A New Time Adjusting Step-size LMS Technique for Noise Cancellation Framework with Mean Square Deviation Analysis

Abstract: - In order to mitigate the negative impact of a non-stationary signal on the adaptive filter coefficients and to improve the trading off between convergence rate and steady-state error at various input levels, a new time adjusting step size LMS algorithm is proposed. The theoretical analysis of its mean square deviation (MSD) is investigated in this paper. Its closed-form expressions of mean square deviation for the transient and steady-state stages are also estimated. This new approach aims to reduce the steady-state MSD of the coefficients at high levels of input power signal without sacrificing on the speed of convergence as common in the conventional LMS and other VSSLMS approaches. We do so by developing an individual time adjusting step size based logarithmic function for each tap of the adaptive filter. It is found that the steady-state MSD depends directly on the minimum step size value when the reduction rate of step size is faster than the changing rate of the optimal coefficients. Based on the implementation of the adaptive noise cancellation, simulation results show the superiority of the proposed technique in terms of possessing the lowest MSD at various input variances compared with others. Moreover, the proposed technique outperforms the compared algorithms in the matter of tracking a time-vary noise channel.

Key-Words: - Adaptive noise canceller, LMS algorithm, Variable step size LMS algorithms, Mean square Deviation MSD.

1. Introduction

Adaptive filtering has drawn attention much research interest in both theoretical and implemented aspects for a long time. In the context of voice communication applications, eliminating the noise from the desired speech is the important operation. This problem arises in many situations, such as helicopters, airplanes, and automobiles, where the speech and audio signals are corrupted by noise. Adaptive Noise cancellation (ANC) has attracted much attention as a model to eliminate noise added in the speech signal and improve the quality of speech and audio signals. Removing the noise can be achieved by two approaches, the two-microphone approach, and the single microphone approach. Two-microphone approach can deliver a better performance in term of noise cancelling than a single microphone approach [1-2]. However, the performance of such a two-microphone approach is limited by several conditions such as reverberation which arises from high miss-adjustment errors, and signal distortion which initiates from a correlation of some components of the input filter and the original speech signal [3-4]. There has been a tremendous amount of research paper published on adaptive noise cancellation using both single and two-microphones approaches [5-9].

Many algorithms have been proposed for adaptive filters to adapt their impulse response. The LMS algorithm, which is characterized by a low complexity, ease of implementation, and operational stability, is deemed to be one of the most attractive algorithms in the adaptive signal processing area [3, 10]. However, this algorithm has some drawbacks such as slow convergence rate, a high value of steady-state miss-adjustment, and fixed learning rate. The learning rate, which is sometimes named step size $\mu$, defines the convergence time of the filter coefficients and also the steady-state mean square deviation. The convergence time of this algorithm is adversely proportional to the step size. A large step size is usually significant in situations contained nonstationary input signals but on the account of increasing the steady-state mean square deviation. This essential drawback leads to the need for a compromise to take place between the speed of convergence and steady-state miss-adjustment [11-12]. The fixed step size of LMS makes the compromise between these factors is complicated [13]. To overcome the negative effect of the fixed step size LMS-based algorithms, many approaches
proposed. These approaches are based on utilizing a large learning rate values when the filter coefficients are far from the optimal coefficients, thus accelerating the convergence rate of the algorithm. When the filter coefficients are near from the optimal coefficients, small learning rate values are utilized to attain a low level of misadjustment [13-20]. However, these approaches need to be enhanced to deliver a better performance according to several essential challenges associated with these approaches. In [14], the authors proposed a variable step size LMS algorithm that uses the sign of the gradient component of the squared error to control the value of the step size. But this approach suffers from a step size instability in high noise environments. In [16], the author proposed a time-varying step size using the Sigmoid function, the approach utilizes a large step size in the initial stage of adaption process to speed up the convergence time and then the step size is modified to a smaller value progressively, however, this algorithm introduces a high value of steady-state MSD. In [17], the step size updating equation counts on the square of the error signal resulting in a very high sensitivity to the value of error signal. A high input noise impacts substantially on the final outcome of this approach. In [18], the step size parameter is adjusted according to the square of the time-averaged estimate of the autocorrelation of error function $e(n)$ and $e(n-1)$. The main defects of this approach are that the demanding for more parameters to be modified to get the optimum values, and preserving these optimum values for various levels of input power signal without modifying gives a poor performance. In addition to that, this technique introduces a significant degradation in its steady-state error when the measured noise is highly correlated making the possibility of tackling various situations is impossible. In [19], the step size of this approach based on decaying the traditional LMS step size in an exponential pattern; therefore, this algorithms inherits the defects of the previous approaches. Moreover, the parameters of this algorithm demand a high concentration to achieve a good fulfillment at a certain input signal. In addition to that, the output signal of this approach goes into the instability area in high noise situations. Motivating by these intrinsic challenges and others, a new approach attempts to overcome some or all of these challenges was proposed.

