Investigation and Reduction of Large-Scale Dynamical Systems

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Abstract: -We present an efficient implementation of the balance truncation approximation method for large-scale dynamical system, which a benchmark collection Inclusive of some needful real-world examples. In this paper we proposed a new procedure the reduction method based balance truncation is explored for getting structure preserving reduced order model of a large-scale dynamical system, we have considered model order reduction of higher order LTI systems. That aims at finding Error estimation and $H\infty$ and H2 norm using Approximation of original and reduced system. Hence necessary to effortlessness the analysis of the system using approximation Algorithms. The response evaluation is considered in terms of response constraints and graphical assessments. It is reported that the different states of reduced order model compare using a numerical methods is almost alike in performance to that of with original systems. all simulation results have been obtained via MATLAB based novel software (sssMOR toolbox).

Key-Words:- Benchmarks Example, reduced Order model (ROM), Error estimation, Balanced Truncation, International space station (ISS**).**

1 Introduction

As engineering systems become successively more and more complicated so do the mathematical representations[1]. These mathematical methods give an extensive clarification of within the form of a higher order model. The most important problem in any appearance at concerning the dynamic behaviors of the Higher dimensional system are all around and exist in multiple fields such as complicated transportation device, ecological systems, electrical power device aeronautics, hydraulic. And adequately of extra and so on. Or a hybrid of those, a system is said to be big if it could be decoupled into a number of the coordinated device or low-scale system for both computational or practical reasons [2] Otherwise, a system is a huge scale while its size are so huge, such that the traditional strategies of modeling, analysis, manage system design and enumeration fail to present correct solutions with reasonable computational efforts. The analysis of such physical device starts through building up of the model which can be taken into consideration as an enthusiastic representation of such structures, the mission of control engineer instigates with the formulation of a model and evaluation, Consequently, the mission of MOR can be change to outcome suitable characteristics on the original model to be preserved. In this work, we introduce a high stage negotiation on computational technological know-how, and the essential for dense models of occurrence observed in perspective and engineering[3]. We argue that rather more sophisticated complications can be addressed by way of using latest soft technology and innovative algorithms, however, that there is a want for MOR so that you can deal with even greater complex issues(Schilders,2008)[4].

Approximation Techniques play an essential foreword nowadays Large-Scale layout. Direct numerical simulation of high order dynamical system is commonly used to validate the design and examine the behavior of the region of the system earlier than consultation and implementation. One of the principal studies regions in the large-scale real-global system[5].The Dynamical system simulation is to apply mathematical modeling to forecast the manners of a real-world system. A system is typically represented in the form of partial differential equations is known as PDEs for large system and other Distributed parameter structures or ordinary differential equations is ODEs- Lumped Parameter Systems. So, the dynamical system simulation without a doubt includes solving large-scale dynamical (LSD) ODEs, which now and again takes numerous days or maybe weeks. Thus, rapidly and unique system reproduction algorithms are proper to quicken the simulation cycle[6].

The MOR is numerical method for order Decrease of large -scale system to enhance the simulation with a suitably easy system, which reproduction the main characteristics of the original character one, which reduces the complication of the higher large-scale system and produces a ROM (reduced order model) to characterize the original one[7].in this present basically three MOR (model order reduction) methods, however, there may be no technique that gives the quality consequences for all of the structures. So, each system makes use of the quality approach in keeping with its application. So, there is nevertheless want for novel MOR approach [7].

This paper gives innovative MOR procedures based on fully MATLAB environment for the linear systems and non-linear. we give some a numerical mathematics investigational outcome to explain the effectiveness of this method. The experiment performed on a computer of Intel i5 core at 2.5GHz and 8GB RAM. The ssMOR toolbox in the build through the MATLAB R2014b and perform the ISS model of reduced order model. MOR is one area wherein "approximation-based techniques" have tested effective applications. We propose MATLAB Software (sssMOR toolbox)[8] -based totally algorithms for model order reduction that dynamically studies the outstanding explanation. We put in strength our method and compare its techniques. Our method to progress run-time without any offensive the ordinary accuracy.

2 System Representation

In this section, explain the system representation & implementation for MOR. we will go deal with the problem of interpretation or model order reduction (MOR) of The large-scale dynamical system.

2.1 Brief descriptions

This brief discussion is focused on computing the reduced-order approximation estimation for linear dynamical systems are primarily considered over their input-output plot δ : **u** →**y**, mapping inputs **u** to outputs **y** via a state-space realization given as LTI system

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \tag{1}$$

with a related transfer function matrix of the Original system

$$G(s) = C(sI_n - A)^{-1}B + D$$

(2)

where $x \in \mathbb{R}^n$ is the system state, $y \in \mathbb{R}^p$ is the system output, u(t) is the control input, A $\in \mathbb{R}^{n\times n}$, B $\in \mathbb{R}^{n\times m}$, C $\in \mathbb{R}^{p\times n}$ and $D \in \mathbb{R}^{p\times m}$. The systems of all states stable and minimal phase and the number of state variables n is thus the order of the system[9].

