Adaptive Control for a Two-Compartment Respiratory System

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Abstract: - Respiratory failure is a typical clinical issue which needs immediate help with mechanical ventilation while the hidden reason is recognized and treated. Mechanical ventilation can damage the lungs if the applied pressure is too high. It is desirable to provide the desired blood levels of CO_2 and oxygen with limited pressure to avoid causing the lung injury. This paper uses the adaptive inverse dynamics control technique to control a two-compartment modelled respiratory system. Based on the nonlinear respiratory model and desired respiratory pressures, the adaptive inverse dynamics control scheme consisting of a control law and an adaptation law is then applied. The control law has the structure of the two-compartment inverse dynamical model but uses estimates of the dynamics parameters in the computation of pressure applied to the lungs. The adaptation law uses the tracking error to compute the parameter estimates for the control law, stops updating a given parameter when it reaches its known bounds, and resumes updating as soon as the corresponding derivative changes sign. Computer simulations to evaluate the control technique were conducted. Our results indicate that the tracking errors can be improved if the parameter values associated with the adaptation law are properly chosen, and the performance is also robust despite relatively large deviations in the initial estimates of the system parameters.

Key-Words: - Adaptive inverse dynamics control, respiratory system.

1 Introduction

Respiration is the trading of oxygen and carbon dioxide (CO_2) between the environment and the body cells. In human body, this procedure incorporates spark and termination, dispersion of oxygen from alveoli to the blood and of CO_2 from the blood to the alveoli, and the vehicle of oxygen to and CO_2 from the body cells by method for the circulatory framework. Respiratory failure, that is, the lacking trade of CO_2 and oxygen by the lungs, is a typical clinical issue which needs immediate help with mechanical ventilation while the hidden reason is recognized and treated. For example, a patient with pneumonia may require mechanical ventilation while the pneumonia is being dealt with anti-toxins, which will in the end adequately cure the disease. Since the lungs are vulnerable against discriminating disease and respiratory failure is regular, backing of patients with mechanical ventilation is important in the intensive care unit. The objective of mechanical ventilation is to provide an adequate exchange of oxygen and CO_2 in order for the lungs to function normally. However,

without proper control, mechanical ventilation can damage the lungs if the applied ventilation pressure is too high. Therefore, it is desirable to provide the desired blood levels of CO_2 and oxygen with limited pressure to avoid causing the lung injury, either by inflating the lungs to excessive volumes or by applying excessive pressures to inflate the lungs.

A single compartment respiratory lung model, characterized by its compliance (i.e., volume/pressure) and the resistance to air flow into the compartment [1-3], is the most commonly used model. In this paper, we use a two-compartment model [4-6] to mirror the way that there are two lungs although a more complicated multi-compartment respiratory model has also been considered by some other researchers [7-10]. The mechanical ventilation via various control techniques such as model predictive control, classical calculus of variations minimization technique, adaptive sliding model control, etc. can be found in the literature (e.g., [8], [10]). The inverse dynamics (or computed torque) control is a

well known technique for the robot motion control ([11-14]). However, to the best knowledge of the authors, the adaptive inverse dynamics control technique applying to a lung-rib-cage system has not been reported, even though its effectiveness in the biped locomotion control has been reported [15, 16]. Therefore, it is interesting and worthwhile to investigate whether the adaptive inverse dynamics based methods can still be effectively used to a nonlinear respiratory system despite various other approaches that have been reported in the literature.

2 A Two-Compartment Respiratory System Model

In this section, a two-compartment model is briefly described. The motion of one complete breathing cycle can be divided into two phases: inspiration and expiration. Starting from a parent airway, we assume that each airway unit branches into two airway units of the subsequent generation (i.e., a dichotomy architecture is considered). At time t = 0, a driving pressure $p_{in}(t)$ is applied to the opening of the parent airway by the respiratory muscles or a mechanical ventilator over the time interval $0 \leq t \leq T_{in}$, with T_{in} the inspiration duration time. At $t = T_{in}$, the applied airway pressure is released and expiration takes place passively during the interval $T_{in} \leq t \leq T_{in}+T_{ex}$, where T_{ex} is the duration of expiration. Let x_i ($i = 1, 2$) be the lung volume in the ith compartment, $c_i^{in}(x_i)$, $i = 1, 2$ ($c_i^{ex}(x_i)$) be the compliance of the compartment i at inspiration (respectively, expiration) which is a nonlinear of x_i, $R_{j,i}^{in}$, j = 0, 1 ($R_{j,i}^{ex}$, j = 0, 1) be the resistance to air flow of the ith airway in the jth generation during the inspiration (respectively, expiration) phase with $R_{0,1}^{in}$ ($R_{0,1}^{ex}$) the inspiration of the 0th generation (parent) airway, then the equations for a two compartmental lung model can be expressed as follows:

Inspiration Phase:

$$R^{in}\,\dot{x}(t) + C^{in}x(t) = p_{in}(t), 0 \leq t \leq T_{in}; x(0) = x_0^{in}$$

Expiration Phase:

$$R^{ex}\,\dot{x}(t) + C^{ex}x(t) = p_{ex}(t), T_{in} \leq t \leq T_{in} + T_{ex}; x(T_{in}) = x_0^{ex}$$

where $x = [x_1, x_2]^T$ (the superscript T means the transpose), and the diagonal compliance matrix C^{in} (C^{ex}) is

$$C^{in} = \begin{pmatrix} \frac{1}{C_1^{in}} & 0 \\ 0 & \frac{1}{C_2^{in}} \end{pmatrix}, C^{ex} = \begin{pmatrix} \frac{1}{C_1^{ex}} & 0 \\ 0 & \frac{1}{C_2^{ex}} \end{pmatrix}$$

and

$$R^{in} = \begin{pmatrix} R_{0,1}^{in} + R_{1,1}^{in} & R_{0,1}^{in} \\ R_{0,1}^{in} & R_{0,1}^{in} + R_{1,2}^{in} \end{pmatrix}, R^{ex} = \begin{pmatrix} R_{0,1}^{ex} + R_{1,1}^{ex} & R_{0,1}^{ex} \\ R_{0,1}^{ex} & R_{0,1}^{ex} + R_{1,2}^{ex} \end{pmatrix}$$

In order for the system to achieve ideal performance, a set of volume and airflow pattern (i.e., trajectories) corresponding to the inspiration and expiration for both the phases will be used as our reference trajectories.

3 Adaptive Inverse Dynamics Control

Since linearized system equations cannot always be trusted to accurately predict the responses of real (nonlinear) systems, we directly consider nonlinear control and briefly review the adaptive control scheme [11-14] to be used in our respiratory system.

Consider a nonlinear dynamical robotic system described as

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau \qquad (1)$$

where q is the n×1 vector of robot joint coordinates, τ is the n×1 vector of applied joint torques (or forces). $D(q)$ is the n×n symmetric positive definite inertia matrix, $C(q,\ddot{q})\dot{q}$ is the n×1 vector of centrifugal and Coriolis torques, and $g(q)$ is the n×1 vector of gravitational torques. It is well known that by the property of linearity in the parameters [12-14] the dynamical equation can be written as

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = Y(q,\dot{q},\ddot{q})p \qquad (2)$$

where $Y(q,\dot{q},\ddot{q})$ is an n×m matrix of known functions, known as the regressor, and $p = [p_1, p_2, \cdots, p_m]^T$ is an m-dimensional vector of parameters.

Inspecting (1) we see that if the (nonlinear) control τ is chosen as

$$\tau = D(q)a + C(q,\dot{q})\dot{q} + g(q) \qquad (3)$$

then, by substituting (3) into (1) and using the property of $D(q)$ we obtain

$$\ddot{q} = a \tag{4}$$

The vector term a can be defined in terms of a given linear compensator K as

$$a = \ddot{q}^d - Ke \tag{5}$$

with the tracking error $e = q - q^d$, where $q^d(t)$ is an n-dimensional vector of desired joint trajectories. Substituting (5) into (4) leads to the linear error equation in the s-domain as

$$[s^2 I_n + K(s)]e(s) = 0 \tag{6}$$

where I_n is an n×n identity matrix. Letting $K(s) = K_v s + K_p$ leads to the familiar second-order error equation (in the time-domain)

$$\ddot{e} + K_v \dot{e} + K_p e = 0 \tag{7}$$

If the gain matrices K_v and K_p are chosen as diagonal matrices with positive diagonal elements then the closed-loop system is linear, decoupled, and exponentially stable.

The above approach is based on exact cancellation of all nonlinearities in the system. However, in any physical system there is a degree of uncertainty regarding the values of various parameters. There will always be inexact cancellation of the nonlinearities in the system due to this uncertainty and also due to computational round-off, etc. In addition, the burden of computing the complete model may be prohibitively expensive or impossible within the bounds imposed by the available computer architecture. In such cases, it is desirable to simplify the equations of motion as much as possible by ignoring certain of the terms in the equations in order to speed the computation of the control law. Therefore, it is much more reasonable to suppose that, instead of (3), the nonlinear control law is actually of the form

$$\tau = \hat{D}(q)a + \hat{C}(q,\dot{q})\dot{q} + \hat{g}(q)$$

$$a = \ddot{q}^d - K_v \dot{e} - K_p e \tag{8}$$

where $\hat{D}$, $\hat{C}$, and $\hat{g}$ are the estimates of D, C, and g, respectively. Assume that $\hat{D}, \hat{C}$, and $\hat{g}$ have the same functional form as D, C, and g with estimated parameters

$\hat{p}_1, \hat{p}_2, ..., \hat{p}_m$, then

$$\hat{D}(q)\ddot{q} + \hat{C}(q,\dot{q})\dot{q} + \hat{g}(q) = Y(q,\dot{q},\ddot{q})\hat{p} \tag{9}$$

where $\hat{p} = [\hat{p}_1, \hat{p}_2, ..., \hat{p}_m]^T$ is the vector of the estimated parameters.

Substituting (8) into (1) gives

$$D\ddot{q} + C\dot{q} + g = \hat{D}(\ddot{q}^d - K_v \dot{e} - K_p e) + \hat{C}\dot{q} + \hat{g} \tag{10}$$

Adding and subtracting $\hat{D}\ddot{q}$ on the left-hand side of (10) and using (9), we obtain

$$\hat{D}(\ddot{e} + K_v \dot{e} + K_p e) = \tilde{D}\ddot{q} + \tilde{C}\dot{q} + \tilde{g} = Y(q,\dot{q},\ddot{q})\tilde{p} \tag{11}$$

where $(\tilde{\cdot}) := (\hat{\cdot}) - (\cdot)$. Finally, the error dynamics may be written as

$$\ddot{e} + K_v \dot{e} + K_p e = \hat{D}^{-1}Y\tilde{p} = \Phi\tilde{p} \tag{12}$$

The system (12) can be expressed in state space as

$$\dot{x} = Ax + B\Phi\tilde{p} \tag{13}$$

where

$$A = \begin{pmatrix} 0 & I_n \\ -K_p & -K_v \end{pmatrix}, B = \begin{pmatrix} 0 \\ I_n \end{pmatrix}, x = \begin{pmatrix} e \\ \dot{e} \end{pmatrix} \tag{14}$$

Based on (13) and (14), we choose the update law

$$\dot{\tilde{p}} = -\Gamma^{-1}\Phi^T B^T P x \tag{15}$$

where $\Gamma = \Gamma^T > 0$ and P is the unique symmetric positive definite solution to the Lyapunov equation

$$A^T P + PA = -Q \tag{16}$$

for a given symmetric, positive definite Q. Since the parameter vector p is constant, $\dot{\tilde{p}} = \dot{\hat{p}}$. Assume that $\ddot{q}$ is measurable and $\hat{D}^{-1}$ is bounded, then the solution x of (13) satisfies $x \to 0$ as $t \to \infty$ with all signals remaining bounded (for proof, see [12]).

There are several different versions of the above technique. For example, the boundedness of the estimated inertia $\hat{D}^{-1}$ is removed in [13], while in [17] the requirement on measurement of $\ddot{q}$ is

removed but still needs the boundedness of $\hat{D}^{-1}$. Several recent papers have been devoted to the implementation of the above adaptive inverse dynamics method without measuring $\ddot{q}$. For example, estimate $\ddot{q}$ from $\dot{q}$ via a first-order filter. In practice, this approach should be expected to work well.

4 Results

Based on the desired volume pressures and the two-compartment model equation, the adaptive inversed dynamics control scheme is used to control the pressure parameters.

The values of the inspiratory and expiratory lung resistance constants and compliances for the two-compartment lung model were taken from [9] and they are: $R_{0,1}^{in} = 9$ cm $H_2O/l/s$, $R_{1,1}^{in} = R_{1,2}^{in} = 16$ cm $H_2O/l/s$, $R_{0,1}^{ex} = 18$ cm $H_2O/l/s$, $R_{1,1}^{ex} = R_{1,2}^{ex} = 32$ cm $H_2O/l/s$. The expiratory resistance is assumed two times higher than the inspiratory resistance. The lung compliance is chosen to be 0.1 $l/$cm H_2O. The inspiration duration time $T_{in} = 2$ s and the expiration time $T_{ex} = 3$ s. The desired air pressures were taken from [8-9]. During the adaptive inverse dynamics control process, the total number of parameters to be estimated is six. And the estimates of the three resistances and two compliances over time during one breathing cycle are shown in Fig.1. Figure 2 shows the desired lung compartment 1 volume changes with time for over the 12 seconds period, while its input pressure is given in Fig. 3. This is also the same when applied to the 2nd compartment. The input pressure in compartment 1 is shown in Fig. 3. Finally, Fig. 4 shows the tracking errors; for instance, e$_2$ is the difference between the desired and actual pressures entering the 2nd compartment. Overall, the tracking errors are reasonable small.
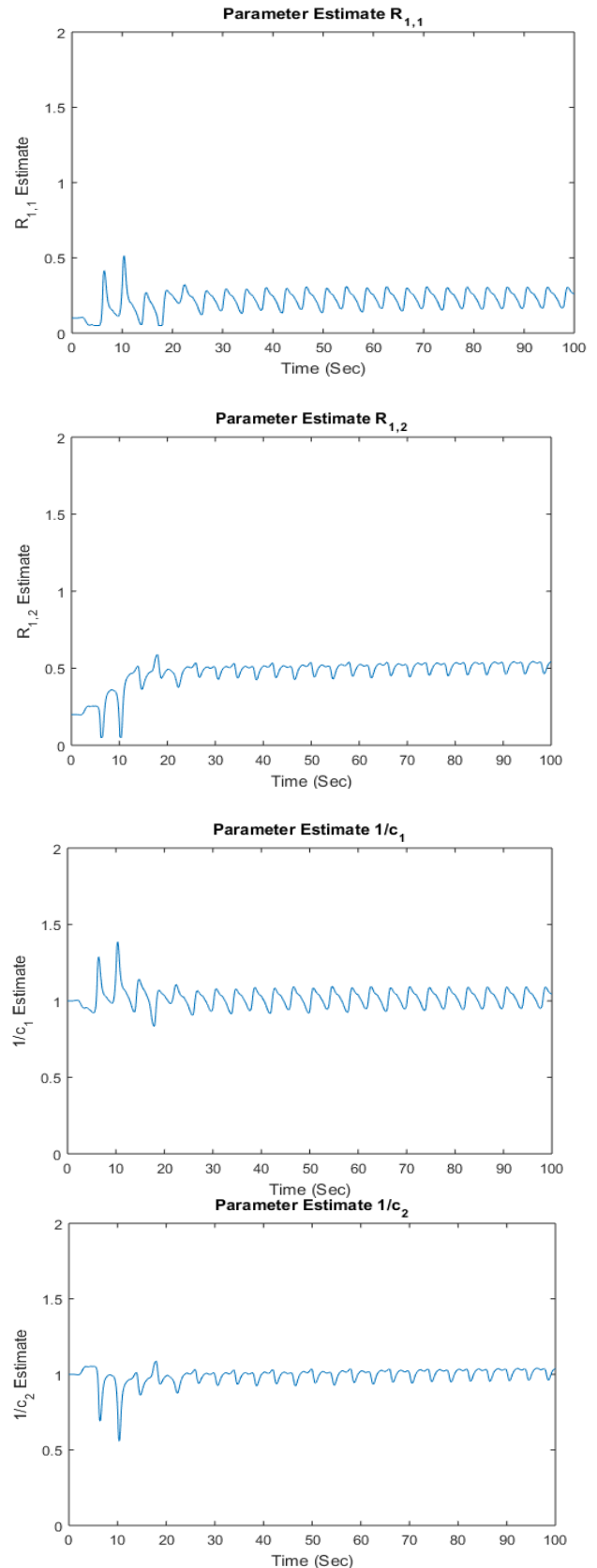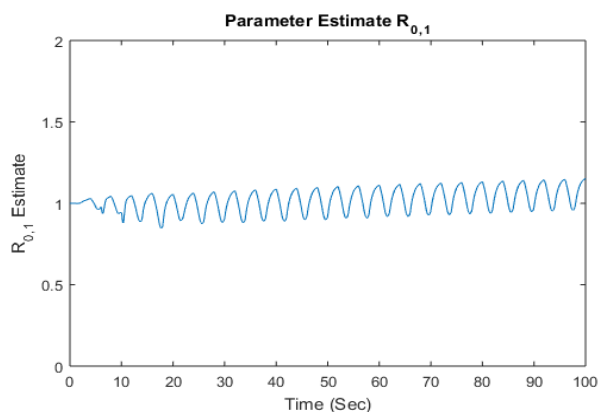

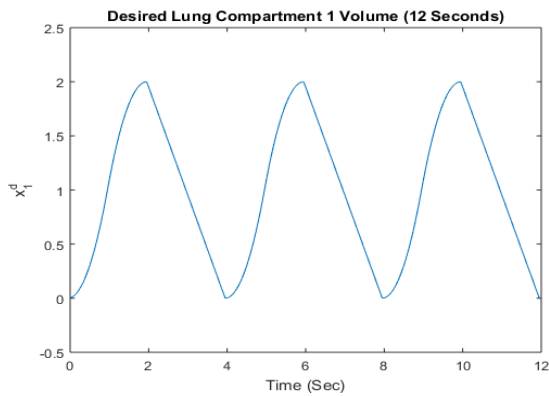Fig. 1. Parameter estimates during one breathing cycle.

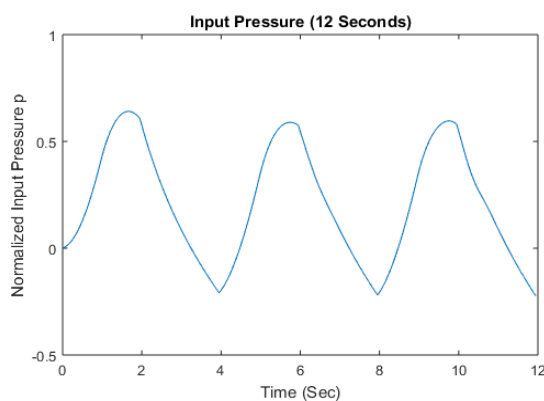Fig. 2 Desired volume trajectory over the specified time period.

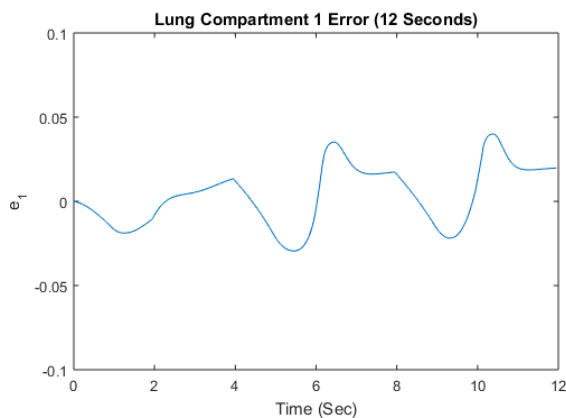Fig. 3 Input pressure in compartment 1 over the specified time period.

(a)

computation of pressure applied to the lungs. The adaptation law uses the tracking error to compute the parameter estimates for the control law, stops

(b)

Fig. 4 Tracking errors.

updating a given parameter when it reaches its known bounds, and resumes updating as soon as the corresponding derivative changes sign. The advantage of using the inverse dynamics control method is that it formulates a globally convergent adaptive controller which does not require approximations such as local linearization, time-invariant, or decoupled dynamics to guarantee the tracking convergence. Simulations show that the tracking errors are acceptably small. The future work includes the robustness study of the control method to the multi-compartment model.
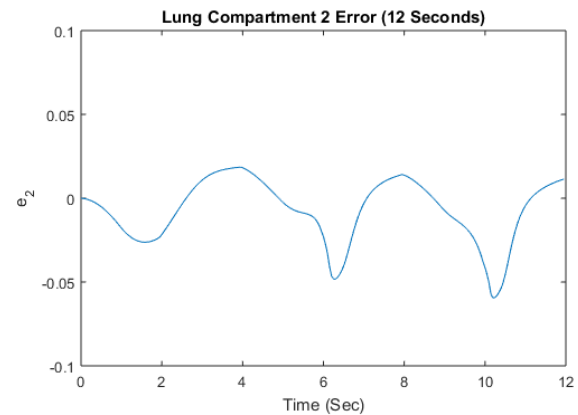
5 Conclusion

We have applied the adaptive inverse dynamics control method to a two-compartment respiratory system. The implementation of the control scheme consists of a control law and an adaptation law. The control law has the structure of the two-compartment inverse dynamics servo but uses estimates of the dynamics parameters in the

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