In this work, we propose a new time adjusting step size LMS algorithm that is essentially modified according to the logarithmic function of the power of input excitation. The purpose of this technique was mainly focused on solving two intrinsic challenges faced by LMS and other variable step size algorithms. Firstly, maintaining a low level of mean square deviation at high levels of input power and, secondly, improving the trading-off between achieving fast convergence rate and low steady-state mean square deviation (maladjustment) simultaneously. The idea behind the proposed approach is to dedicate a particular step size for each coefficient of the adaptive filter that adapts according to the logarithmic value of two main factors, which are total input signal and input excitation associated to that coefficient.

A valuation between our proposed, LMS, NLMS [10], VSSLMS [17], RVSSLMS [18], and TVLMS [19] algorithms is carried out to establish a fair evaluation by comparing our technique with other approaches utilized either input or error signals in their step size update equations. Computer simulations are realized using real speech signal. White Gaussian noise with various variances is mixed with the speech signal after passing through low pass fitters that mimics the actual system of noise path.

2. The Concept of Adaptive Noise Cancellation (ANC)

A typical two-microphone adaptive noise cancellation, depicted in Fig. 1, consists of two inputs, primary input, and reference input. The primary input signal, $d(n)$, consists of the original speech signal, $s(n)$, corrupted by an additive noise $v(n)$. The reference input signal, $x(n)$, is the input to the adaptive filter and noise path channel $h(n)$. The noise signal, $x(n)$, is uncorrelated with the signal $s(n)$ but correlated with the noise, $v(n)$, received by primary input sensor. This correlation is important for the adapting algorithm in order to remove the noise from the speech signal. The noise reference, $x(n)$, is adjusted by filter coefficients to generate an output signal, $y(n)$, that approximates, $v(n)$, signal as possible as. This output is subtracted from the primary input, $d(n) = s(n) + v(n)$, to produce the error signal $e(n)$ [10].

$$e(n) = s(n) + v(n) - y(n) \quad (1)$$

Then, the error signal is employed to adjust the weights of the adaptive filter using a LMS-based adaptive algorithm such that the error signal is progressively minimized. By squaring and taking the expectation $E[.]$ of both sides of (1), and realizing that $s(n)$ is uncorrelated with $y(n)$, $v(n)$, and $v(n) - y(n)$, yields [10].
\[ E[e^2(n)] = E[s^2(n)] + E[(v(n) - y(n))^2] \quad (2) \]

The signal power \( E[s^2(n)] \) will be unaltered by filtering process; therefore, the filter is adjusted to minimize \( E[e^2(n)] \) by minimizing the correlated noises \( E[(v(n) - y(n))^2] \). The output filter \( y(n) \) is then a best least-squares estimate of the primary noise \( v(n) \).

![Adaptive noise canceller model.](image)

**3. Traditional LMS**

The LMS algorithm is a kind of adaptive algorithm known as stochastic gradient-based algorithms as it utilizes the gradient vector of the filter weights to converge on the optimal wiener solution. The filter coefficients of the adaptive filter are iteratively adapted in each iteration according to the following formula [21].

\[ W(n + 1) = W(n) + 2\mu e(n)X(n) \quad (3) \]

where \( X(n) \) is the input vector such that \( X(n) = [x(n) x(n-1) x(n-2) \ldots x(n-L+1)]^T \). The vector \( W(n) = [w_0(n) w_1(n) w_2(n) \ldots w_{L-1}(n)]^T \) denotes to the coefficients of the adaptive filter. The step size parameter, \( \mu \), is usually a small positive constant and the adaption process affected by the value of this step size parameter. In order to assure stability (or convergence) of the LMS algorithm; the step size parameter is bounded by the following equation [10]:

\[ 0 < \mu < \frac{2}{Tr[R]} \quad (4) \]

where \( Tr[R] \) is the trace of the input autocorrelation matrix \( E[X(n)X^T(n)] \).

**4. Variable Step size LMS Algorithms**

The step size adjusting equations of the compared algorithms are summarized in Table 1.

