Privatization and government preference in an international Cournot triopoly

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Abstract: - This paper presents an analysis of the relationship between the privatization of a state-controlled firm and government preferences for tax revenue, by using a mixed Cournot model with domestic and foreign firms. We assume that the government fixes a tax rate on the domestic production and a tariff for imported goods. The state-controlled firm aims to maximize the sum of consumer and producer surplus; the government's objective function is a weighted sum between social welfare and tax revenue.

Key-Words: - Privatization, import tariff, government preference, Cournot triopoly

1 Introduction

Studies on privatization policies have been developed using mixed oligopoly theories. Competition between public and private firms exists in a range of industries like telecommunications, electricity, natural gas, airlines industries, as well as services including hospitals, banking and education. Some authors studied mixed duopoly markets and some others considered markets with more than two firms.

DeFraja and Delbono [1] is one of the pioneering papers on mixed oligopoly, where a welfaremaximizing state-controlled firm (SCF) competes with several profit-maximizing private firms. Fjell and Pal [2] considered an international Cournot model, in which a domestic SCF competes with both domestic and foreign private firms, to study the effects of an open door policy and foreign acquisitions. Fjell and Heywood [3] studied these fields under a Stackelberg model, in which a statecontrolled leader firm competes with both domestic and foreign private follower firms. Pal and White [4] adopt Fiell and Pal's model to study the effects of privatization and strategic policy taking the form of domestic production subsidies and import tariffs. Ferreira and Ferreira [5] studied, in both mixed and privatized markets, the effects of environmental and trade policies in an international duopoly serving two countries, with pollution abatement. Ferreira and Ferreira [6] analysed domestic and international competitions with one public leader firm and one follower private firm, producing complementary goods and competing on prices. Ferreira and Ferreira [7] studied the relationship between the

privatization of a public firm and government preferences for environmental tax revenue, in a market with two firms competing à la Cournot.

Kato [8] used a mixed Cournot model to study the relationship between privatization and government preferences for tax revenue. He assumes that the government prefers tax revenue to the social welfare, whereas the public firm only cares about the social welfare. Ferreira and Ferreira [9] did a similar analysis in a Stackelberg duopoly model with the public firm as the leader.

Ferreira and Ferreira [10] also studied the relationship between privatization and government preferences for tax revenue, but by considering a Cournot triopoly, instead of a Cournot duopoly. Furthermore, the authors considered that firms produce differentiated goods, instead of homogenous goods. They showed that privatization policies depend on the government preference for the tax revenue and on the differentiation of the goods.

Tomaru [11] investigated how decision-making upon cost-reducing R&D investment by a domestic public firm is affected by privatization and entry of a foreign firm. The author showed that privatization deteriorates domestic social welfare.

In this paper, we analyse the relationship between the privatization of a state-controlled firm and government preferences for tax revenue, by using a mixed Cournot model with domestic and foreign firms.

The paper is organized as follows. In Section 2, we introduce and describe the model. Then, we study the mixed market in Section 3 and the privatized market in Section 4. In Section 5, we analyse the effects of privatization. Section 6 presents the results for the domestic competition, and in Section 7 we compare some results in the domestic model with the ones in the international model. Section 8 concludes the paper.

2 Description of the model

We consider a triopoly model, with one statecontrolled firm (SCF) F_0 , one domestic private firm F_1 and one foreign private firm F_2 . Assume that firms produce a homogenous good and the market inverse demand is given by¹

$$p=1-Q,$$

where *p* is the price of the good and $Q = \sum_{i=0}^{2} q_i$, where q_i is the quantity produced by firm F_i , is the total production of all firms. All firms have the same cost function, $C(q_i) = q_i^2/2$, i = 0,1,2. We also assume that the government fixes a tax rate *t* on the domestic production and a tariff μ on the imported goods. Thus, the firms F_i ' profit functions π_i are given by

$$\pi_{i} = (p-t)q_{i} - \frac{q_{i}^{2}}{2}, \ i = 0,1$$
$$\pi_{2} = (p-\mu)q_{2} - \frac{q_{2}^{2}}{2}.$$

As usual, social welfare W is defined as the sum of consumer surplus CS and producer surplus $W = CS + \pi_0 + \pi_1$,

where

$$CS = \frac{1}{2} (q_0 + q_1)^2.$$

The government's payoff is given by

 $U = W + (1 + \alpha)R,$

where $R = t(q_0 + q_1) + \mu q_2$ is the total tax revenue, and α is the parameter that represents the weight of the government preference for the tax revenue. We consider $\alpha \ge 0$, which means that the government puts a larger weight on *R* than on *W*.

