Finite-time stabilization for uncertain neural networks with time-varying delay

MENGXIAO DENG, YALI DONG*, MENGYING DING
School of Mathematical Sciences
Tiangong University
Tianjin 300387
CHINA

Abstract: - In this paper, the problems of finite-time boundedness and control design for uncertain neural networks with time-varying delay is considered. By constructing Lyapunov-Krasovskii function and using the matrix inequality method, sufficient conditions for finite-time boundedness of a class of neural networks with time-varying delay are established. Then, we proposed a criterion to ensure that the neural networks with time-varying delay is finite-time stabilizable. A numerical example is given to verify the validity of the results.

Key-Words: - Finite-time boundedness, stabilization, uncertain neural networks, Lyapunov-Krasovskii functional, time-varying delay.

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1 Introduction

In the past decades, networked control systems have attracted considerable attention due to the rapid development of network communication technologies [1]. As is known to all, only when the system is stable, on the basis of the analyses other performance of the control system is of practical significance. The stability analysis and control design are very important research topics for neural networks. Nowadays, the study of the stability analysis of neural networks has gained popularity among researchers, and some remarkable results have been reported in the literature [2-4].

In practical industrial control systems, uncertainty and time delay are widespread, which may lead to system instability and system performance degradation, which also aggravate the difficulty and complexity of system analysis and synthesis. Therefore, delayed neural networks have been proposed and have received a great deal of attention [4-8]. In [2], the stability analysis of delayed cellular neural networks was given. In [4], Wu et al. investigated the exponential stability of neural networks with time-varying delay. In [7], Xia et al. studied the robust stability for neutral-type uncertain neural networks with Markovian jumping parameters and time-varying delays. In [8], Ma et al. considered the stabilization of networked switched linear systems. However, from practical considerations, there exist some systems, whose behavior may be only defined over a finite time interval or state variables are required to be within specific bounds. For this case, it is very important to study s the finite-time stability and stabilization [9-17]. In [9], Stojanovic dealt with the robust finite-time stability of discrete time systems with interval time-varying delay. In [11], the finite-time boundedness of uncertain time-delayed neural network with Markovian jumping parameters was considered. In [13], Dong et al. considered finite-time boundedness analysis and H_∞ control for switched neutral systems with mixed time-varying delays. In [15], Lv et al, considered the finite time stability and controller design for nonlinear impulsive sampled-data systems. To the best of our knowledge, the problem of robust finite-time stabilization for uncertain neural networks with time-varying delays has rarely been studied.

This paper studies the finite-time boundedness and stabilization for uncertain neural network with time-varying delay. We developed the sufficient conditions of finite-time boundedness for a class of neural networks with time-varying delay. Then, we proposed the sufficient conditions of finite-time stabilization for neural networks with time-varying delay. We provide a numerical example to demonstrate the proposed results in this paper.

The rest of the paper is organized as follows. In Section 2, the system description, necessary definitions and lemmas are given. In Section 3, the sufficient conditions are derived to ensure finite-time stabilization of the uncertain delayed neural networks. In Section 4, an numerical example is given to demonstrate the validity of the proposed method. Finally, some conclusions are given in Section 5.

Notation. N is a set of all natural numbers, $R^{n\times n}$ represent the set of $n\times n$ real matrices, A^T is the transpose of A, and A^{-1} is the inverse of A. $X>0$ ($X\geq 0$) means X is a positive definite (semi-positive definite) matrix. * represents the elements below the main diagonal of a symmetric matrix. $\lambda_{\min}(M)$ and $\lambda_{\max}(M)$ stand for the minimum and maximal eigenvalue of a matrix M , respectively.