**Table 1. Variable step size LMS Algorithms**

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Step size update equation</th>
</tr>
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<tbody>
<tr>
<td>NLMS[8]</td>
<td>( \beta + \chi^T(n)X(n) )  ( B &gt; 0 ) and ( 0 &lt; \mu &lt; 1 )</td>
</tr>
<tr>
<td>VSSLMS[17]</td>
<td>( \mu_{n+1} = \alpha_{n} + \gamma_{n} e^2(n) ) ( 0 &lt; \alpha &lt; 1 ) ( \gamma &gt; 0 )</td>
</tr>
<tr>
<td>RVSSLMS[18]</td>
<td>( p(n) = \beta p(n-1) + (1-\beta)e(n)e(n-1) ) ( 0 &lt; \alpha &lt; 1 ) ( \gamma &gt; 0 ) ( 0 &lt; \beta &lt; 1 )</td>
</tr>
<tr>
<td>TVLMS[19]</td>
<td>( \mu_n = \alpha(n)\mu_0 ) ( \alpha(n) = \frac{1}{C(1+\alpha n)} ) ( C, \alpha, b ) are positive constants ( C &gt; 1 ) and ( \mu_0 ) is LMS step size</td>
</tr>
</tbody>
</table>

**5. Proposed Algorithm**

This paper proposes a new strategy to adjust the step size of the standard LMS algorithm by utilizing log-based time-varying step size. This new variable step size, which is called Absolute Weighted Input using Log function (AWILOG), is proposed to function with a consistent performance for a feasible range of variances of the input excitation. This approach directly infers from the input signal to make the coefficients adjusting equation is adversely influenced by the power of the input signal as will be demonstrated in the step size adaption equation. It utilized a variable step size that will be adapted based on the absolute value of the current and the previous value of the input signal with a variable parameter \( \phi(n) \) that depends on a logarithmic function as given in equations (5), (6) of Table 2. Table 2 illustrates the iterative steps of the proposed technique as a flowchart scheme.

Equation (5) exhibits that, each coefficient of the adaptive filter has a specific step size adapts according to the current step size and the corresponding input signal related to that coefficient. It also shows that initiating with a large step size is essential to increase the convergence speed and progressively decreasing it to achieve its minimum value, to obtain a low level of mean square deviation (sometimes called maladjustment). The subtraction process is necessary to force the next step \( \mu_n(n+1) \) size to be smaller than the current step size \( \mu_n(n) \). To assure decreasing in the next step size and attaining the minimum step size \( \mu_{min} \) value with a lower number of iterations the absolute function is important. The adaptive filtering process with a particular step size for each coefficient is intended to enhance the weight tracking ability of the adaptive filter of the actual system by delivering various scales of step size for each coefficient.
The variable \( q(n) \) consists of two effective factors, the first one is the constant factors \((\zeta, \gamma)\) and the second one is the variable factor \( X^T(n)X(n) \). A small value of \( q(n) \) increases the number of iteration required to achieve the minimum step size \((\mu_{min})\). On the contrary, a large value of \( q(n) \) decreases the number of iteration needed to reach minimum step size \((\mu_{min})\).

\[ \begin{align*}
W(n) &= [w_0(n), w_1(n), w_2(n), \ldots, w_{N-1}(n)]^T \\
\text{INPUT VECTOR} &\equiv x(n-1)x(n-2)\ldots x(n-L+1)^T \\
\text{WEIGHT VECTOR} &\equiv W(n) \\
\text{INITIALIZATION VECTOR AND CONSTANT} &\equiv [0 0 \ldots 0]^T \\
\text{Initial Step size for Each Tap} &\equiv \mu_{init}(n) = [\mu_{max,0}, \mu_{max,1}, \mu_{max,2}, \ldots, \mu_{max,L-1}] \\
\text{Constant} &\equiv 0 < \zeta < 1
\end{align*} \]

\[ x(n) = X^T(n)W(n) \]
\[ e(n) = d(n) - y(n) \]
\[ W(n + 1) = W(n) + 2\mu(n)e(n)X(n) \]
\[ M(n) = \text{diag}(\mu_0, \mu_1, \mu_2, \ldots, \mu_{L-1}) \]

At Each Iteration
\[ y(n) = X^T(n)W(n) \]
\[ e(n) = d(n) - y(n) \]
\[ W(n + 1) = W(n) + 2\mu(n)e(n)X(n) \]
\[ M(n) = \text{diag}(\mu_0, \mu_1, \mu_2, \ldots, \mu_{L-1}) \]

Step size Update
\[ \mu_i(n + 1) = \mu_i(n) - |x(n-i)\cdot q(n)| \]
\[ q(n) = \zeta \cdot |\log((y + X^T(n)X(n)))| \]

