Selection of Control Vector Switching Points for Circuit Optimization

ALEXANDER ZEMLIAK
Department of Physics and Mathematics
Autonomous University of Puebla
Av. San Claudio y Rio Verde, Puebla, 72570
MEXICO
Institute of Physics and Technology
National Technical University of Ukraine
UKRAINE
azemliak@yahoo.com

Abstract: - The process of optimizing the circuit is formulated as a controlled dynamic system. A special control vector has been defined to redistribute the computational cost between circuit analysis and parametric optimization. This redistribution will help solve the task of optimizing the circuit for minimum CPU time. In this case, the task of optimizing the circuit with minimal CPU time can be formulated as the classical optimal control problem for minimizing some functional. The concept of the Lyapunov function of a controlled dynamic system is used to analyze the main characteristics of the design process. An analysis of the Lyapunov function and its time derivative makes it possible to predict the optimal structure of the control vector for constructing an optimal or quasi-optimal circuit design algorithm. The results are based on the previously discovered effect of accelerating the design process. In this case, the optimal structure of the control vector is determined, which minimizes processor time.

Key-Words: - Time-optimal design algorithm, control theory formulation, Lyapunov function.

1 Introduction

The problem of reducing computer time in the design of large systems is one of the significant problems in the overall improvement of design quality. In addition to the traditionally used ideas of sparse matrix methods and decomposition methods defined decades ago [1–5], other methods have been proposed for reducing the total computer design time [6–8]. A generalized approach to the design of analogue systems based on the formulation of control theory was developed in some previous works, for example [9-10]. This approach serves to determine the design algorithm with minimal CPU time. On the other hand, this approach makes it possible to analyse the design process with great clarity when moving along a trajectory in the design space. The main concept of this approach is the introduction of special control functions, which, on the one hand, generalize the design process, and on the other hand, allow you to control the design process to achieve the optimal value of the cost function in a minimum CPU time. This possibility appears because an almost infinite number of different design strategies exists within the framework of this approach. Different design strategies have different numbers of operations and different CPU times. Within this concept, a traditional design strategy is just one representative of a vast array of different design strategies. As shown in [9], the potential gain in computer time, which can be obtained using a new formulation of the design problem, increases with increasing size and complexity of the system. However, this is realized only when the algorithm is built on the basis of the optimal design strategy. Finding optimal structure of control functions that implement the optimal design strategy is the goal of this work. The construction of an optimal strategy consisting of several strategies allows reducing processor time by several orders of magnitude.

2 Problem Formulation

The optimization process for any analog system can be defined in discrete form as the problem of the generalized cost function $F(X,U)$ minimization by means of the system (1) with the constraints (2):

$$x_i^{s+1} = x_i^s + t_s \cdot f_i(X,U) \quad i = 1,2,...,N \qquad (1)$$

$$(1 - u_j) g_j(X) = 0 \qquad j = 1,2,...,M \qquad (2)$$

where $X \in R^N$, $X = (X',X'')$, $X' \in R^K$ is the vector of the independent variables and the vector

$X'' \in R^M$ is the vector of dependent variables ($N = K + M$). All the functions $g_j(X)$ for all j presents the network model, s is the iterations number, t_s is the iteration parameter, $t_s \in R^1$, $H \equiv H(X, U)$ is the direction of the generalized cost function $F(X, U)$ decreasing, U is the vector of the special control functions $U = (u_1, u_2, ..., u_m)$, where $u_j \in \Omega$; $\Omega = \{0;1\}$. The functions $f_i(X, U)$ for example for the gradient method are defined as:

$$f_i(X, U) = -\frac{\delta}{\delta x_i} F(X, U) \quad i = 1, 2, ..., K \quad (3)$$

$$f_i(X, U) = -u_{i-K} \frac{\delta}{\delta x_i} F(X, U) + \frac{(1 - u_{i-K})}{t_s} \{-x_i^s + \eta_i(X)\} \quad (3')$$