Assumption. To assure that all the firms are active in the market, we assume that $\alpha > 1/3$.

The game runs as follows:

- In the first stage, the government fixes the tax rate *t* and the import tariff μ ;
- In the second stage, all firms simultaneously decide their outputs.

To obtain a subgame perfect equilibrium, the game will be solved by backward induction.

3 The mixed international triopoly

Starting from the last stage, the SCF solves the optimization problem $\max_{a_0} W$, and the private firms

solve the optimization problems $\max_{q_i} \pi_i$, with

$$i = 1, 2.$$

By solving the system

$$\begin{cases} \frac{\partial W}{\partial q_0} = 1 - 2q_0 - q_1 - q_2 - t = 0\\ \frac{\partial \pi_1}{\partial q_1} = 1 - q_0 - 3q_1 - q_2 - t = 0\\ \frac{\partial \pi_2}{\partial q_2} = 1 - q_0 - q_1 - 3q_2 - \mu = 0 \end{cases}$$

we find the optimal quantities q_0 and q_i produced, respectively, by the SCF and each private firm, as function of *t* and μ :

$$q_0 = \frac{2 - 3t + \mu}{6}$$
, $q_1 = \frac{2 - 3t + \mu}{12}$ and $q_2 = \frac{2 + 3t - 5\mu}{12}$.

Therefore, the price of the good, also as function of the tax rate and the import tariff, is given by

$$p = \frac{2+3t+\mu}{6}$$

Furthermore, firms' profits are as follows:

$$\pi_0 = \frac{\left(2 - 3t + \mu\right)^2}{72},$$
$$\pi_1 = \frac{\left(2 - 3t + \mu\right)^2}{96}$$

and

$$\pi_2 = \frac{\left(2+3t-5\mu\right)^2}{96},$$

and social welfare is given by

$$W = \frac{\left(2 - 3t + \mu\right)^2}{18}.$$

Next proposition presents the comparative static analysis.

PROPOSITION 1. Production of stated-controlled firm and of private domestic firm decreases in tax rate and increases in import tariff; production of the private foreign firm increases in tax rate and

¹ This demand function results from maximization of a semi-linear utility function of a representative consumer.

decreases in import tariff. Price increases in both tax rate and import tariff.

PROOF. The results follow from the following:

$$\begin{aligned} \frac{\partial q_0}{\partial t} &= -\frac{1}{2} < 0, \ \frac{\partial q_1}{\partial t} = -\frac{1}{4} < 0 \ \text{and} \ \frac{\partial q_2}{\partial t} = \frac{1}{4} > 0; \\ \frac{\partial q_0}{\partial \mu} &= \frac{1}{6} > 0, \ \frac{\partial q_1}{\partial \mu} = \frac{1}{12} > 0 \ \text{and} \ \frac{\partial q_2}{\partial \mu} = -\frac{5}{12} < 0; \\ \frac{\partial p}{\partial t} &= \frac{1}{2} > 0, \ \frac{\partial p}{\partial \mu} = \frac{1}{6} > 0. \end{aligned}$$

Now, solving

$$\begin{cases} \frac{\partial U}{\partial t} = 0\\ \frac{\partial U}{\partial \mu} = 0 \end{cases}$$

gives

$$t^{I,M} = \frac{9\alpha - 1}{6(3\alpha + 1)}$$
 and $\mu^{I,M} = \frac{1}{2}$

REMARK 1. Since we are assuming $\alpha > 1/3$, the optimal tax rate is positive.

By the above results, we can easily get the subgame perfect equilibrium, as stated in the next proposition.