2 Problem Formulation

We consider the uncertain neural networks with time-varying delay as follows:

$$\begin{cases}\dot{x}(t)=(-A+\Delta A(t))x(t)+(C+\Delta C(t))x(t-\tau(t))\\ \quad +Bu(t)+(W_0+\Delta W_0(t))f(x(t))\\ \quad +(W_1+\Delta W_1(t))f(x(t-\tau(t))),\\ x(t)=\varphi(t),\quad t\in[-h,0],\end{cases}\tag{1}$$

where $x(t)=[x_1(t),x_2(t),\cdots,x_n(t)]\in R^n$ denotes the state vector with n neurons. $u(t)\in R^m$ is the control input, $\varphi(t)\in R^m$ is the initial condition. $\tau(t)$ is the time-varying delay and satisfies

$$0\leq\tau(t)<h,\quad 0\leq\dot{\tau}(t)<\mu,$$

W_0 and W_1 are the connection weight matrix and the delayed connection weight matrix, respectively, $A,W_0,W_1,B,C,$ are constant matrices with appropriate dimension, and $A=diag(a_1,a_2,\cdots,a_n)$. The parameter uncertainties $\Delta A,\Delta W_0,\Delta W_1,\Delta C$ satisfy

$$\begin{aligned}&\Delta A=H_0F_0(t)E_0,\quad \Delta C=H_1F_1(t)E_1,\\ &\Delta W_0=H_2F_2(t)E_2,\quad \Delta W_1=H_3F_3(t)E_3,\end{aligned}\tag{2}$$

where $H_i,E_i,i=0,1,2,3,$ are known real constant matrices. $F_i(t),i=1,2,3,4,$ are unknown matrix which satisfy

$$F_i^T(t)F_i(t)\leq I.$$

$f^T(x(t))=[f_1(x_1(t)),f_2(x_2(t)),\cdots,f_n(x_n(t))]\in R^n$ is the neuron activation function with $f(0)=0$ and satisfies

$$|f_i(\alpha)-f_i(\beta)|\leq\gamma_i|\alpha-\beta|,\quad \forall\alpha,\beta\in R,i=1,2,\cdots,n,\tag{3}$$

where $\gamma_i,i=1,2,\cdots,n,$ are known constants.

To obtain the main results, the following lemmas and definitions are necessary.

Lemma 1 [18]. For matrices D, E and Y of appropriate dimensions and $Y^T=Y$, then

$$Y+DFE+E^TF^TD^T<0,$$

holds for all matrix F satisfying $F^TF\leq I$, if and only if there exists a constant $\varepsilon>0$, such that

$$Y+\varepsilon DD^T+\varepsilon^{-1}E^TE<0.$$

Lemma 2 [19]. For a given matrix $B\in R^{p\times m}$ with rank $(B)=p$, assume that $X\in R^{m\times m}$ is a symmetric matrix, then there exists a matrix $\hat{X}\in R^{p\times p}$ such that $BX=\hat{X}B$, if and only if $X=Vdiag(\hat{X}_{11},\hat{X}_{22})V^T$, where $\hat{X}_{11}\in R^{p\times p}$ and $\hat{X}_{22}\in R^{(m-p)\times(m-p)}$.

Definition 1. Given three positive constants c_1,c_2,T, with $c_1<c_2$, and a positive definite matrix R. The uncertain neural networks (1) with $u(t)=0$ is said to be finite-time bounded with respect to (c_1,c_2,T,R), if

$$\sup_{-h\leq\theta\leq 0}\{x^T(\theta)Rx(\theta)\}\leq c_1\Rightarrow x^T(t)Rx(t)<c_2,\forall t\in[0,T].\tag{4}$$

We consider the full-state feedback controller,

$$u(t)=-Kx(t),\tag{5}$$

where K is the controller gain.