$$i = K + 1, K + 2, ..., N$$

where the operator $\frac{\delta}{\delta x_i}$ hear and below means

$$\frac{\delta}{\delta x_i} \varphi(X) = \frac{\partial \varphi(X)}{\partial x_i} + \sum_{p=K+1}^{K+M} \frac{\partial \varphi(X)}{\partial x_p} \frac{\partial x_p}{\partial x_i}, \quad x_i^s \text{ is}$$

equal to $x_i(t - dt)$; $\eta_i(X)$ is the implicit function ($x_i = \eta_i(X)$) that is determined by the system (2). The generalized cost function $F(X, U)$ can be defined for example as:

$$F(X, U) = C(X) + \psi(X, U) \quad (4)$$

where $C(X)$ is the non negative cost function of the design process, and $\psi(X, U)$ is the additional penalty function:

$$\psi(X, U) = \frac{1}{\varepsilon} \sum_{j=1}^{M} u_j \cdot g_j^2(X) \quad (5)$$

This formulation of the design process permits the redistribution of the computer time expense between the solution of the problem (2) and the optimization procedure (1) for the function $F(X, U)$. The control vector U is the main tool for the redistribution process in this case. Practically an infinite number of the different design strategies are produced because the vector U depends on the optimization procedure current step. The traditional design strategy (TDS) is formulated in this case as a strategy with all functions u_j equal to 0. The problem of the optimal design strategy search is formulated now as the typical problem for the functional minimization of the control theory. The functional that needs to minimize is the total CPU time T of the design process. This functional depends directly on the operations number and on the design strategy that has been realized. The main difficulty of this definition is unknown optimal dependencies of all control functions u_j. It is necessary to find the optimal behavior of the control functions u_j during the design process to minimize the total design computer time.

Now the process for analog network design is formulated as a dynamic controllable system. The minimal-time design process can be defined as the dynamic system with the minimal transition time In this case. So, we need to find the special conditions to minimize the transition time for this dynamic system.

3 Lyapunov Function of the Optimization Process

On the basis of the analysis in previous section we can conclude that the minimal-time algorithm has one or some switch points in control vector where the switching is realize among different design strategies. As shown in [10] it is necessary to switch the control vector from like modified traditional design strategy (MTDS) when all u_j equal to 1 to like traditional design strategy (TDS) with some adjusting. Some principal features of the time-optimal algorithm were determined previously. These are: 1) an additional acceleration effect that appeared under special circumstances [11]; 2) the start point special selection outside the separate hyper-surface to guarantee the acceleration effect, at least one negative component of the start value of the vector X is can be recommended for this; 3) an optimal structure of the control vector with the necessary switch points. The two first problems were discussed in [10-11]. The third problem is discussed in the present paper.

The main problem of the time-optimal algorithm construction is unknown optimal sequence of the switch points during the design process. We need to define a special criterion that permits to realize the optimal or quasi-optimal algorithm by means of the optimal switch points searching. A Lyapunov function of dynamic system serves as a very informative object for any system analysis in limits of the control theory. Traditionally this function is used for the stability analysis of dynamic systems. We propose to use a Lyapunov function of the design process for the optimal algorithm searching, particularly for the optimal switch points detect. The

properties of the Lyapunov function give possibility to solve this problem.

There is a freedom of the Lyapunov function choice because of a non-unique form of this function. Let us define the Lyapunov function of the design process (1)-(5) by the following expression:

$$V(X) = \sum_i (x_i - a_i)^2 \qquad (6)$$

where a_i is the stationary value of the coordinate x_i, in other words the set of all the coefficients a_i is the main objective of the design process. The function (6) satisfies all of the conditions of the standard Lyapunov function definition for the variables $y_i = x_i - a_i$.