We mention to δ as a SISO system -single-input/single-output system once $m= p = 1$ (scalar-valued input, scalar-valued output) and as a multiple -input/ multiple -output -MIMO system [10]. The Systems of the form (1) express with enormously the higher state-space dimension n follow in numerous orders; see and for an assembly of such examples. In spite of high order dimension, as a rule, the state space directions, x(t), slash near subspaces with expressively lower[11] the size and develop in behaviors that do not completely acquire the state space[3]. We need the reduced input-output plot, δ_r : $\mathbf{u} \to y_r$, to be close to δ in an appropriate sense[12]. Being a smaller version of the LSD system , the input-output plot δ_r is described by the reduced system in state-space form as in model reduction, one attempts to the invention a ROM of the system[13].

The essential knowledge behind model order reduction is to originate a low-order approximation $G_r(s)$ for a given higher-order system $G(s)$ ROM- Reduced order model can be used expressed as given below:

$$\begin{aligned} \dot{x}_r &= A_r x_r + B_r u_r \\ y_r &= C_r x_r + D_r u_r \end{aligned} \tag{3}$$

of the order $n_r \ll n$ such that the TF of the original System or also say LSD -large scale dynamical system.

$$G_r = C_r(sI_{n_r} - A_r)^{-1}B_r + D_r \tag{4}$$

approaches original system G(s) in a specific sense, and model reduction approaches diverge characteristically in the error estimated that is being diminished. Throughout this brief, subscript "r " is used to denote parameters related to the ROMs[14].

- A is the matrix of the large system so the eigen-values (or at least the nearby ones to the $j\omega$ axis), which are also the poles of large-scale system G(s)
- control theory to describe controllability Gramian G_c and observability Gramian G_o of the higher-order system, the explanations of the Lyapunov equations is defined as

$$AG_c + G_cA^T + BB^T = 0, \tag{5}$$

$$A^TG_o + G_oA + CC^T = 0 \tag{6}$$

- singular values of the system are a Hankel plot so-called state contributions, this plot introduce the system states that dominant area which are similarly the square-roots of the eigen-values of

Balanced truncation Algorithm[19][23]

1.Compute $X = RR^T$ and $Y = LL^T$

2.Compute the SVD

$$L^T R = [U_1, U_2]\begin{bmatrix} \Sigma_1 & \\ & \Sigma_2 \end{bmatrix}[V_1, V_2]^T,$$

With $\Sigma_1 = diag(\xi_1, ..., \xi_l)$,

$\Sigma_2 = diag(\xi_{l+1}, ..., \xi_n)$.

3.Compute the Reduced order model

$(A_r, B_r, C_r, D_r) = (W^T AT, W^T B, CT, D)$

With

$W = LU_1 \Sigma_1^{-1/2} \in \mathbb{R}^{n\times l}, T = RV_1\Sigma_1^{-1/2} \in \mathbb{R}^{n\times l}$.

Properties

- (A_r, B_r, C_r, D_r) is asymptotically stable
- Error bound:
  $\|G_r - G\|_{H_\infty} \le 2(\xi_{l+1} + ... + \xi_n)$
- Need to solve large -scale Lyapunov equations

H_∞-Norm of Difference System (G).

Absolute: $H_\infty = \|G_1 - G_2\|_{H_\infty}$

Relative: $H_\infty = \dfrac{\|G_1 - G_2\|_{H_\infty}}{\|G_1\|_{H_\infty}}$

H_2-Norm of Difference System (G).

Absolute: $H_2 = \|G_1 - G_2\|_{H_2}$

Relative: $H_2 = \dfrac{\|G_1 - G_2\|_{H_2}}{\|G_1\|_{H_2}}$

controllability Gramian, observability Gramian ($G_c G_o$)[15].

- frequency response is also known as TF for the largest singular value.

$$\sigma(\omega) = \| G(j\omega) \| 2 \qquad (7)$$

These appearances can be associated with those of the reduced system $G_r(s)$, When they are presented, in this brief, a benchmark example is considered as a test system. Such models are represented by linear MIMO with the dynamical system, we give all of the overhead assets for the benchmark examples. For each example, we afford the matrix model $\{A, B, C, and, D\}$ [16][17].