Definition 2. Given three positive constants c_1,c_2,T, with $c_1<c_2$, and a positive definite matrix R. The uncertain neural networks (1) with controller (5) is said to be finite-time stabilizable with respect to (c_1,c_2,T,R), if

$$\sup_{-h\leq\theta\leq 0}\{x^T(\theta)Rx(\theta)\}\leq c_1\Rightarrow x^T(t)Rx(t)<c_2,\forall t\in[0,T].$$

3 Main results

We consider the following neural networks

$$\begin{cases}\dot{x}(t)=(-A+\Delta A(t))x(t)+(C+\Delta C(t))x(t-\tau(t))\\ \quad +(W_0+\Delta W_0(t))f(x(t))\\ \quad +(W_1+\Delta W_1(t))f(x(t-\tau(t))),\\ x(t)=\varphi(t),\quad t\in[-h,0].\end{cases}\tag{6}$$

Firstly, we give the sufficient conditions for finite-time boundedness of neural networks (6).

Theorem 1. For given a matrix $R>0$, and positive scalars $c_1\leq c_2,T$, the system (6) is finite-time bounded with respect to (c_1,c_2,T,R), if there exist positive definite symmetric matrices $P,Q,S,\tilde{T},Z$, and diagonal matrices $X>0,V>0$, and positive scalars $\varepsilon,\alpha,\lambda_1,k_1,k_2,k_3,k_4,k_5$, such that

$$\Phi=\begin{bmatrix}\Phi_{11} & \Phi_{12}\\ * & \Phi_{22}\end{bmatrix}<0,\tag{7}$$

$$c_1(k_1+hk_2+hk_3+\frac{h^3}{2}k_4+\frac{h^5}{12}k_5)<\lambda_1c_2e^{-\alpha T},\tag{8}$$

where

$$\Phi_{11}=\begin{bmatrix}\bar{\Phi}_{11} & PC & PW_0+V & PW_1 & 0\\ * & \bar{\Phi}_{22} & 0 & 0 & 0\\ * & * & \bar{\Phi}_{33} & 0 & 0\\ * & * & * & \bar{\Phi}_{44} & 0\\ * & * & * & * & -\tilde{T}\end{bmatrix},$$

$$\Phi_{12}=\begin{bmatrix}0 & PH_0 & PH_1 & PH_2 & PH_3\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\end{bmatrix},$$

$$\Phi_{22}=diag(-Z,-\varepsilon I,-\varepsilon I,-\varepsilon I,-\varepsilon I,),$$

$$\bar{\Phi}_{11}=-PA-A^TP+Q+h^2\tilde{T}+(\frac{h^2}{2})^2Z+\Gamma X\Gamma$$
$$+2\Gamma V-\alpha P+\varepsilon E_0^TE_0,$$
$$\bar{\Phi}_{22}=-(1-\mu)Q+\varepsilon E_1^TE_1,$$
$$\bar{\Phi}_{33}=S-X+\varepsilon E_2^TE_2,$$
$$\bar{\Phi}_{44}=-(1-\mu)S+\varepsilon E_3^TE_3,$$
$$\Gamma=diag(\gamma_1,\gamma_2,\cdots,\gamma_n),$$
$$\bar{P}=R^{-\frac{1}{2}}PR^{-\frac{1}{2}},\bar{Q}=R^{-\frac{1}{2}}QR^{-\frac{1}{2}},\bar{W}=R^{-\frac{1}{2}}WR^{-\frac{1}{2}},$$
$$\bar{T}=R^{-\frac{1}{2}}\tilde{T}R^{-\frac{1}{2}},\bar{Z}=R^{-\frac{1}{2}}ZR^{-\frac{1}{2}},\lambda_1=\lambda_{\min}\left(\bar{P}\right),$$
$$k_1=\lambda_{\max}\left(\bar{P}\right),k_2=\lambda_{\max}\left(\bar{Q}\right),k_3=\lambda_{\max}\left(\bar{W}\right),$$
$$k_4=\lambda_{\max}\left(\bar{T}\right),k_5=\lambda_{\max}\left(\bar{Z}\right),W=\Gamma S\Gamma.$$