For \( i = 0, 1, 2, 3, \ldots, L-1 \)
And
\[ \mu_i(n + 1) = \begin{cases} 
\mu_{max} & \text{if } \mu_i(n+1) > \mu_{max} \\
\mu_{min} & \text{if } \mu_i(n+1) < \mu_{min} \\
\mu_i(n + 1) & \text{otherwise}
\end{cases} \]  

The presence of the parameter \( \gamma \) in the step size adjusting equation is to avoid the restrictive values of the natural logarithmic function. The constant \( \zeta \) is involved to preserve the coefficients adjusting process in the stable area of the adaptation process and prevent it to take place in the divergent region. The output accomplishment of the proposed technique delivers an approximately coherent performance for a reasonable range of constant \( \zeta \). These abilities are based on the tackling of several input scales in the logarithmic function of the step size adjusting equation.

6. Mean Square Deviation Analysis of Proposed Algorithm

The steady-state mean square deviation (MSD) has been addressed by many researchers such as in [3], [10], and [22]. In these works, the conventional LMS and NLMS algorithms are presented. To illustrate our work using MSD, we substitute \( \mu(n) \) in place of \( \mu \) of the conventional algorithms since the \( \lim_{n \to \infty} E[\mu(n + 1)] \to \mu_{min} \) at the steady-state.

Furthermore, we assume a time-invariant transfer function for the noise path of ANC to facilitate the derivation of the MSD of the proposed algorithm. The MSD is defined in [1], [2], and [23] as:

\[ MSD(n) = Tr\{E[|\epsilon(n)|^2]\} \]  

where \( \epsilon(n) = W(n) - W_{opt} \) and substituting this into equation (7), we obtain,

\[ MSD(n) = Tr\left\{ E\left[ (W(n) - W_{opt}) (W(n) - W_{opt})^T \right]\right\} \]  

The output of the canceller system can be expressed in terms of the error weight vector, \( \epsilon(n) \),

\[ e(n) = s(n) + X^T(n)\epsilon(n) \]  

The updated weight vector for the proposed algorithm can be expressed in term of the error weight vector, given in equation (7) after substituting in equation (3) with variable step size instead of constant step size:

\[ \epsilon(n + 1) = \epsilon(n) - M(n)X(n)[s(n) + X^T(n)\epsilon(n)] \]  

where \( M(n) \) is a diagonal matrix of the step size values at each tap of the adaptive filter as given in equation (6). Post-multiply equation (10) by \( \epsilon(n + 1)^T \) and take the expectation of both side we get:

\[ \begin{align*}
E\{\epsilon(n + 1)\epsilon(n + 1)^T\} &= E\{\epsilon(n)\epsilon(n)^T\} - 2E\{M(n)\}E\{X(n)X^T(n)\}E\{\epsilon(n)\epsilon(n)^T\}X(n)X^T(n) + E\{M^2(n)\}E\{X(n)X^T(n)\}E\{s^2(n)\}I \\
\text{where } M(n) &\text{ is a diagonal matrix of the step size values at each tap of the adaptive filter as given in equation (6). Post-multiply equation (10) by } \epsilon(n + 1)^T \text{ and take the expectation of both side we get:}
\end{align*} \]
Assuming \( E[M^2(n)] = M^2(n) \) and \( E[M(n)] = M(n) \), this assumption is valid in the transient phase of updating step size equation because our proposed algorithm starts initially with maximum step size \( \mu_{\text{Max}} \) and gradually decreases to minimum step size \( \mu_{\text{Min}} \) with a few numbers of iterations based on input power signal. Therefore, equation (11) becomes

\[
E(\epsilon(n + 1)\epsilon(n + 1)^T) = E[\epsilon(n)\epsilon(n)^T] - 2M(n)E[X(n)X^T(n)] + M^2(n)E[X(n)X^T(n)]E[s^2(n)]\frac{1}{l} \tag{12}
\]

Carrying further simplifications and using statically independent assumptions, we produce:

\[
K'(n + 1) = K'(n) - M(n)[D.K'(n) + K'(n).D] + 2M^2(n).D.K(n).D + M^2(n).D.T{\text{tr}}D.K(n).D + M^2(n).E[s^2(n)].D \tag{13}
\]

where \( D \) is a diagonal matrix of eigenvalues of the \( E[X(n).X^T(n) = TDT^T, T^T = 1 \), \( K'(n + 1) = E[(\epsilon'(n + 1)\epsilon'(n + 1)^T) \) and \( \epsilon'(n + 1) = T^T\epsilon(n + 1) \). \( MSD(n) \) equals to the trace of the above equation \( \text{tr}[K'(n + 1)] \).