PROPOSITION 2. The subgame perfect equilibrium for the international mixed triopoly is as follows:

$$q_0^{I,M} = \frac{\alpha + 1}{2(3\alpha + 1)}, \ q_1^{I,M} = \frac{\alpha + 1}{4(3\alpha + 1)},$$
$$q_2^{I,M} = \frac{3\alpha - 1}{12(3\alpha + 1)}, \ Q^{I,M} = \frac{3\alpha + 2}{3(3\alpha + 1)}.$$

The price clearing the market at equilibrium is:

$$p^{I,M} = \frac{6\alpha + 1}{3(3\alpha + 1)}.$$

SCF's profit is given by

$$\pi_0^{I,M} = \frac{(\alpha+1)^2}{8(3\alpha+1)^2};$$

domestic private firm's profit is

$$\pi_1^{I,M} = \frac{3(\alpha+1)^2}{32(3\alpha+1)^2};$$

and foreign private firm's profit is

$$\pi_2^{I,M} = \frac{(3\alpha - 1)^2}{96(3\alpha + 1)^2}.$$

Social welfare and government's payoff are, respectively, as follows:

$$W^{I,M} = \frac{(\alpha+1)^2}{2(3\alpha+1)^2}, \ U^{I,M} = \frac{3\alpha^2 + 5\alpha + 2}{6(3\alpha+1)}$$

Next proposition states a static analysis.

PROPOSITION 3. In the mixed market, and with respect to government preference for the tax revenue, we have the following:

- (i) Production of stated-controlled firm and of domestic private firm decreases;
- (ii) production of the foreign private firm increases;
- (iii) aggregate quantity in the market decreases;
- (iv) Price increases;
- (v) The profits of both domestic public firm and domestic private firm decrease;
- (vi) foreign private firm's profit increases (resp., decreases) for high (resp., low) values of government preference for the tax revenue;
- (vii) Social welfare decreases;
- (viii) Government's payoff increases (resp., decreases) for high (resp., low) values of government preference for the tax revenue.

PROOF. The results follow from the following:

$$\frac{\partial q_0}{\partial \alpha} = -\frac{1}{(3\alpha+1)^2} < 0, \quad \frac{\partial q_1}{\partial \alpha} = -\frac{1}{2(3\alpha+1)^2} < 0,$$
$$\frac{\partial q_2}{\partial \alpha} = \frac{1}{2(3\alpha+1)^2} > 0, \quad \frac{\partial Q}{\partial \alpha} = -\frac{1}{(3\alpha+1)^2} < 0,$$
$$\frac{\partial p}{\partial \alpha} = \frac{1}{(3\alpha+1)^2} > 0,$$
$$\frac{\partial \pi_0}{\partial \alpha} = -\frac{\alpha+1}{2(3\alpha+1)^3} < 0, \quad \frac{\partial \pi_1}{\partial \alpha} = -\frac{3(\alpha+1)}{8(3\alpha+1)^3} < 0,$$
$$\frac{\partial \pi_2}{\partial \alpha} = \frac{3\alpha-1}{8(3\alpha+1)^3} \begin{cases} <0, \text{ se } \alpha < \frac{1}{3} \\ \ge 0, \text{ se } \alpha \ge \frac{1}{3} \end{cases}$$
$$\frac{\partial W}{\partial \alpha} = -\frac{2(\alpha+1)}{(3\alpha+1)^3} < 0,$$
$$\frac{\partial U}{\partial \alpha} = \frac{9\alpha^2 + 6\alpha - 1}{6(3\alpha+1)^2} \begin{cases} <0, \text{ se } \alpha < \frac{\sqrt{2}-1}{3} \\ \ge 0, \text{ se } \alpha \ge \frac{\sqrt{2}-1}{3} \end{cases}.$$

4 The privatized international triopoly

Now, suppose that the public firm is privatized. So, all firms aim to maximize their own profits. First

order conditions
$$\frac{\partial \pi_i}{\partial q_i} = 0$$
 give
 $q_i = \frac{2 - 3t + \mu}{10}$, with $i = 0, 1$,

and

 $q_2 = \frac{1+t-2\mu}{5}$.

Thus, we get

$$p = \frac{2 + 2t + \mu}{5},$$

$$\pi_i = \frac{3(2 - 3t + \mu)^2}{200}, \text{ with } i = 0,1,$$

$$\pi_2 = \frac{3(1 + t - 2\mu)^2}{50}$$

and

$$W = \frac{\left(2 - 3t + \mu\right)^2}{20}$$

Now, solving

$$\begin{cases} \frac{\partial U}{\partial t} = 0\\ \frac{\partial U}{\partial \mu} = 0 \end{cases}$$

gives

$$t^{I,P} = \frac{4\alpha - 1}{2(4\alpha + 1)}$$
 and $\mu^{I,P} = \frac{1}{2}$.

REMARK 2. Since we are assuming $\alpha > 1/3$, the optimal tax rate is positive.

By the above results, we can easily get the subgame perfect equilibrium, as stated in the next proposition.

PROPOSITION 4. The subgame perfect equilibrium for the international privatized triopoly is as follows:

$$q_0^{I,P} = q_1^{I,P} = \frac{2(\alpha+1)}{5(4\alpha+1)}, \ q_2^{I,P} = \frac{4\alpha-1}{10(4\alpha+1)},$$
$$Q^{I,P} = \frac{12\alpha+7}{10(4\alpha+1)}.$$

The price clearing the market at equilibrium is:

$$p^{I,P} = \frac{28\alpha + 3}{10(4\alpha + 1)}$$

Domestic private firms' profits are given by

$$\pi_0^{I,P} = \pi_1^{I,P} = \frac{6(\alpha+1)^2}{25(4\alpha+1)^2};$$

and foreign private firm's profit is

$$\pi_2^{I,P} = \frac{3(4\alpha - 1)^2}{200(4\alpha + 1)^2}.$$

Social welfare and government's payoff are, respectively, as follows:

$$W^{I,P} = \frac{4(\alpha+1)^2}{5(4\alpha+1)^2}, \ U^{I,P} = \frac{(\alpha+1)(12\alpha+7)}{20(4\alpha+1)}$$

Next proposition states a static analysis.

PROPOSITION 5. In the privatized market, and with respect to government preference for the tax revenue, we have the following:

- (i) Production of domestic private firms decreases;
- (ii) production of the foreign private firm increases;
- (iii) aggregate quantity in the market decreases;
- (iv) Price increases;
- (v) The profits of both domestic public firm and domestic private firm decrease;
- (vi) foreign private firm's profit increases;
- (vii) Social welfare decreases;
- (viii) Government's payoff increases.

PROOF. The results follow from the following:

$$\begin{aligned} \frac{\partial q_0^{I,P}}{\partial \alpha} &= \frac{\partial q_1^{I,P}}{\partial \alpha} = -\frac{6}{5(4\alpha+1)^2} < 0, \\ \frac{\partial q_2^{I,P}}{\partial \alpha} &= \frac{4}{5(4\alpha+1)^2} > 0, \ \frac{\partial Q^{I,P}}{\partial \alpha} = -\frac{8}{5(4\alpha+1)^2} < 0, \\ \frac{\partial p^{I,P}}{\partial \alpha} &= \frac{8}{5(4\alpha+1)^2} > 0, \\ \frac{\partial \pi_0^{I,P}}{\partial \alpha} &= \frac{\partial \pi_1^{I,P}}{\partial \alpha} = -\frac{36(\alpha+1)}{25(4\alpha+1)^3} < 0, \\ \frac{\partial \pi_2^{I,P}}{\partial \alpha} &= \frac{6(4\alpha-1)}{25(4\alpha+1)^3} > 0, \\ \frac{\partial W^{I,P}}{\partial \alpha} &= -\frac{24(\alpha+1)}{5(4\alpha+1)^3} < 0, \end{aligned}$$

$$\frac{\partial U^{I,P}}{\partial \alpha} = \frac{3(16\alpha^2 + 8\alpha - 3)}{20(4\alpha + 1)^2} > 0.$$

5 Effects of privatization

First, we compare the optimal tax and import tariff before and after privatization, in order to investigate the effect of privatization on these strategic policies. From the results above, we conclude that privatization decreases taxes $(t^{I,M} > t^{I,P})$ and does not affect the import tariff $(\mu^{I,M} = \mu^{I,P})$. Furthermore, under optimal production tax and import tariff, privatization of a SCF worsens (resp., raises) social welfare, for low (resp., high) values of the government preference for the tax revenue:

$$W^{I,M} - W^{I,P} = \frac{(\alpha+1)(8\alpha^2 - 8\alpha - 3)}{10(3\alpha+1)^2(4\alpha+1)^2} \begin{cases} <0, \text{ if } \alpha < \frac{2+\sqrt{10}}{4} \\ \ge 0, \text{ if } \alpha \ge \frac{2+\sqrt{10}}{4} \end{cases}$$

Since

$$U^{I,M} - U^{I,P} = \frac{12\alpha^3 + 23\alpha^2 + 10\alpha - 1}{60(3\alpha + 1)(4\alpha + 1)} > 0,$$

we conclude that the government has an incentive to privatize the SCF.

6 Domestic competition

Here, we suppose that all the firms F_0 , F_1 and F_2 are domestic. In this case, the tax rate *t* is applied to the production of the three firms. Social welfare is given by

 $W = CS + \pi_0 + \pi_1 + \pi_2$,

where

$$CS = \frac{1}{2} (q_0 + q_1 + q_2)^2,$$

and the government's payoff is given by

$$U = W + (1 + \alpha)R,$$

where $R = t(q_0 + q_1 + q_2)$.

From the results presented by Ferreira and Ferreira [10], we get the following propositions 6 and 7.

PROPOSITION 6. In the domestic competition with mixed competition, the subgame perfect equilibrium is given by:

$$t^{D,M}=\frac{12\alpha-1}{24\alpha+11},$$

$$q_{0}^{D,M} = \frac{4(\alpha+1)}{24\alpha+11}, \ q_{i}^{D,M} = \frac{2(\alpha+1)}{24\alpha+11}, \text{ with } i = 1,2,$$
$$Q^{D,M} = \frac{8(\alpha+1)}{24\alpha+11}, \ p^{D,M} = \frac{16\alpha+3}{24\alpha+11},$$
$$\pi_{0}^{D,M} = \frac{8(\alpha+1)^{2}}{(24\alpha+11)^{2}},$$
$$\pi_{i}^{D,M} = \frac{6(\alpha+1)^{2}}{(24\alpha+11)^{2}}, \text{ with } i = 1,2,$$
$$W^{D,M} = \frac{52(\alpha+1)^{2}}{(24\alpha+11)^{2}}, \ U^{D,M} = \frac{4(\alpha+1)^{2}}{24\alpha+11}.$$

PROPOSITION 7. In the domestic competition with privatized competition, the subgame perfect equilibrium is given by:

Since, in the domestic competition,

$$U^{D,M} - U^{D,P} = \frac{(\alpha+1)^2 (8\alpha-1)}{4(5\alpha+2)(42\alpha+11)},$$

the government will privatize the SCF if $\alpha < 1/8^2$.

7 Comparisons: domestic *versus* international competition

Now, let us compare some results in the domestic model with the ones in the international model. Here, we have to assume $\alpha > 1/3$.

As we saw above, in the international competition, the government always has an incentive to privatize the SCF. The same does not occur in the domestic competition: the government only privatize the SCF if it has low preferences for the tax revenue.

² We note that in the domestic competition we do not impose any restriction in the values of $\alpha \ge 0$.

In both mixed and privatized models, the tax revenue in the domestic competition is higher (resp., lower) than in the international competition, if the government has low (resp., high) preferences for the tax revenue:

$$R^{D,M} - R^{I,M} = \frac{873 + 42\alpha - 369\alpha^2}{6(3\alpha + 1)^2 (24\alpha + 11)^2} \begin{cases} <0, \text{ if } \frac{1}{3} < \alpha < \frac{21 + 13\sqrt{2}}{123} \\ \ge 0, \text{ if } \alpha \ge \frac{21 + 13\sqrt{2}}{123} \end{cases}, \\ R^{D,P} - R^{I,P} = \frac{3(7 + 8\alpha - 44\alpha^2)}{20(4\alpha + 1)^2 (5\alpha + 2)^2} \begin{cases} <0, \text{ if } \frac{1}{3} < \alpha < \frac{1}{2} \\ \ge 0, \text{ if } \alpha \ge \frac{1}{2} \end{cases}.$$

Social welfare is always higher in the domestic competition than in the international competition:

$$W^{D,M} - W^{I,M} = \frac{(\alpha+1)^2 (360\alpha^2 + 96\alpha - 17)}{2(3\alpha+1)^2 (24\alpha+11)^2} > 0,$$

$$W^{D,P} - W^{I,P} = \frac{(\alpha+1)^2 (320\alpha^2 + 40\alpha - 19)}{20(4\alpha+1)^2 (5\alpha+2)^2} > 0.$$

8 Conclusion

In this paper, we analysed the relationship between the privatization of a state-controlled firm and government preferences for tax revenue, by using a mixed Cournot model with domestic and foreign firms

We concluded that privatization decreases taxes and does not affect the import tariff. Furthermore, under optimal production tax and import tariff, privatization of a SCF worsens (resp., raises) social welfare, for low (resp., high) values of the government preference for the tax revenue.

Furthermore, we showed that in the international competition, the government always has an incentive to privatize the SCF. The same does not occur in the domestic competition: the government only privatize the SCF if it has low preferences for the tax revenue.

We also proved that the tax revenue in the domestic competition is higher (resp., lower) than in the international competition, if the government has low (resp., high) preferences for the tax revenue. Social welfare is always higher in the domestic competition than in the international competition

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References:

- [1] G. DeFraja and F. Delbono, Alternative Strategies of a Public Enterprise in Oligopoly, *Oxford Economic Papers*, Vol. 41, 1989, pp. 302-311.
- [2] K. Fjell and D. Pal, A mixed oligopoly in the presence of foreign private firms, *Canadian Journal of Economics*, Vol. 29, 1996, pp. 737-743.
- [3] K. Fjell and J. S. Heywood, Public Stackelberg leadership in a mixed oligopoly with foreign firms, *Australian Economic Papers*, Vol. 41, 2002, pp. 267-281.
- [4] D. Pal and M.D. White, Mixed oligopoly, privatization, and strategic trade policy, *Southern Economic Journal*, Vol. 65, 1998, pp. 264-281.
- [5] F. A. Ferreira and F. Ferreira, Environmental taxation: Privatization in an international duopoly with tariffs, *Applied Mathematics and Information Sciences*, Vol. 11, 2017, pp. 13-18.
- [6] F. A. Ferreira and F. Ferreira, *Privatization in a public leadership mixed duopoly*, Proceedings of the International Conference of Numerical Analysis and Applied Mathematics, Rhodes, Greece. American Institute of Physics, Vol. 1738, 2016, pp. 110003-1 110003-4.
- [7] F. A. Ferreira and F. Ferreira, *Government* preference, environmental taxes and privatization, Proceedings of the International Conference on Industrial Engineering and Operations Management, Dubai, United Arab Emirates, 2015.
- [8] H. Kato, Privatization and government preference, *Economics Bulletin*, Vol. 12, 2008, pp. 1-7.
- [9] F. A. Ferreira and F. Ferreira, Privatization and government preferences in a mixed duopoly: Stackelberg versus Cournot, in *Discontinuity* and Complexity in Nonlinear Physical Systems, edited by Tenreiro Machado et al., Springer International Publishing Switzerland, 2014, pp. 421-430.
- [10] F. A. Ferreira and F. Ferreira, *Privatization and* government preference in a differentiated *Cournot triopoly*, Proceedings of the International Conference on Mathematical

Methods & Computational Techniques in Science & Engineering, Cambridge, UK, 2018.

[11] Y. Tomaru, Privatization productive efficiency and social welfare with a foreign competitor, Research in Economics, Vol. 61, 2007, pp. 224-232.