3 Approximation by Balanced Truncation (BT) Algorithm

In this section, the key plan in BT is to change the original stable LTI system into a picture of the same characteristics, but with assets that those states of the transformed system that controllable are also observable and vice versa.[18]describe the methodology of Balanced Truncation, firstly presented by B. C. Moore [19], is constructed on related to control theory system, where one basically examines how a system can be explored and how its response can be found[20]. In such manner, the essential thought of the BT[21]the concept BT is to change the original LTI system to an comparable one is which the states that are hard to reach are also does not make sense to observe . balanced truncation approach basically applies the truncation procedure to the BRT-balanced realization techniques (A, B, C, and D) of an original system G for model order reduction[22].

4 Numerical Experiments

In this paper, we introduced a globalized way for the approximation of a Benchmark Problems. We apply the algorithms mentioned above to a large-scale dynamical system: International space station (ISS)[24]. This is a model of component 1r (Russian service module) from the collection of benchmark example [25].it has 270 states,3 input, and 3 outputs, hence MIMO system. The HSV (state contributions) of the system are shown in below Fig.1. we approximate the system in different-2 states such as 15 states,16 states and 17 states with reduced model's effort quite well. The resultant reduced states ISS model are exposed in below Fig.2.Singular values and Fig.3 in magnitude plot of individual states of ISS model. as seen from this below figure, approximated in this contribution, implementing balance truncation methods mentioned above have been developed MATLAB based, an innovative software used the sssMOR toolbox (Castagnotto *et al*,.2017). a package which are used for a high-scale dynamical system. The popularity of the Balance truncation technique as a suitable model reduction technique exists in it's of a rule to reduce high order structures like the ones springing up for example from the modeling of a large-scale dynamical system[26]. The technique can manage manners with casually damped modes and uniform unstable structures. In the circumstance of actual massive order systems, the Balance truncation is one of the very few suitable techniques[27][3]. Numerous obstacles of the modal adjustment balance problems for contemporary use of this technique. In the original system place, the curtailment of a commonly adapted modal domination assay address prevents the use of this address in abounding cases as-as an analogy if the original system has added than one pole. The present techniques abort occasionally even to appear original appearance (characteristics) non-minimally, that is, poles which are uncontrollable or unobservable. [28][29].

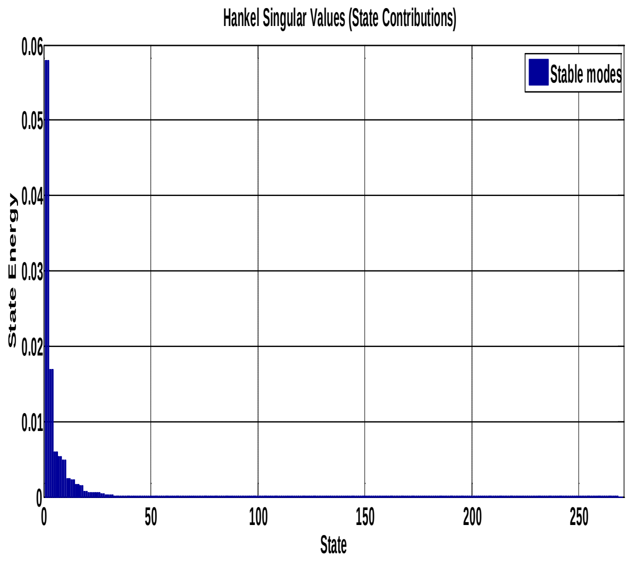

Figure1: Hankel Singular Values of ISS Model

Fig. 1. In the Hankel Singular values (state contributions) are plotted, from these plots, it is understandable that ISS Model is stable. From this figure, we can see that some eigenvalue has the greatest dominating characteristics of the system than other states i.e. These some states are greatest dominating state and the relaxation of the state are much less dominant. Consequently, the system can be reduced into different-2 states such as 15 states,16 states, and 17 states is most dominant states of the system.

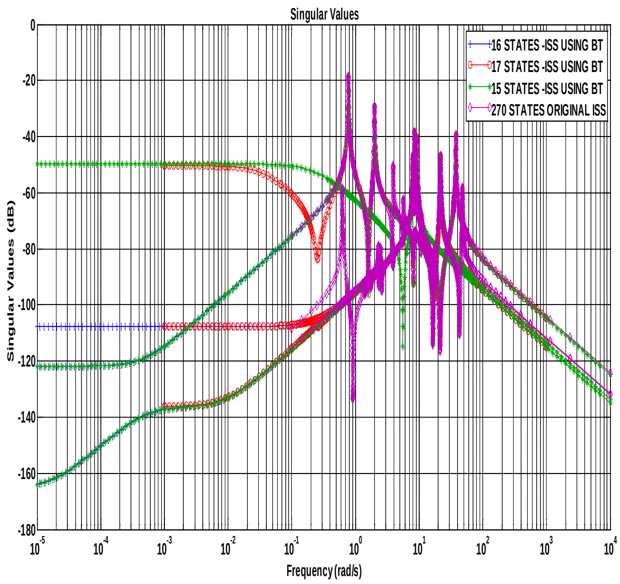

Figure 2: Singular Values of ISS model and Reduced states

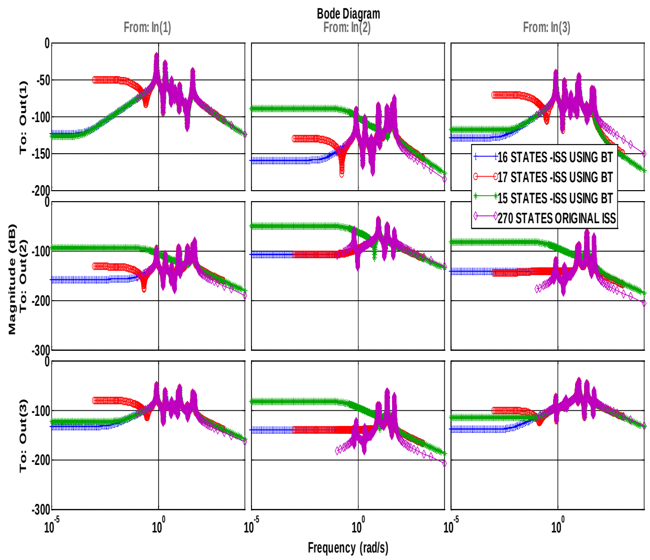

Figure 3: Magnitude Bode Diagram of ISS Model and the Reduced States

It is clear that in Fig.2. and Fig.3. the shows the singular values and magnitude bode diagram of original with the reduced system. After that we can understand that abolish the less dominating states of the system without affecting stability preserving of the original system with less error acceptance.

Table1. ISS for SSMOR-a comparison of Reduced model with the method

Response of Model							
Models	**Selected model**			**Difference model**			
	H_∞	H_2		H_∞ **Norm**		H_2 **Norm**	
	Norm	**Norm**	**Decay Time(s)**	**Absolute**	**Relative**	**Absolute**	**Relative**
270 States Original model	0.1159	0.01006	1477.3	2.942E-18	2.538E-17	3.303E18	3.299E-16
15 States Reduced model	0.1159	0.01001	1188.25	0.00335	0.02869	0.001883	0.1881
16 States Reduced model	0.1159	0.01001	1188.25	0.003024	0.1014	0.001015	0.1014
17 States Reduced model	0.1159	0.1002	11.88.23	0.003023	0.02608	0.0011	0.1097

5 Discussions

In this segment, we give some a numerical mathematics experimental outcome to explain the effectiveness of this approach. The experiment achieved on a computer of Intel i5 core at 2.5GHz and 8GB RAM. The ssMOR toolbox in the build through the MATLAB software version R2014b and done the ISS model of reduced order model. Above Table.1. Details analysis of a Benchmark illustration. In this paper, we present a reasonable study of an approximation algorithm for model order reduction, an error estimation of a globalized method for approximation of Benchmark Problems. We apply the algorithms mentioned above for structural model of LSD systems [25]. The HSV (state contributions) of the system are shown in above Fig.1. we approximate the system in different-2 states such as 15 states,16 states and 17 states with reduced model's effort quite well. The resultant reduced states ISS model is shown in above Fig.2.Singular values and Fig.3 in magnitude plot of different-2 states of ISS model. as seen from this below figure, approximated. this contribution, implementing balance truncation methods mentioned above have been developed MATLAB based, an innovative software used the sssMOR toolbox (Castagnotto *et al*,.2017 a package which is used for LSD system. The control toolbox in MATLAB software is not potentially to achievement this representative and store all matrix as full for this purpose the description of state -space models by these commands

sys=ss (A, B, C, D) or sys=(A, B, C, D, E)

is only feasible up until an order of $0(10^4)$ on a standard computer system. absolutely, the explanation of the full identity matrix of the size 10^5 needed eighty GB storage, while its sparse counterpart requires only 2.4MB to being stored.

In this analysis, we have been used as a balanced truncation algorithm for model order reduction. We find Error estimation using Approximation of Original system. This enables a new evaluation of the error system Provided that the Observability Gramian of the original system has once been considered, an H∞ and H2 norms can be calculated with negligible numerical challenge for any reduced model because of The decreased order demonstrating of a large-scale dynamical machine is compulsory to effortlessness the analysis of the system the usage of approximation techniques.in above Table 1.

6 conclusion

In this contribution, we have shown that a reasonable investigation of the techniques for model order reduction of LSD system a benchmark problem, namely as a balanced truncation algorithm using the sssMOR toolbox. These algorithms have been applied for international space station (ISS) large-scale dynamical systems. In this approach, we compare exact approximation of the original system to some different-2 reduced states with error estimation globalized error bound, H_∞ Norm. and H_2 norm of the systems. The outcomes illustration that as seen from the above figure all reduced states model work quite well.

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