Taking the diagonal elements of \( K'(n + 1) \) and constructing a vector of these elements, we obtain:

\[
k(n + 1) = [P + M^2(n).\lambda.\lambda^T].k(n) + M^2(n).E[s^2(n)].\lambda \tag{14}
\]

where \( k(n + 1) \) is a vector of the variance of each tap of the adaptive filter. \( P \) is a diagonal matrix of elements

\[
\rho_i = 1 - 2\mu(n).\lambda_i + 2\mu_i^2(n).\lambda_i^2
\]

and \( \lambda \) is a vector of the eigenvalues of the \( E[X(n).X^T(n)] \) for all \( i = 0, ..., L - 1 \).

We follow [3] and [10], to simplify equation (14) to get \( MSD(n) \) at steady-state as:

\[
MSD(\infty) = M^2(n).E[s^2(n)].[I - P + M^2(n).\lambda.\lambda^T]^{-1}.\lambda \tag{15}
\]

At the steady-state stage, the step size of the proposed algorithm will be at its minimum, \( \mu_{\text{min}} \). The above equation becomes

\[
MSD(\infty) = M_{\text{min}}^2.n.E[s^2(n)].[I - P + M_{\text{min}}^2.n.\lambda.\lambda^T]^{-1}.\lambda \tag{16}
\]

According to equation (15), when the step size is large, \( M^2(n) \) enlarges \( E[s^2(n)] \) and thus forces \( MSD \) to mimic \( s(n) \). That means a large power of \( s(n) \) will deviate \( W(n) \) from optimal solution and will fall in the unstable region. Conversely, small step size will cause \( MSD \) to converge to a minimum value even with a large value of \( s(n) \).

It is also worthy of noting that this is in contrast to the fact that a small value of the step size increases the convergence time which is a major drawback that most common fixed and variable step size algorithms suffer from. Our proposed algorithm successfully overcomes this drawback by initiating a large value of step size \( M_{\text{Max}}^2 \) to accelerate the convergence rate and end the adaptation process by \( M_{\text{Min}}^2 \) to attain minimum \( MSD(\infty) \).

Equation (16) illustrates the contribution of the proposed algorithm to maintain \( MSD \) at a minimum level at steady-state. We also would like to point that the value of \( MSD(\infty) \) will no more be impacted by the variation of noise power \( x(n) \), or the power of the desired signal \( s(n) \), due to having the minimum value of step size at steady-state. Also, each tap of \( \mu_{\text{Min}} \) step size keeping the adaptation process independent of each other. Consequently, each tap exhibits a different number of iterations to achieve its minimum step size \( \mu_{\text{min}} \), which leads to a fast convergence rate and a low level of misadjustment at steady-state.

The importance of equation (16) will be noticed in a time-varying channel noise where arriving at the minimum step size of each tap individually is truly crucial. This result enables us to control the value of the variance around the optimal solution of the weight vector as will be shown the in simulation results.

It is important to mention that, the step size stability condition for the proposed technique is the same as the conventional LMS algorithm but in mean value and individually as depicted in equation (17):

\[
0 < E[\mu(n)] < \frac{2}{\text{tr}[\rho]} \tag{17}
\]

In order to investigate the mean behavior of the proposed time varying step size, taking the expectation value of equation (5):

\[
E[\mu_i(n + 1)] = E[\mu_i(n)] - E[|x(n - i).\zeta.\log(y + X^T(n)X(n))]\tag{18}
\]

Taylor series can be used to simplify the log-term of equation (18) as follow:

\[
E[\mu_i(n + 1)] = E[\mu_i(n)] - E[|x(n - i).\zeta.(X^T(n)X(n) - \frac{(X^T(n)X(n))}{2})|]\tag{19}
\]
By assuming the statistical independence between individual input signal, $x(n - i)$, and the total input power $X^T(n)X(n)$, equation (19) becomes:

$$E[\mu_i(n + 1)] = E[\mu_i(n)] - E[|x(n - i)\|].Tr.\{TDT^t\} + E\left[\frac{x(n-i)}{2}\right]E[(X^T(n)X(n))^2]$$ (20)

For simplicity, we have used the assumption of, $E[(X^T(n)X(n))^2] = (X^T(n)X(n))^2$

$$E[\mu_i(n + 1)] = E[\mu_i(n)] - E[|x(n - i)\|].Tr.\{TDT^t\} + E\left[\frac{x(n-i)}{2}\right](X^T(n)X(n))^2$$ (21)

Two important points, which conclude the main improvements of proposed technique, can be deduced from equation (21) as the following. Firstly, the eigenvalue of each tapped delay input signal controls, individually, the reduction speed of the underlying step size. Second, the number of iterations required to achieve minimum step size for each coefficient, individually, depends on the amount of the eigenvalue corresponding to that coefficient. Consequently, this action determines the availability of large step sizes during the adaptation process to deliver a fast convergence rate while maintaining low steady-state error at various levels of input excitation.