Inconvenience of the formula (6) is an unknown point $A = (a_1, a_2, ..., a_N)$, because this point can be reached at the end of the design process only. We can use this form of the Lyapunov function if we already found the design solution someway. On the other hand, it is very important to control the stability of the design process during the optimization procedure. In this case we need to construct other form of the Lyapunov function that doesn't depend on the unknown stationary point. Let us define two new forms of the Lyapunov function by the next formulas:

$$V(X, U) = [F(X, U)]^r \qquad (7)$$

$$V(X, U) = \sum_i \left(\frac{\partial F(X, U)}{\partial x_i} \right)^2 \qquad (8)$$

where $F(X,U)$ is the generalized cost function of the design process. The formula (7) can be used when the general cost function is no negative and has zero value at the stationary point A. Other formula can be used always because all derivatives $\partial F / \partial x_i$ are equal to zero in the stationary point A.

We can define now the design process as a transition process for controllable dynamic system that can provide the stationary point (optimal point of the network optimization procedure) during some time. The problem of the construction of the time-optimal design algorithm can be formulated now as the problem of the transition process searching with the minimal transition time. There is a well-known idea [12-13] to minimize the time of the transition process by means of the special choice of the right hand part of the principal system

of equations, in our case these are the functions $f_i(X, U)$. It is necessary to change the functions $f_i(X, U)$ by means of the control vector U selection to obtain the maximum speed of the Lyapunov function decreasing (the maximum absolute value of the Lyapunov function time derivative $\dot{V} = dV/dt$). Normally the time derivative of the Lyapunov function is non-positive for the stable processes. However, we can define now more informative function as a time derivative of Lyapunov function relatively the Lyapunov function: $W = \dot{V}/V$. In this case we can compare the different design strategies by means of the function $W(t)$ behavior and we can search the optimal position for the control vector switch points.

4 Control Vector Optimal Structure

The optimal structure of the control vector U is the principal aim of the analysis of design process based on generalized methodology. This control vector's structure produces optimal or quasi optimal design process that minimizes the computer time. Functions $V(t)$ and $\dot{V}(t)$ were the main objects of the analysis and its behavior has been analyzed during the design process. The behavior of the functions $V(t)$, $\dot{V}(t)$ and $W(t)$ can define the total computer time for each design strategy [14-15]. The analysis of the behavior of these functions gives possibility to determine the optimal position of the switch points of the control vector. These functions serve as a sensitive criterion to detect the optimal switching position for the control vector U.

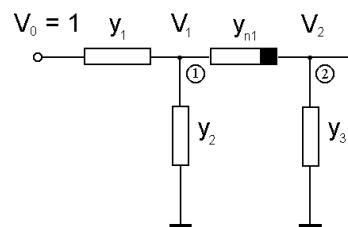The analysis of the design process for two-node passive nonlinear network is presented in Fig. 1.

Fig. 1 Two-node nonlinear passive network

The nonlinear element has the following dependency: $y_{nl} = a_{nl} + b_{nl} \cdot (V_1 - V_2)^2$. The vector X includes five components: $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4 = V_1$, $x_5 = V_2$. The model of this network (2)

includes two equations (*M*=2) and the optimization procedure (5) includes five equations. This network is characterized by two dependent parameters and the control vector includes two control functions: $U=(u_1,u_2)$. Structural basis includes four different strategies with corresponding control vector: (00), (01), (10), and (11). Behavior of the functions *V(t)* and *W(t)* help us to determine the switch point optimal position of the control vector. When using TDS (*U*=(00)), the CPU time is 2.53 seconds.

Taking into account the preliminary reasons about the optimal algorithm structure [15] we have been analyzed the strategy that consists of two parts. The first part is defined by the control vector (11) that corresponds to MTDS and the second part is defined by the control vector (00) that corresponds to TDS. So, the switching is realized between two strategies, (11) and (00).

The behavior of the functions *V(t)* and *W(t)* during the design process after the switch point is shown in Fig.2. The corresponding iteration number and computer time are presented in Table 1.
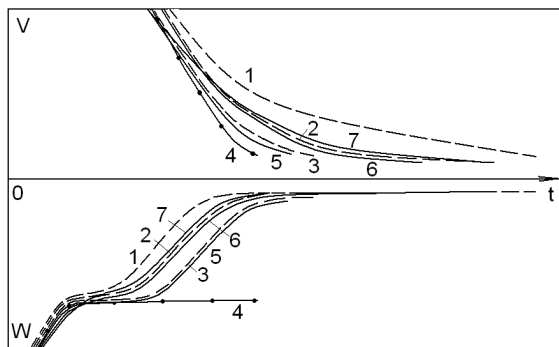

Fig. 2 Behavior of the functions *V(t)* and *W(t)* during the design process for seven different switch points (from 147 to 267)

Table 1. Iterations number and computer time for strategies with different switch points

N	Switch point	Iterations number	Total design time (sec)
1	147	8319	0.221
2	167	6501	0.172
3	187	3697	0.096
4	207	2860	0.073
5	227	3383	0.087
6	247	5429	0.142
7	267	6682	0.175

The analysis shows that the optimal switch point corresponds to the step 207 (graph 4 with dots in Fig. 4). The curves 1, 2, and 3 correspond to the switch point position before the optimal switch point (curve 4), but the curves 5, 6, and 7 correspond to the switch point that lies after the optimal one. There is a decreasing of the computer time from curve 1 to curve 4. On the contrary, the computer time increases from curve 4 to curve 7. It means that curve 4 corresponds to the optimal position of the switch point.

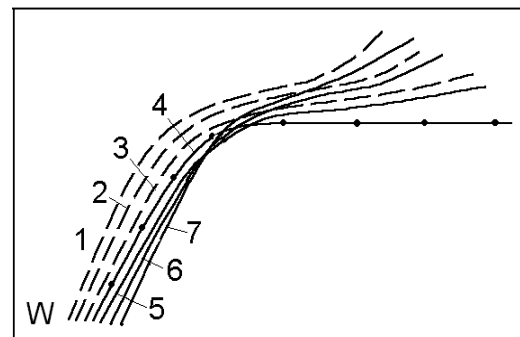The initial parts of *W(t)* dependencies of Fig. 2 are shown in Fig. 3 in large scale.

Fig. 3 Behavior of the functions *V(t)* and *W(t)* during the initial part of design process

We can see that the curves 1, 2, and 3, which correspond to the switch points before the optimal point (4) have not intersections. On the other hand, the curves 5, 6, and 7 that are based on the switch point after the optimal one have intersections and each this curve lies upper the curve 4 till some time point. It means that from this time moment the graph *W(t)* for the optimal switch point lies below all of other graph. So, from one hand the optimal switch point corresponds to a minimal computer time, from the other hand, this point corresponds to the graph of *W(t)* function that lies below all of other graphs. This property serves as a principal criterion for the optimal switch point selection.

The function *W(t)*, which corresponds to the optimal switching point, has a maximum absolute value, starting from the 340th step of integration. This means that at this stage of integration, we can confidently predict the optimal position of the switching point, which leads to a minimum CPU time. The time gain of a complex strategy consisting of MTDS and TDS with an optimal switching point between them at the 207th integration step compared to TDS is 34.5 times.

The analysis of the design process for three-node passive nonlinear network in Fig. 4 is presented below.
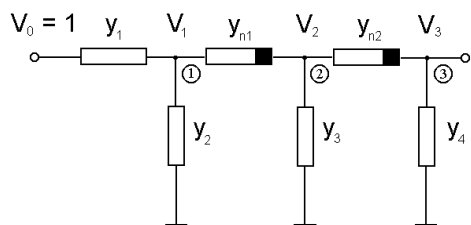
Fig. 4 Three-node nonlinear passive network

The nonlinear elements are defined as: $y_{n1} = a_{n1} + b_{n1} \cdot (V_1 - V_2)^2$, $y_{n2} = a_{n2} + b_{n2} \cdot (V_2 - V_3)^2$. The vector X includes seven components: $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4^2 = y_4$, $x_5 = V_1$, $x_6 = V_2$, $x_7 = V_3$. The model of this network (2) includes three equations (M=3) and the optimization procedure (5) includes seven equations. This network is characterized by three dependent parameters and the control vector includes three control functions: $U = (u_1, u_2, u_3)$. Structural basis includes eight different strategies with corresponding control vector: (000), (001), (010), (011), (100), (101), (110), and (111). When using TDS (U=(000)), the CPU time is 3.88 seconds. Behavior of the functions $W(t)$ can determine the optimal position of switch point.

We will search for the optimal strategy, consisting of two parts MTDS (U=(111)) and TDS (U=(000)). So, the switching is realized between two strategies, (111) and (000).

The optimal switch point was a principal objective of this analysis. The consecutive change of the switch point was realized for the integration step number from 2 to 20.

The behavior of the functions $V(t)$ and $W(t)$ during the design process after the switch point is shown in Fig. 5.
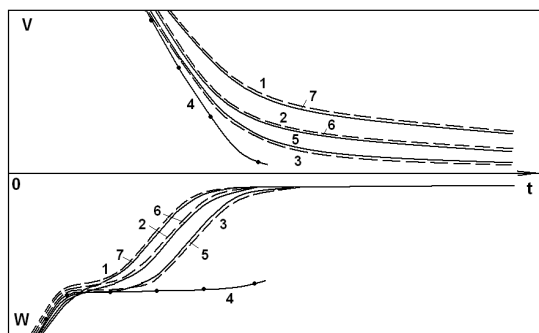

Fig. 5 Behavior of the functions $V(t)$ and $W(t)$ during the design process for seven different switch points (from 6 to 12) for network in Fig. 4

As discussed above, the principal element of the minimal-time design algorithm is the optimal position of the control vector switch point. Fig. 5 shows the behavior of the functions $V(t)$ and $W(t)$ for seven different positions of the switch point. The corresponding total iteration number and computer time are presented in Table 2.

Table 2. Iterations number and computer time for strategies with different switch points for network in Fig. 4

N	Switch point	Iterations number	Total design time (sec)
1	6	8409	0.659
2	7	6408	0.502
3	8	3141	0.246
4	9	1234	0.096
5	10	3310	0.259
6	11	5918	0.464
7	12	7404	0.581

The integration of the system (1) was realized by the constant integration step. The analysis shows that the optimal switch point corresponds to the step 9 (graph 4 with dots in Fig. 5). The curves 1, 2, and 3 correspond to the switch point position before the optimal switch point (curve 4), but the curves 5, 6, and 7 correspond to the switch point that lies after the optimal one. There is a decreasing of the computer time from curve 1 to curve 4. On the contrary, the computer time increases from curve 4 to curve 7. It means that curve 4 corresponds to the optimal position of the switch point.

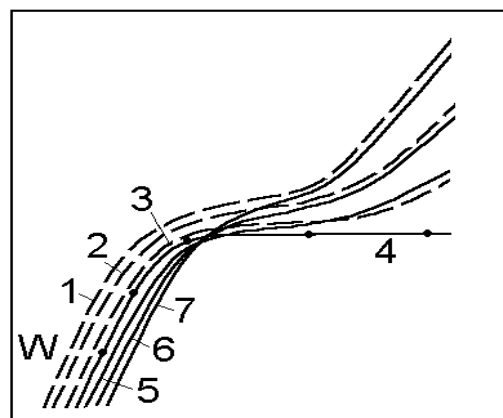The initial part of $W(t)$ dependencies of Fig. 5 are shown in Fig. 6 in large scale.

Fig. 6 Behavior of the functions $V(t)$ and $W(t)$ during the initial part of de sign process for network in Fig. 4

We can see that the curves 1, 2, and 3, which correspond to the switch points before the optimal point (4) have not intersections. On the other hand, the curves 5, 6, and 7 that are based on the switch point after the optimal one have intersections and each this curve lies upper the curve 4 till some time point. It means that from this time moment the graph $W(t)$ for the optimal switch point lies below all of other graph. So, from one hand the optimal switch point corresponds to a minimal computer time, from the other hand, this point corresponds to the graph of $W(t)$ function that lies below all of other graphs. This property anew serves as a principal criterion for the optimal switch point selection. The function $W(t)$ that corresponds to the optimal switch point has a maximum absolute value leading off the 15th integration step. It means that from this integration step we can confidently predict the optimal switch point position that leads to the minimal computer design time. The time gain of a complex strategy consisting of MTDS and TDS with an optimal switching point between them at the 9th integration step compared to TDS is 40.4 times.

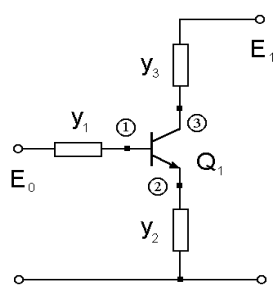Next example corresponds to the one-stage transistor amplifier in Fig. 7.

Fig. 7 One-stage transistor amplifier

The vector X includes ten components: $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4 = V_1$, $x_5 = V_2$, $x_6 = V_3$. The model of this network (2) includes three equations (M=3) and the optimization procedure (5) includes six equations. The total structural basis contains eight different design strategies. The control vector includes five control functions: $U=(u_1, u_2, u_3)$. The Ebers-Moll static model of the transistor has been used [16]. When using TDS (U=(000)), the CPU time is 4.17 seconds.

We will search for the optimal strategy, consisting of two parts MTDS (U=(111)) and TDS (U=(000)).

Fig. 8 shows the behavior of the functions $V(t)$ and $W(t)$ during the design process with different switch points.

The behavior of these functions helps us to determine the optimal position of the control vector switch point. We have been analyzed the strategy that consists of two parts. The first part is defined by the control vector (111) that corresponds to MTDS and the second part is defined by the control vector (000) that corresponds to TDS. The optimal switch point was an aim of the analysis. The consecutive change of the switch point was realized for the integration step number from 2 to 50. The behavior of the functions $V(t)$ and $W(t)$ for the switch points from 33 to 39 are shown in this figure and the data, which correspond to these graphs, are presented in Table 3.
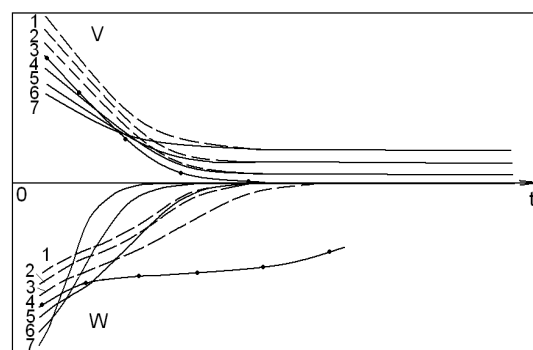

Fig. 8 Behavior of the functions $V(t)$ and $W(t)$ during the design process for seven different switch points (from 33 to 39) for network in Fig. 7

Table 3. Iterations number and computer time for strategies with different switch points for one-stage transistor amplifier

N	Switch point	Iterations number	Total design time (sec)
1	33	2433	0.404
2	34	2180	0.361
3	35	1748	0.289
4	36	61	0.01
5	37	1705	0.281
6	38	2111	0.349
7	39	2349	0.389

The analysis shows that the optimal switch point corresponds to the step 36 (graph with dots). The computer design time has a minimal value for this step. The function $W(t)$ has a maximum absolute value for the optimal switch step (number 4) leading off the 55th integration step. It means that from this integration step we can confidently

predict the optimal switch point position that leads to the minimal computer design time. The time gain of a complex strategy with an optimal switching point at the 36th integration step compared to TDS is 417 times.

The last example corresponds to the two-stage transistor amplifier in Fig. 9.
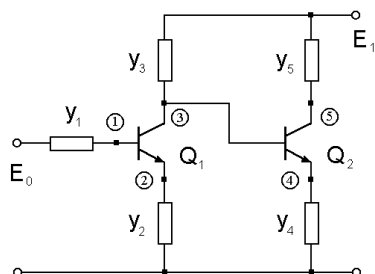

Fig. 9 Two-stage transistor amplifier

The vector X includes ten components: $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4^2 = y_4$, $x_5^2 = y_5$, $x_6 = V_1$, $x_7 = V_2$, $x_8 = V_3$, $x_9 = V_4$, $x_{10} = V_5$. The model of this network (2) includes five equations ($M=5$) and the optimization procedure (5) includes ten equations. The total structural basis contains 32 different design strategies. The control vector includes five control functions: $U = (u_1, u_2, u_3, u_4, u_5)$. When using TDS ($U=(00000)$), the CPU time is 967.4 sec.

We will search for the optimal strategy, consisting of three parts MTDS ($U=(11111)$), TDS ($U=(00000)$) and MTDS ($U=(11111)$) with two switch points.

Fig. 10 shows the behavior of the functions $V(t)$ and $W(t)$ for some design strategies with different switch points including the optimal one.

The data, which correspond to these graphs, are presented in Table 4.
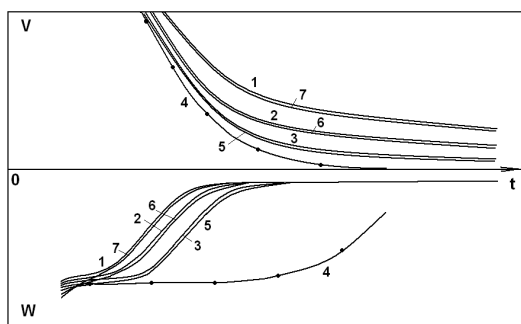

Fig. 10 Behavior of the functions $V(t)$ and $W(t)$ during the design process for seven different switch points (from 7 to 13) for network in Fig. 9

The integration of the system (5) was realized by the optimal integration step.

Table 4. Iterations number and computer time for strategies with different switch points for network in Fig. 9

N	Switch point 1	Switch point 2	Iterations number	Total design time (sec)
1	7	8	4900	9.912
2	8	9	4486	9.113
3	9	10	3785	7.691
4	10	11	1354	2.742
5	11	12	3618	7.341
6	12	13	4424	8.981
7	13	14	4882	9.893

In this case the quasi-optimal control vector includes two switching points. We changed the control vector from (11111) to (00000) and from (00000) to (11111). The consecutive change of the switching point was realized for the integration step's number from 2 to 20.

The behavior of the functions $V(t)$ and $W(t)$ for the optimal switch steps and some steps near the optimal confidently detect the optimal position of the switch points.

The time gain of a complex strategy with optimal switching points at the 10th and 11th integration steps compared to TDS is 352.8 times.

We observe a specific behavior of the function $W(t)$ near the optimal switch point's position. Before the optimal switching points the function $W(t)$ graphs are "parallel". Function $W(t)$ has the maximum negative value for the optimal switch points. The graphs of the function $W(t)$ that correspond to the optimal switch point's position (number 4) and before the optimal position (1, 2 and 3) have not intersection. After the optimal points the graphs of the function $W(t)$ intersect the graphs that correspond to the optimal switch point and before the optimal one. It means that we can detect the optimal position of the switch points during the initial design interval.

Thus, the optimal structure of the control vector, that is, the structure of the time-optimal design strategy, can be determined by analyzing the relative time derivative of the Lyapunov function during the initial time interval of the design process.