Proof: Choose the following Lyapunov-Krasovskii functional:

$$\bar{V}\left(t\right)=V_1\left(t\right)+V_2\left(t\right)+V_3\left(t\right)+V_4\left(t\right),\qquad(9)$$

where

$$V_1(t)=x^T(t)Px(t),$$
$$V_2(t)=\int_{t-\tau(t)}^{t}x^T(\theta)Qx(\theta)d\theta+\int_{t-\tau(t)}^{t}f^T(x(\theta))Sf(x(\theta))d\theta,$$
$$V_3(t)=h\int_{t-h}^{t}\int_{\theta}^{t}x^T(s)\tilde{T}x(s)dsd\theta,$$
$$V_4(t)=\frac{h^2}{2}\int_{-h}^{0}\int_{\theta}^{0}\int_{t+s}^{t}x^T(u)Zx(u)dudsd\theta.$$

Taking the derivative of $V_i\left(t\right),i=1,2,3,4$ along the trajectory of system (6), we have

$$\dot{V}_1(t)=x^T(t)(-PA-A^TP+P\Delta A(t)+\Delta A^T(t)P)x(t)$$
$$+x^T(t)(PC+P\Delta C(t))x(t-\tau(t))+x^T(t-\tau(t))$$
$$\times(\Delta C^T(t)P+C^TP)x(t)+x^T(t)(PW_0$$
$$+P\Delta W_0(t))f(x(t))+f^T(x(t))(W_0^TP$$
$$+\Delta W_0^T(t)P)x(t)+x^T(t)(PW_1+P\Delta W_1(t))$$
$$\times f(x(t-\tau(t)))$$
$$+f^T(x(t-\tau(t)))(W_1^TP+\Delta W_1^T(t)P)x(t),$$
$$\dot{V}_2(t)\le x^T(t)Qx(t)-(1-\mu)x^T(t-\tau(t))Qx(t-\tau(t))$$
$$-(1-\mu)f^T(x(t-\tau(t)))Sf(x(t-\tau(t)))$$
$$+f^T(x(t))Sf(x(t)),$$
$$\dot{V}_3(t)\le h^2x^T(t)\tilde{T}x(t)-\left(\int_{t-h}^{t}x\left(\theta\right)d\theta\right)^T\tilde{T}\left(\int_{t-h}^{t}x\left(\theta\right)d\theta\right),$$
$$\dot{V}_4(t)\le(\frac{h^2}{2})^2x^T(t)Zx(t)-\left(\int_{-h}^{0}\int_{t+\theta}^{t}x\left(s\right)dsd\theta\right)^T$$
$$\times Z\left(\int_{-h}^{0}\int_{t+\theta}^{t}x\left(s\right)dsd\theta\right)\qquad(10)$$

For diagonal matrices $X>0,V>0,$ using (3), one has

$$f^T(x(t))Xf(x(t))\le x^T(t)\Gamma X\Gamma x(t),$$
$$-2x^T(t)Vf(x(t))\le 2x^T(t)\Gamma Vx(t),\qquad(11)$$

and from (10) and (11), we get

$$\dot{\bar{V}}(t)-\alpha\bar{V}(t)\le\xi^T(t)\Psi\xi(t),\qquad(12)$$

where

$$\xi^T(t)=[x^T(t),x^T(t-\tau(t)),f^T(x(t)),f^T(x(t-\tau(t))),$$
$$\left(\int_{t-h}^{t}x\left(\theta\right)d\theta\right)^T,\left(\int_{-h}^{0}\int_{t+\theta}^{t}x\left(s\right)dsd\theta\right)^T],$$

$$\Psi=\begin{bmatrix}\psi_{11}&\psi_{12}&\psi_{13}&\psi_{14}&0&0\\ *&-(1-\mu)Q&0&0&0&0\\ *&*&S-X&0&0&0\\ *&*&*&-(1-\mu)S&0&0\\ *&*&*&*&-\tilde{T}&0\\ *&*&*&*&*&-Z\end{bmatrix},$$

$$\psi_{11}=-PA-A^TP+PH_0F_0(t)E_0+E_0^TF_0^T(t)H_0^TP$$
$$+Q+h^2\tilde{T}+(\frac{h^2}{2})^2Z+\Gamma X\Gamma+2\Gamma V-\alpha P,$$
$$\psi_{12}=PC+PH_1F_1(t)E_1,$$
$$\psi_{13}=PW_0+PH_2F_2(t)E_2+V,$$
$$\psi_{14}=PW_1+PH_3F_3(t)E_3,$$