7. Simulation Results

In the context of adaptive noise cancellation shown in Fig. 1, the achievement of the proposed, traditional LMS and others VSSLMS algorithms are evaluated in this section. This evaluation is performed in terms of mean square deviation, noise path tracking, the error between the original speech signal and cleaned speech signal, and MSE learning curves of the list of compared algorithms. The parameters of the approaches are selected by doing many simulations in order to confirm a fair comparison and achieve a better accomplishment in terms of fast convergence rate and low maladjustment for compared algorithms. The facility of the proposed algorithm creates a flexible strategy that makes the selecting of the constant $\zeta$ is easy because any value for this parameter in the reasonable range presents an acceptable achievement of the proposed technique. Conversely, parameters selection with respect to other algorithms (LMS, VSSLMSs) demands an optimization process to choose the appropriate value. We do so because those algorithms cannot produce a good performance with various variances of input (reference noise here) without adjusting their parameters in each input power (this reason makes the proposed technique is truly very robust).

The optimum step size for the standard LMS and NLMS [10] was chosen by trial and error to be 0.09 and 0.025 respectively. The parameter values for the proposed algorithm of $\mu_{max}$ and $\mu_{min}$ were chosen to be 0.09 and 0.0005 respectively and the constants ($\zeta=0.00015$, $\gamma=0.8$). The constant values for the VSSLMS [17] of $\mu_{max}$ and $\mu_{min}$ were chosen to be 0.09 and 0.0005 respectively and the constants ($\gamma$, $\alpha$) to be (0.007, 0.99) respectively. The optimum constant values ($\mu_{max}$, $\mu_{min}$, $\gamma$, $\alpha$, $\beta$) for the RVSSLMS [18] was picked up to be (0.09, 0.0005, 0.0038, 0.97, 0.99) respectively. The optimum constant values ($C$, $\alpha$, $b$) for the TVLMS [19] was selected to be (2, 0.01, 0.7) respectively. The order of FIR adaptive filter (L) for all simulation was eight. The noise source used for simulation was white Gaussian noise with zero mean and various variances $\sigma^2$. The impulse response of the noise path $h(n)$ was randomly chosen with eight coefficients. The utilized SNR is ($s(n)/v(n)$) and by changing the variance of the input signal, $x(n)$, we change the value of SNR.

7.1. Mean Square Deviation

In this section, the performance of MSD of the list of compared algorithms is evaluated. The aim of this evaluation is to investigate the performances of the algorithms with various scales of the variance of the input filter. We do so, by simulating each algorithm for 20000 number of samples and dedicating the first 5000 samples to estimate the transient MSD value as well as the last 15000 samples to estimate the steady-state MSD value. Fig. 2 and Fig.3 show the estimated transient and steady-state MSD for various SNR respectively. This action demonstrates the convergence speed and steady-state of the adaption process for each algorithm.

As shown in these figures, except our proposed technique, the performance of each algorithm is inconsistent. Generally, each algorithm introduces a good performance at a particular power of input signal, which coincides with its parameters, and exhibits a poor performance at other levels of input power if compared with others. For instance, the NLMS algorithm has the lowest transient and steady-state MSD at SNR equals -1dB but it has the highest transient and steady-state at SNR equals 12dB. Other examples, the outcomes of LMS and VSSLMS algorithms are also inconsistent. At 3dB the
VSSLMS algorithm outperforms the LMS algorithm in transient and steady-state MSD.

However, LMS outperforms the former algorithm in transient MSD value at (12dB, 9dB) and so on to the other approaches. On the other hand, the proposed technique introduces a consistent achievement at various scales (SNRs) of input excitation. Moreover, the proposed algorithm outperforms the compared algorithms at all signal to noise ratios and its performance improved gradually as the input power increase. Because, as the input power increases the number of iterations required to achieve its minimum step size decreases. Consequently, this action provides fast convergence rate (due to the high input power) and maintains a low steady-state value of MSD (due to the minimum step size).