Summarizing all the results obtained, we can conclude that the behavior of the time derivative of the Lyapunov function of the design process allows

us to determine the optimal switching points of the control vector, that is, the optimal or quasi-optimal structure of the control vector. This means that the optimal structure of the control vector can be obtained during the initial interval of the design process.

5 Conclusion

The task of constructing a minimum-time design algorithm can be adequately solved on the basis of control theory. The design process in this case is formulated as a controlled dynamic system. The Lyapunov function of the design process and its time derivative contain sufficient information to select more promising design strategies from the infinite number of different design strategies that exist in the generalized design methodology. A special function W (t) was proposed to predict a time-optimal design strategy. This function can be used as the main tool for constructing the optimal sequence of control vector switching points. The solution to this problem allows you to build a system design algorithm in minimal CPU time. Moreover, the time gain of the optimal strategy in comparison with the traditional strategy is 2–3 orders of magnitude.

References:
[1] J.R. Bunch and D.J. Rose, (Eds.), *Sparse Matrix Computations*, Acad. Press, N.Y., 1976.

[2] O. Osterby and Z. Zlatev, *Direct Methods for Sparse Matrices*, Springer-Verlag, N.Y., 1983.

[3] F.F. Wu, Solution of Large-Scale Networks by Tearing, *IEEE Trans. Circuits Syst.*, Vol. CAS-23, No. 12, 1976, pp. 706-713.

[4] A. Sangiovanni-Vincentelli, L.K. Chen and L.O. Chua, An Efficient Cluster Algorithm for Tearing Large-Scale Networks*, IEEE Trans. Circuits Syst.,* Vol. CAS-24, No. 12, 1977, pp. 709-717.

[5] N. Rabat, A.E. Ruehli, G.W. Mahoney and J.J. Coleman, A Survey of Macromodeling, *Proc. of the IEEE Int. Symp. Circuits Systems*, April, 1985, pp. 139-143.

[6] I.S. Kashirskiy and I.K. Trokhimenko*, General Optimization for Electronic Circuits*, Tekhnika, Kiev, 1979.

[7] V. Rizzoli, A. Costanzo and C. Cecchetti, Numerical optimization of broadband nonlinear microwave circuits, *IEEE MTT-S Int. Symp.*, Vol. 1, 1990, pp. 335-338.

[8] E.S. Ochotta, R.A.Rutenbar and L.R. Carley, Synthesis of High-Performance Analog Circuits in ASTRX/OBLX, *IEEE Trans. on CAD*, Vol.15, No. 3, 1996, pp. 273-294.

[9] A.M. Zemliak, Design of Analog Networks by Control Theory Methods, Part 1, Theory, *Radioelectronics and Communications Systems*, Vol. 47, No. 5, 2004, pp. 11-17.

[10] A. Zemliak, Analog circuit optimization on basis of control theory approach, *COMPEL: The International Journal for Computation and Mathematics in Electrical and Electronic Engineering*, Vol. 33, No. 6, 2014. pp. 2180-2204.

[11] A.M. Zemliak, Acceleration Effect of System Design Process, *IEICE Trans. on Fundam.*, Vol. E85-A, No. 7, 2002, pp. 1751-1759.

[12] E.A. Barbashin, *Introduction to the Stability Theory*, Nauka, Moscow, 1967.

[13] N. Rouche, P. Habets and M. Laloy, *Stability Theory by Liapunov's Direct Method*, Springer-Verlag, N.Y, 1977.

[14] A.M. Zemliak, Analysis of Dynamic Characteristics of Process of Designing Analogue Circuits, *Radioelectronics and Communications Systems*, Vol. 50, No. 11, 2007, pp. 603-608.

[15] A. Zemliak, T Markina, Behaviour of Lyapunov´s function for different strategies of circuit optimization, *International Journal of Electronics*, Vol. 102, No. 4, 2015, pp. 619-634.

[16] G. Massobrio G. and P. Antognetti, *Semiconductor Device Modeling with SPICE*, N.Y.: Mc. Graw-Hill, Inc., 1993.