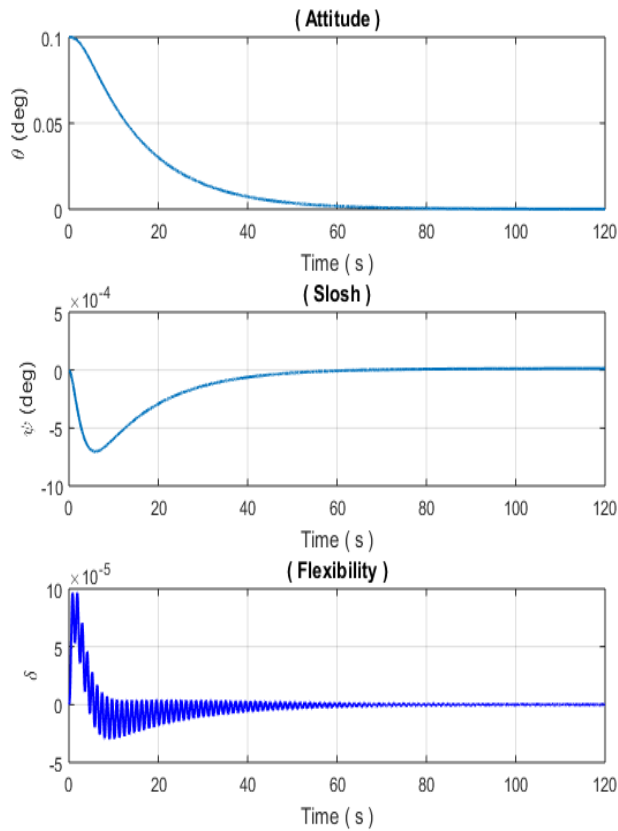


Fig. 9 Simulation of the states

Figure 9 and 10 show the time evolution of the states: rigid body angular displacement and velocity, pendulum angular displacement and velocity and the flexible displacement and its rate, respectively. For this response of the initial condition these states are stabilized around 60 s. The perturbation of the initial condition generates a small perturbation in the slosh and flexible states.

The Figure 11 shows the control signal of an internal torque M and an external force f . The internal torque may have been generated by a reaction wheel and the external force may have been generated by a pair of thrusters. The signal for the external actuator has a overshoot of $2.3 \cdot 10^{-3}$ N and for the internal actuator the overshoot was 0.023 Nm.

By the results of the simulations, the control law design was able to provide a stable answer of the

states, using accepted values for the actuators. This residual value is due to the coupling of flexibility with rigid body and sloshing movements.

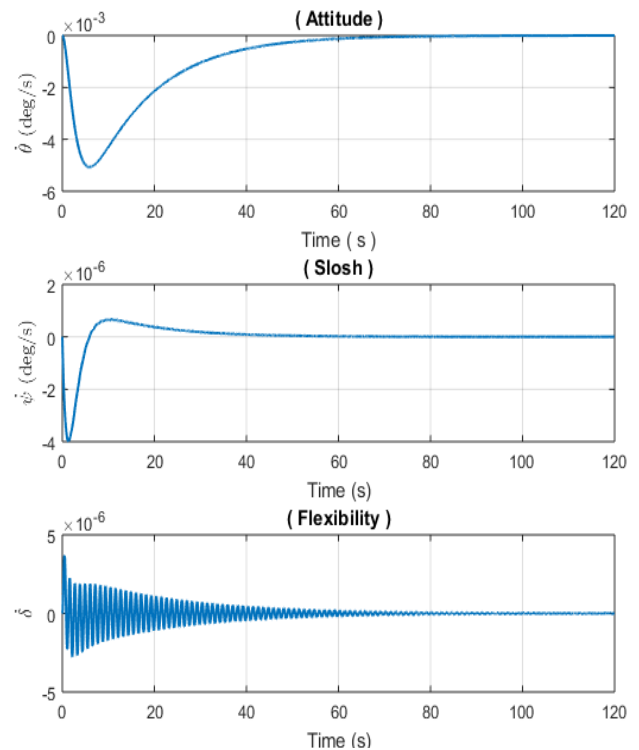


Fig. 10 Simulation of the derivatives of the states

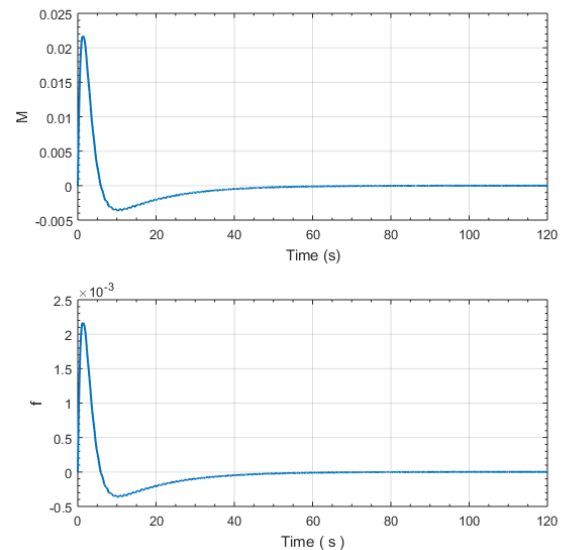


Fig. 11 Control signal

5 Conclusion

In this paper, it was described briefly the concepts of the sloshing phenomenon which is associated with the dynamics of a liquid moving into a partially fills reservoir.

The dynamic equations were deduced using a Lagrangian approach, admitting dissipation energy in the equation of motion. The dissipation energy was considered in the flexible movement and in the sloshing. The coupling of movements exists between the rigid body, the flexibility and the liquid slosh.

The control law is designed using the H-infinity method with weight functions acting in the sensitivity function S and complementary sensitivity function T. The weight function design was based on the distance between the bandwidth frequency with flexibility and slosh frequencies, thus avoiding resonance effects.

The simulations showed the good performance of the H-infinity controller, since it controlled and stabilized the attitude angle minimizing the effects of the interaction between sloshing and flexibility. In addition, the actuator remains stable and active, which means that they have been consuming the minimum energy. This result can be seen in the control signal due to the residual movement caused by flexibility, which in turn is coupled to all other controlled states, which forced the need for continuous action on the system. But as the magnitude of this disturbance is very small, in the case of implementing the real actuators, it would be impossible to generate such a low level of effort, so the result obtained in this article is only numerical.

For future works, a more complex model with more vibration modes and non-linearities will be used to investigate the performance and robustness of the H-infinity control law. It is also suggested to include the model of the shape actuators, to check the effectiveness of this system designed to control a spaceship. It is also intended to carry out an experimental investigation using hardware in the loop system HIL as in [13].

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