7.2. Noise Channel Tracking

The tracking ability to a time-varying unknown system for the list of compared algorithms is presented in this section. Eight coefficients noise channel changes every 2500 samples is generated in order to investigate the achievement of each algorithm at a high rate of changing of optimal coefficients (noise channel). Fig. 4, Fig. 5, Fig. 6, Fig. 7, Fig. 8 and Fig. 9 show the coefficients update of the proposed algorithm, VSSLMS [17], LMS, NLMS [10], RVSSLMS [18], and TVLMS [19] at signal to noise ratio equals (-1dB) respectively. As shown in these figures, the proposed technique outperforms the compared algorithms in terms of achieving fast-tracking to the noise path and also maintaining smooth filter coefficients around the actual coefficients of the noise path. This smoothness comes from the step size updating equation that dedicates a particular step size for each coefficient. This step size introduces its minimum value after each transition of the noise path maintaining a very low mean square deviation.
On the other hand, the other approaches exhibit high fluctuations in filter coefficients around the actual coefficients of the noise path. These high fluctuations arise mainly from the common step size for each coefficient that shares the same amount of error signal or input power to all coefficients, consequently, this action increases the mean square deviation. In addition to that, some of the compared algorithms commence to mimic the original speech signal and introduce fluctuations which are proportional to the amplitude of the speech signal. This action represents an intrinsic challenge for these approaches.

### 7.3. Error between Original Speech Signal and Cleaned Speech Signal

In this section, the achievement of each algorithm is evaluated in terms of its ability to reduce the error between Original Speech Signal (OSS), which is $s(n)$ here, and the Cleaned Speech Signal (CSS), which is $e(n)$ in our application. Fig. 10, Fig. 11, and Fig. 12 show error between the OSS and CSS for the list of compared algorithms at a signal to noise ratio equal to 9dB. As shown in these figures, the proposed approach outperforms the compared algorithms in terms of attaining the lowest difference or error between OSS and CSS. This action also manifests why the filter coefficients of the proposed approach are smooth and closed to actual noise channel. Conversely, other approaches are significantly...
impacted by the non-stationarity of the speech signal and they commence to introduce a high error between OSS and CSS.

Fig. 10. (a) Shows error between OSS and Cleaned Speech for AWILOG and NLMS at 9dB SNR, (b) shows error between OSS and Cleaned Speech for AWILOG and VSSLMS at 9dB SNR.

Fig. 11. (a) Shows error between OSS and Cleaned Speech for AWILOG and LMS at 9dB SNR, (b) shows error between OSS and Cleaned Speech for AWILOG and TVLMS at 9dB SNR.

Fig. 12. (a) Shows error between OSS and Cleaned Speech for AWILOG, and RVSSLMS at 9dB SNR, (b) shows OSS and Error of AWILOG, (c) Error between OSS and Cleaned Speech for AWILOG, VSSLMS, and RVSSLMS at 9dB SNR.

Fig. 13, Fig. 14, and Fig. 15 show error between the OSS and CSS for the list of compared algorithms at a signal to noise ratio equal to 3dB. These figures again illustrate the consistency of the proposed technique to maintain a low error between OSS and CSS at a low signal to noise ratio. On the other hand, the compared algorithms exhibit a considerable deterioration in their outcomes as the signal to noise ratio decreases. For instance, LMS and VSSLMS approaches have the lowest performances and this emerges from their step size methods which significantly impact by a high noise input.

Fig. 13. (a) Shows error between OSS and Cleaned Speech for AWILOG and NLMS at 3dB SNR, (b) shows error between OSS and Cleaned Speech for AWILOG and VSSLMS at 3dB SNR.

Fig. 14. (a) Shows error between OSS and Cleaned Speech for AWILOG and LMS at 3dB SNR, (b) shows error between OSS and Cleaned Speech for AWILOG and TVLMS at 3dB SNR.

Fig. 15. (a) Shows error between OSS and Cleaned Speech for AWILOG, and RVSSLMS at 3dB SNR, (b) shows OSS and Error of AWILOG, (c) Error between OSS and Cleaned Speech for AWILOG, VSSLMS, and RVSSLMS at 3dB SNR.

7.4. MSE Learning Curve

This section demonstrates the MSE learning curves of the list of compared algorithms. The MSE was calculated based on the difference between OSS and CSS in order to deeply demonstrate the effecting of the non-stationary speech signal on the final results. Fig. 16 and Fig. 17 show the MSE learning curve.
curves of the compared algorithms at SNRs equal to 9dB and 3dB respectively.

As obvious in these figures, the proposed algorithm possesses the lowest steady-state error and the fastest convergence time compared with other. Moreover, the MSE learning curve of the proposed technique shows a consistent decreasing toward zero as SNR decreases. On the contrary, other algorithms start to mimic the mean square (MS) of the original speech signal as SNR decreases and consequently, this leads to very high fluctuations of the adaptive filter coefficients around the actual coefficient of noise channel.