From (2), the matrix Ψ can be rewrite

$$\Psi=\tilde{\Omega}+\Sigma\Theta\Pi+\Pi^T\Theta^T\Sigma^T,$$

where

$$\tilde{\Omega}=\begin{bmatrix}\bar{\psi}_{11}&\bar{\psi}_{12}&\bar{\psi}_{13}&\bar{\psi}_{14}&0&0\\ *&-(1-\mu)Q&0&0&0&0\\ *&*&S-X&0&0&0\\ *&*&*&-(1-\mu)S&0&0\\ *&*&*&*&-\tilde{T}&0\\ *&*&*&*&*&-Z\end{bmatrix},$$

$$\Sigma=\begin{bmatrix} PH_0 & PH_1 & PH_2 & PH_3 & 0 & 0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\end{bmatrix},$$

$$\Theta=diag(F_0(t),F_1(t),F_2(t),F_3(t),0,0),$$
$$\Pi=diag(E_0,E_1,E_2,E_3,0,0),$$
$$\bar{\psi}_{11}=-PA-A^TP+Q+h^2\tilde{T}+(\frac{h^2}{2})^2Z+\Gamma X\Gamma+2\Gamma V-\alpha P,$$
$$\bar{\psi}_{12}=PC,$$
$$\bar{\psi}_{13}=PW_0+V,$$
$$\bar{\psi}_{14}=PW_1.$$

From Lemma 1, $\Psi<0$ if

$$\Omega=\tilde{\Omega}+\varepsilon^{-1}\Sigma\Sigma^T+\varepsilon\Pi^T\Pi=\begin{bmatrix}\tilde{\psi}_{11} & \bar{\psi}_{12} & \bar{\psi}_{13} & \bar{\psi}_{14} & 0 & 0\\ * & -(1-\mu)Q+\varepsilon E_1^TE_1 & 0&0&0&0\\ * & * & \tilde{\psi}_{33} & 0&0&0\\ *&*&*&\tilde{\psi}_{44}&0&0\\ *&*&*&*&-\tilde{T}&0\\ *&*&*&*&*&-Z\end{bmatrix}<0, \quad (13)$$

where

$$\tilde{\psi}_{11}=-PA-A^TP+Q+h^2\tilde{T}+(\frac{h^2}{2})^2Z+\Gamma X\Gamma+2\Gamma V-\alpha P+\varepsilon E_0^TE_0+\varepsilon^{-1}(PH_0H_0^TP+PH_1H_1^TP+PH_2H_2^TP+PH_3H_3^TP),$$
$$\tilde{\psi}_{33}=S-X+\varepsilon E_2^TE_2,$$
$$\tilde{\psi}_{44}=-(1-\mu)S+\varepsilon E_3^TE_3.$$

By using the Schur complement lemma, (13) is equivalent the following matrix:

$$\Phi=\begin{bmatrix}\Phi_{11} & \Phi_{12}\\ * & \Phi_{22}\end{bmatrix}<0,$$

where

$$\Phi_{11}=\begin{bmatrix}\bar{\Phi}_{11} & PC & PW_0+V & PW_1 & 0\\ * & \bar{\Phi}_{22} & 0&0&0\\ *&*&\bar{\Phi}_{33}&0&0\\ *&*&*&\bar{\Phi}_{44}&0\\ *&*&*&*&-\tilde{T}\end{bmatrix},$$

$$\Phi_{12}=\begin{bmatrix}0 & PH_0 & PH_1 & PH_2 & PH_3\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\end{bmatrix},$$

$$\Phi_{22}=diag(-Z,-\varepsilon I,-\varepsilon I,-\varepsilon I,-\varepsilon I,),$$
$$\bar{\Phi}_{11}=-PA-A^TP+Q+h^2\tilde{T}+(\frac{h^2}{2})^2Z+\Gamma X\Gamma+2\Gamma V-\alpha P+\varepsilon E_0^TE_0,$$
$$\bar{\Phi}_{22}=-(1-\mu)Q+\varepsilon E_1^TE_1$$
$$\bar{\Phi}_{33}=S-X+\varepsilon E_2^TE_2,$$
$$\bar{\Phi}_{44}=-(1-\mu)S+\varepsilon E_3^TE_3.$$

From condition (7), we know that

$$\dot{\bar{V}}(t)-\alpha\bar{V}(t)\le 0, \quad (14)$$

So, we have

$$\dot{\bar{V}}(t)\le\alpha\bar{V}(t), \quad (15)$$

Integrating (15) from 0 to t, $t\in[0,T]$, yields,

$$\bar{V}(t)<e^{\alpha t}\bar{V}(0), \quad (16)$$

It then follows that

$$\lambda_{\min}(\bar{P})x^T(t)Rx(t)\le\bar{V}(t), \quad (17)$$

According the condition (8) , we can obtain

$$\begin{aligned}\bar{V}(0)&=x^T(0)Px(0)+\int_{-\tau(0)}^0 x^T(\theta)Qx(\theta)d\theta\\&+\int_{-\tau(0)}^0 f^T(x(\theta))Sf(x(\theta))d\theta\\&+h\int_{-h}^0\int_\theta^0 x^T(s)\tilde{T}x(s)dsd\theta\\&+\frac{h^2}{2}\int_{-h}^0\int_\theta^0\int_s^0 x^T(u)Zx(u)dudsd\theta\\&\le\lambda_{\max}(\bar{P})x^T(0)Rx(0)\\&+h(\lambda_{\max}(\bar{Q}))\sup_{-h\le\theta\le 0}\{x^T(\theta)Rx(\theta)\}\\&+h(\lambda_{\max}(\bar{W}))\sup_{-h\le\theta\le 0}\{x^T(\theta)Rx(\theta)\}\\&+\frac{h^3}{2}(\lambda_{\max}(\bar{T}))\sup_{-h\le\theta\le 0}\{x^T(\theta)Rx(\theta)\}\\&+\frac{h^5}{12}(\lambda_{\max}(\bar{Z}))\sup_{-h\le\theta\le 0}\{x^T(\theta)Rx(\theta)\}\\&\le(k_1+hk_2+hk_3+\frac{h^3}{2}k_4+\frac{h^5}{12}k_5)\sup_{-h\le\theta\le 0}\{x^T(\theta)Rx(\theta)\}\\&\le c_1(k_1+hk_2+hk_3+\frac{h^3}{2}k_4+\frac{h^5}{12}k_5).\end{aligned} \quad (18)$$

From (8) , (17) and (18), one get that

$$x^T(t)Rx(t) \le \frac{\bar{V}(t)}{\lambda_{\min}(\bar{P})} \le \frac{c_1(k_1 + hk_2 + hk_3 + \frac{h^3}{2}k_4 + \frac{h^5}{12}k_5)}{\lambda_1} e^{\alpha T} < c_2.$$

Thus, the uncertain neural networks (6) is finite-time bounded with respect to (c_1, c_2, T, R). The proof is completed.

Under control (5), the closed-loop system of (1) is

$$\begin{cases} \dot{x}(t) = (-A + \Delta A(t) - BK)x(t) + (C + \Delta C(t))x(t-\tau(t)) \\ \quad +(W_0 + \Delta W_0(t)) f(x(t)) \\ \quad +(W_1 + \Delta W_1(t)) f(x(t-\tau(t))), \\ x(t) = \varphi(t), \quad t \in [-h, 0]. \end{cases} \tag{19}$$

Next, we will design controller (3), such that the system (19) is finite-time bounded with respect to (c_1, c_2, T, R).

Theorem 2. For given a matrix $R > 0$, and positive scalars $c_1 \le c_2, T$, the neural networks (1) under the control (5) is finite-time stabilizable with respect to (c_1, c_2, T, R), if there exist matrices $P > 0, Q > 0, S > 0, \tilde{T} > 0, Z > 0, Y$ and diagonal matrices $X > 0, V > 0$, and positive scalars $\varepsilon, \alpha, \lambda_1, k_1, k_2, k_3, k_4, k_5$, such that

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} \\ * & \Xi_{22} \end{bmatrix} < 0, \tag{20}$$

$$c_1(k_1 + hk_2 + hk_3 + \frac{h^3}{2}k_4 + \frac{h^5}{12}k_5) < \lambda_1 c_2 e^{-\alpha T}, \tag{21}$$

where

$$\Xi_{11} = \begin{bmatrix} \bar{\Xi}_{11} & PC & PW_0 + V & PW_1 & 0 \\ * & \bar{\Xi}_{22} & 0 & 0 & 0 \\ * & * & \bar{\Xi}_{33} & 0 & 0 \\ * & * & * & \bar{\Xi}_{44} & 0 \\ * & * & * & * & -\tilde{T} \end{bmatrix},$$

$$\Xi_{12} = \begin{bmatrix} 0 & PH_0 & PH_1 & PH_2 & PH_3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Xi_{22} = diag(-Z, -\varepsilon I, -\varepsilon I, -\varepsilon I, -\varepsilon I,),$$

$$\bar{\Xi}_{11} = -PA - A^T P - BY^T - YB^T + Q + h^2\tilde{T} + (\frac{h^2}{2})^2 Z + \Gamma X \Gamma + 2\Gamma V - \alpha P + \varepsilon E_0^T E_0,$$

$$\bar{\Xi}_{22} = -(1-\mu)Q + \varepsilon E_1^T E_1,$$

$$\bar{\Xi}_{33} = S - X + \varepsilon E_2^T E_2,$$

$$\bar{\Xi}_{44} = -(1-\mu)S + \varepsilon E_3^T E_3,$$

$$P = V^T \begin{bmatrix} \hat{P}_{11} & \\ & \hat{P}_{22} \end{bmatrix} V, \quad PB = B\hat{P}.$$

Furthermore, the controller gains are given by

$$K = \hat{P}^{-T} Y^T.$$

Proof. Replacing A with $A + BK$, from Theorem 1, the system (19) is finite-time bounded if (21) and the following matrix inequality hold

$$\tilde{\Phi} = \begin{bmatrix} \tilde{\Phi}_{11} & \Phi_{12} \\ * & \Phi_{22} \end{bmatrix} < 0, \tag{22}$$

where

$$\tilde{\Phi}_{11} = \begin{bmatrix} \hat{\Phi}_{11} & PC & PW_0 + V & PW_1 & 0 \\ * & \hat{\Phi}_{22} & 0 & 0 & 0 \\ * & * & \bar{\Phi}_{33} & 0 & 0 \\ * & * & * & \bar{\Phi}_{44} & 0 \\ * & * & * & * & -\tilde{T} \end{bmatrix},$$

$$\hat{\Phi}_{11} = -PA - A^T P - PBK - K^T B^T P + Q + h^2\tilde{T} + (\frac{h^2}{2})^2 Z + \Gamma X \Gamma + 2\Gamma V - \alpha P + \varepsilon E_0^T E_0,$$

$$\hat{\Phi}_{22} = -(1-\mu)Q + \varepsilon E_1^T E_1.$$

Let $Y = \hat{P}K$. Using Lemma 2, $PB = B\hat{P}$, and (20), we get

$$\tilde{\Phi} < 0.$$

According to Theorem 1, the system (19) is finite-time bounded with respect to (c_1, c_2, T, R), and the controller gains are given by $K = \hat{P}^{-T} Y^T$.

Remark 1. It is worth noting that, if $\alpha < 0$, the neural networks (1) is global exponential stable, if $\alpha = 0$, then the neural networks is asymptotically stable.

3 Numerical examples

Consider the neural networks (1) with the following parameters

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}, W_0 = \begin{bmatrix} -1 & 3 \\ 4 & -3 \end{bmatrix}, W_1 = \begin{bmatrix} -5 & 4 \\ -4 & 5 \end{bmatrix},$$

$$B = \begin{bmatrix} 7 & 3 \\ 6 & 9 \end{bmatrix}, C = \begin{bmatrix} 7 & 2 \\ 4 & 5 \end{bmatrix}, E_0 = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix},$$

$$H_0 = \begin{bmatrix} -5 & 0 \\ 0 & 4 \end{bmatrix}, E_1 = \begin{bmatrix} 3 & 0 \\ 0 & 6 \end{bmatrix}, E_2 = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix},$$

$$E_3 = \begin{bmatrix} 5 & 0 \\ 0 & 6 \end{bmatrix}, H_1 = \begin{bmatrix} -8 & 0 \\ 0 & 6 \end{bmatrix}, H_2 = \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix},$$

$$H_3 = \begin{bmatrix} -4 & 0 \\ 0 & 3 \end{bmatrix}, \tau(t) = |0.1\sin t|,$$

$$f(x(t-\tau(t))) = \begin{bmatrix} \tanh(0.4x_1(t-\tau(t))) \\ \tanh(0.2x_2(t-\tau(t))) \end{bmatrix},$$

$$f(x(t)) = \begin{bmatrix} \tanh(0.6x_1(t)) \\ \tanh(0.3x_1(t)) \end{bmatrix}, \Gamma = \begin{bmatrix} 0.6 & 0 \\ 0 & 0.3 \end{bmatrix},$$

Take

$c_1 = 1, T = 2, R = I, \alpha = 0.5, h = 0.3, \mu = 0.3, \varepsilon = 0.2.$

By using Matlab LMI control Toolbox to solve inequalities (20) and (21), we have

$$P = \begin{bmatrix} 0.0488 & -0.0020 \\ -0.0020 & 0.0967 \end{bmatrix}, \quad Q = \begin{bmatrix} 19.0294 & 0.1126 \\ 0.1126 & 22.3732 \end{bmatrix},$$

$$S = \begin{bmatrix} 15.8267 & 0.3130 \\ 0.3130 & 13.2863 \end{bmatrix}, \quad \tilde{T} = \begin{bmatrix} 15.2725 & 0.0000 \\ 0.0000 & 15.2725 \end{bmatrix},$$

$$Z = \begin{bmatrix} 15.2725 & 0.0000 \\ 0.0000 & 15.2725 \end{bmatrix}, \quad V = \begin{bmatrix} 6.4562 & 0 \\ 0 & 5.8017 \end{bmatrix},$$

$$X = \begin{bmatrix} 28.7279 & 0 \\ 0 & 25.3255 \end{bmatrix}, \quad c_2 = 487.3830.$$

According to Theorem 2, the system (1) is finite-time stabilizable with respect to (c_1, c_2, T, R). The control gain is

$$K = \begin{bmatrix} -1.5562 & -3.1520 \\ 0.2417 & -0.2820 \end{bmatrix} \times 10^4.$$

4 Conclusion

In this paper, the finite-time stabilization problem of neural networks with uncertainty and time delay is studied. By constructing the Lyapunov-Krasovskii function and matrix inequality method, the sufficient conditions of finite-time stabilize for the uncertain neural networks with time-varying delay are obtained. The controller gain design method is given. Finally, a numerical example is given to show the effectiveness the theoretical results.

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