8. Proposed Algorithm Performance vs Constant Parameters (\(\zeta, \gamma\))

In this section, the performance of the proposed algorithm is investigated against its constant parameters (\(\zeta, \gamma\)). The investigation is based on examining its MSE learning curve performance with respect to various values of constant parameters (\(\zeta, \gamma\)).

8.1. Constant Parameter (\(\zeta\))

The effect of the constant parameter \(\zeta\) on the MSE error learning curve of the proposed algorithm is demonstrated in this section. Fig. 18 shows MSE learning curve of the proposed approach for different values of constant parameter \(\zeta\) at SNR equals 9dB and \(\gamma = 0.8\). Although, \(\zeta = 0.00015\) has the highest MSE it still shows the best achievement through all these simulations. As shown in this figure, \(\zeta = 0.0006\) has the best performance as compared to other values.

Fig. 15. (a) Shows error between OSS and Cleaned Speech for AWILOG, and RVSSLMS at 3dB SNR, (b) shows OSS and Error of AWILOG, (c) Error between OSS and Cleaned Speech for AWILOG, VSSLMS, and NLMS at 3dB SNR.

Fig. 16. Shows MSE learning curves of the list of compared algorithms at 3dB SNR.

Fig. 17. Shows MSE learning curves of the list of compared algorithms at 3dB SNR.

Fig. 18. Shows MSE learning curves of the proposed algorithm for various (\(\zeta\)) values at 9dB SNR.

Fig. 19 shows an important feature that characterizes our technique from others. As the SNR decreases to 3dB the variance of the input filter excitation increases making the variable
factor $X^T(n)X(n)$ of $\varphi(n)$ handles the significant role in the step size updating equation and leaves the constant ($\zeta$) with no a considerable effect. This action simplifies the process of selecting the proper value of constant $\zeta$. As shown in this figure, after $\zeta=0.0003$ the MSE learning curves approximately introduces a congruent behavior. That means the variable factor $X^T(n)X(n)$ has the predominant role at high input excitation.

Fig. 19. Shows MSE learning curves of the proposed algorithm for various ($\zeta$) values at 3dB SNR.

8.2. Constant Parameter ($\gamma$)

The effect of the constant parameter $\gamma$ on the MSE error learning curve performance of the proposed technique is manifested in this section. Fig. 20 and Fig. 21 show MSE learning curve of the proposed approach for various values of constant parameter $\gamma$ at 9dB and 3dB SNRs respectively. The value of constant $\zeta$ in this test is (0.00015). As shown in these figures, the behaviors of the learning curves are somewhat related to Fig. 18 and Fig. 19.

Fig. 20. Shows MSE learning curves of the proposed algorithm for various $\gamma$ values at 9dB SNR.

Fig. 21. Shows MSE learning curves of the proposed algorithm for various $\gamma$ values at 3dB SNR.

The essential role of the parameter $\gamma$ is to avoid the restricted value of log function and consequently, it works like a regulator factor. Its value shifts up or down the curve of log function. The similarity between Fig. 18 and Fig. 20 provides a very important mechanism that expedites the process of selecting parameters. This can be easily accomplished by fixing ($\gamma$) and changing ($\zeta$) to achieve the lowest error. That means one constant parameter needs to be optimized not more like other variable step size approaches. The same inferences can be applied to Fig. 19 and Fig. 21.

9. Conclusions

This work concentrates on improvement performance of conventional LMS and other VSSLMS approaches by proposing a new time varying step size LMS algorithm. The mean square deviation performance of a new time varying step size for different variances of white reference inputs in the context of adaptive noise cancellation was introduced. The new algorithm adopts the idea of utilizing a log-based time adjusting step size for each tap of the adaptive filter attempting to get better performance than LMS and other VSSLMS at various variances of the input signal. Each step size was adjusted according to the current and previous input excitation associated with each tap in a logarithmic behavior. We evaluated the proposed approach by comparing its transient and steady-state mean square deviation with other approaches at various variances of input excitation. Moreover, the evaluation is also done in term of the capability of an algorithm to track the coefficients of the actual system and the amount of deviation from these coefficients. In addition, the difference between the original speech signal and cleaned speech signal was
also considered in this work. Theoretical and computer simulation results show the superiority of the proposed approach in terms of attaining the fastest convergence time and maintaining the lowest MSD compared with other approaches. Moreover, the proposed algorithm exhibits the ability to maintain a consistent low value of MSD at high variances of the input signal when compared with others.

References: