Nonlinear Observer Using Mean Value Theorem And Lsqnonlin Matlab Algorithm

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Abstract: In this article, we propose a new nonlinear observer concept. The basic idea for our observer's design is to use mean value theorem (MVT) and lsqnonlin algorithm to determine the estimation error ($e = x - \hat{x}$) and MVT parameters β_i (between 0 and 1) . The stability study is carried out thanks to the quadratic function of Lyapunov. Two numerical examples are provided to show the performance of the proposed approach. The first studies a chaotic system with a linear term ($\dot{x} = f(x, u)$) and the second deals with a linear system ($\dot{x} = Ax + f(x, u)$).

Key–Words: lsqnonlin algorithm;Nonlinear observer; Nonlinear system; mean value theorem; State estimation

1 Introduction

State observation for non-linear has been an active area of research. The goal of this work is to design observers able to reconstruct the non-measurable state of the nonlinear system. Several types of observers have been designed to solve this problem, and many outstanding results have been obtained. Despite significant progress, the main objective remains unresolved, who consist to find generalized observer for all nonlinear systems. We mention just a few: The Nonlinear Luenberger observer approach [1-3], sliding mode observers [4-6] and adaptive observers [7-9].
Several other researchers have worked on observers for nonlinear Lipschitz systems using different approaches. However, a major limitation for this system type is that most of them only work for small values of Lipschitz constant. The major problem is that if this constant is greater than an appropriate value, the design methods cannot be applied. In some recent works [10-12], The mean value theorem is used to write the state estimation error as a linear system with uncertain parameters. The use of this theorem provides a solution even for a large Lipschitz constant.
In this paper, we propose a new nonlinear observer design without unknown inputs. The basic idea is the determination of the estimation error to introduce it into observer structure, based on an error feedback mechanism. The design of this process is done using mean value theorem and lsqnonlin algorithm. The stability analysis is investigate by using a quadratic Lyapunov function. Two numerical examples are provided to show the performance of the proposed approach. The first studies a chaotic system without a linear term ($\dot{x} = f(x, u)$) and the second deals with a nonlinear system with a linear term ($\dot{x} = Ax + f(x, u)$) .

2 Preliminaries

In this section, we present some tools that are important for the next section: The mean value theorem and lsqnonlin algorithm.

2.1 Mean-Value Theorem

Lemma 1: Mean Value Theorem for a Vector Function [13-14]
Let $f : R^n \to R$ is differentiable at each point of the line segment $[a \quad b]$, then there exists on that line segment a point $c = b + \beta(a - b)$ between $[a \quad b]$ and $\beta \in [0 \quad 1]$ such that

$$f(a) - f(b) = \nabla f(c)(a - b) \tag{1}$$

Note that β is a variable that changes continuously with the values of a and b. To use the Mean-Value Theorem, it is necessary to determine at each iteration the value of β. In this paper, we have used lsqnonlin algorithm to solve this problem.

3 Problem formulation and main results

The nonlinear system can be described as following:

$$\begin{cases} \dot{x}(t) = f(x(t), u(t)) \\ y(t) = Cx(t) \end{cases} \tag{2}$$

where $u(t) \in R^k$ and $y(t) \in R^m$ are the input and output vectors. $f(x(t); u(t))$ is supposed to be continuously differentiable,and $C \in R^{m\times n} = [I_m \quad 0]$ are known constant matrices of appropriate dimensions and I_m is an identity matrix. We assume that $rank(C) = m$.

3.1 Proposition of Nonlinear Observer

In this section, we will first present the structure and necessary and sufficient condition for existence of the proposed nonlinear observer. This observer has the following structure:

$$\dot{\hat{x}}(t) = f_{\hat{x},u} + \overline{G}_{\hat{x},u,e} e_1(t) \tag{3a}$$

$$\dot{e}(t) = (D_x(f_{\hat{x},u,e})(c_i) - \overline{G}_{\hat{x},u,e})e(t) \tag{3b}$$

Now we can give a sufficient condition under which the observer given by (3) is indeed a nonlinear observer.

Theorem 1

If there exist positive constants $\eta \geq 1$ *such that:*

$$D_x(f_{\hat{x},u,e})(c_i) - \overline{G}_{\hat{x},u,e} < 0 \tag{4}$$

$\overline{G}_{\hat{x},u,e} \in R^{n\times n}$ is matrix which have to be designed such that $\hat{x}$ asymptotically converges to x, $c_i = \hat{x} + \beta_i e$ with $\beta_i \in [\ 0 \quad 1\ ]$. $\overline{G}_{\hat{x},u,e}$ is a diagonal matrix, which can be written as follows

$$\overline{G}_{\hat{x},u,e} = \eta \begin{bmatrix} \alpha_1(\hat{x},u,e) & 0 & \cdots & 0 \\ 0 & \alpha_2(\hat{x},u,e) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \alpha_n(\hat{x},u,e) \end{bmatrix} \tag{5}$$

with

$$\alpha_k(\hat{x},u,e) = \sum_{j=1}^{n} |g_{kj}| + \sum_{i=1}^{n} |g_{ik}| \tag{6}$$

With $k = \{1, 2, ..., n\}$, g_{ij} represent the coefficients of the matrix $D_x(f_{\hat{x},u,e})(c_i)$, η is positive constant $\eta \geq 1$, $e(t) \in R^n$ is the vector of estimation error determined by observer, $y(t) - \hat{y}(t) \in R^m$ is the measurable estimation error and $e_1(t) \in R^n$ is vector that includes measurable estimation errors and the errors determined by observer such that $e_1(t) = [y_1(t) - \hat{y}_1(t), \ldots, y_m(t) - \hat{y}_m(t), e_{m+1}(t), \ldots, e_n(t)]^T$.

3.2 Proof of condition (3):

In order to investigate the stability and the convergence of the observer the state estimation error dynamics is analyzed:

$$\dot{e}(t) = \dot{x}(t) - \dot{\hat{x}}(t) = f_{x,u} - (f_{\hat{x},u} + \overline{G}_{\hat{x},u,e} e_1(t)) \tag{7}$$

We consider that $f(x, u)$ is expressed according to the known terms, such that $e(t)$ and $\hat{x}(t)$ will be determined by observer (3), c_i using lsqnonlin optimization algorithm (11-12) and the input system $u(t)$:

$$f_{x,u} = f_{e+\hat{x},u} = f_{\hat{x},u} + D_x(f_{\hat{x},u,e})(c_i)e \tag{8}$$

With $c_i = \hat{x} + \beta_i(x - \hat{x}) = \hat{x} + \beta_i e$ such that $\beta \in [0 \quad 1]$ and $i = \{1, 2, ..., n\}$. D_x is the differential operator defined by

$$D_x(f_{\hat{x},u,e})(c_i) = \frac{\partial f_{x,u,e}(c_i)}{\partial x}|_{x=c_i} \tag{9}$$

Using the mean value theorem and dynamic error equation becomes:

$$\dot{e}(t) = (D_x(f_{\hat{x},u,e})(c_i) - \overline{G}_{\hat{x},u,e})e(t) \tag{10}$$

3.2.1 Determination of the parameters β_i

For the determination of the β_i parameters, we use lsqnonlin algorithm and Simulink block "Interpreted MATLAB Function" (figure 2). based on mean value theorem (1), we consider the following system of equations:

$$X(\beta_i) = f_{\hat{x}+e,u} - f_{\hat{x},u} - D_x(f_{\hat{x},u,e})(\hat{x} + \beta_i e)e = 0 \tag{11}$$

$X(\beta_i) = [X_1, \ X_2, \ldots, X_n]^T$, $f_{\hat{x}+e,u} = [f_{1\hat{x}+e,u}, f_{2\hat{x}+e,u}, \cdots,$ $f_{n\hat{x}+e,u}]^T$, $f_{\hat{x},u} = [f_{1\hat{x},u}, f_{2\hat{x},u}, \ldots, \ f_{n\hat{x},u}]^T$, $D_x(f_{\hat{x},u,e})(\hat{x} + \beta_i e) = [\nabla f_{1\hat{x},u,e}(\hat{x} + \beta_i e), \nabla f_{2\hat{x},u,e}\hat{x} + \beta_i e), \ldots, \nabla f_{n\hat{x},u,e}\hat{x} + \beta_i e]^T$ and $\beta_i = [\beta_1, \beta_2, \ldots, \beta_n]^T$.

The parameter estimation β_i is transformed into an optimization problem. A performance objective function is defined for the minimization; f_{obj} is used as objective function and is given by:

$$\min_{\beta_i \in [0 \quad 1]} f_{obj}(\beta_i) = \min_{\beta_i \in [0 \quad 1]} \left(\sqrt{\sum_{j=1}^{n} (X_j(\beta_i))^2} \right) \tag{12}$$

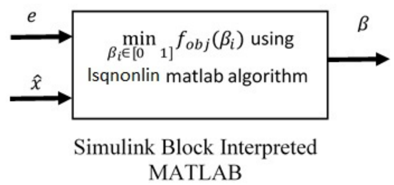

Figure 1: Determination of β_i parameters

3.3 Proof of theorem 1

The aim is to define the matrix $\overline{G}_{\hat{x},u,e}$ so that the error of estimation converges asymptotically to zero. Let

$$V(e) = \frac{1}{2} e^T e \tag{13}$$

The dynamic Lyapunov function can be writing as follows:

$$\dot{V}(e) = e^T (D_x(f_{\hat{x},u,e})(c_i) - \overline{G}_{\hat{x},u,e})e \tag{14}$$

To ensure the asymptotic convergence of e to zero, the derivative of V must be negative, to satisfy this condition. The term $e^T D_x(f_{\hat{x},u,e})(c_i)e$ must be increased as follows:

$$\begin{aligned} & e^T D_x(f_{\hat{x},u,e})(c_i)e < |e^T D_x(f_{\hat{x},u,e})e| \\ & = \sum_{i,j=1}^{n} |D_x(f_{\hat{x},u,e})||e_i e_j| \end{aligned} \tag{15}$$

We assume that $G_{\hat{x},u,e} = D_x(f_{\hat{x},u,e})$ and $|e_i e_j| \leq \frac{1}{2}(e_i^2 + e_j^2)$, consequently the inequality (15) becomes

$$e^T G_{\hat{x},u,e} e \leq \sum_{i,j=1}^{n} |g_{ij}||e_i e_j| \tag{16}$$

with g_{ij} represent the coefficients of the matrix $G_{\hat{x},u,e}$.Therefore, we obtain

$$e^T G_{\hat{x},u,e} e < \sum_{k=1}^{n} \left(\sum_{j=1}^{n} |g_{kj}| + \sum_{i=1}^{n} |g_{ik}| \right) e_k^2 \tag{17}$$

Following this latest development, we can conclude that for all $(e, \hat{x}) \in R^n$, we find:

$$e^T G_{\hat{x},u,e} e < e^T \overline{G}_{\hat{x},u,e} e \tag{18}$$

with $\overline{G}_{\hat{x},u,e}$ is a diagonal matrix, which can be written as follows:

$$\overline{G}_{\hat{x},u,e} = \eta \begin{bmatrix} \alpha_1(\hat{x},u,e) & 0 & \cdots & 0 \\ 0 & \alpha_2(\hat{x},u,e) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \alpha_n(\hat{x},u,e) \end{bmatrix}$$

where

$$\alpha_k(\hat{x},u,e) = \sum_{j=1}^{n} |g_{kj}| + \sum_{i=1}^{n} |g_{ik}| \tag{20}$$

Such that $k = \{1, 2, ..., n\}$.
we obtain the following expression:

$$\dot{V}(e) = e^T (D_x(f_{\hat{x},u})(c_i) - \overline{G}_{\hat{x},u,e})e \tag{21}$$

The previous expression becomes negative if $\eta \geq 1$.

4 Illustrative example

- Example 1 : Nonlinear system without linear term

The numerical simulation example provided to verify the effectiveness of the proposed approach is represented by ordinary differential equations. We consider the chaotic system without linear term [18]:

$$\begin{aligned} \dot{x}(t) &= \begin{pmatrix} ln(0.1 + exp(x_2 - x_1)) \\ x_1 x_3 \\ 0.2 - x_1 x_2 \end{pmatrix} \\ y(t) &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} x(t) \end{aligned} \tag{22}$$

with $u(t) = 0$, We consider the initial conditions given by:
$x_0 = [-2 \quad -1 \quad -1]^T, \hat{x}_0 = [-1 \quad -0.5 \quad -1.5]^T$ and $e_0 = x_0 - \hat{x}_0 = [-1 \quad -0.5 \quad 0.5]^T$.
Two case studies are considered:
In the first, we assume that the proposed observer has the initial estimation error of (3b) equal at $e_0 = x_0 - \hat{x}_0 = [1 \quad -0.5 \quad 0.5]^T$ and $e_0 = e_{01} = [1 \quad -0.5 \quad 0.2]^T$ for the second case and we choose $\eta = 1$.

4.0.1 Determination of β_i parameters

Based on the mean value theorem, lsqnonlin algorithm and equation (12). We note that the functions $X(\beta_i) \simeq 0$ (figure 4) at each moment of the simulation for the β_i (figure 3) parameters that evolve between 0 and 1, such that $i = 1, 2, 3$.

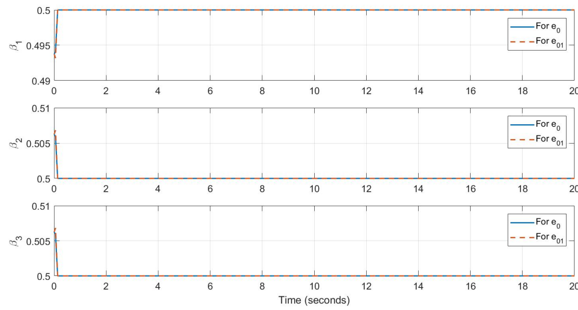

Figure 2: Evolution of β_i parameters

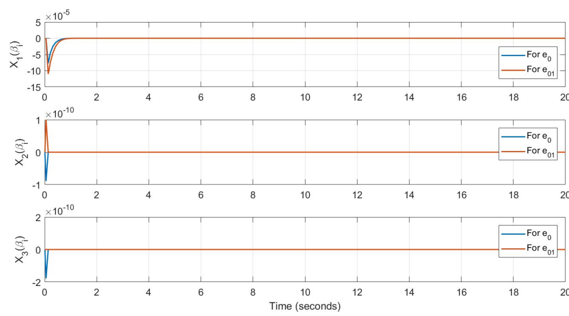

Figure 3: Evolution of $X(\beta_i)$

4.0.2 Comparison between observer's error for e_0, e_{01} and real error

As shown fig.5, we notice that both observer's errors for e_0 and real errors are confused, the observer's error for e_{01} converge to real errors. Which validates the proposed development in the paper.

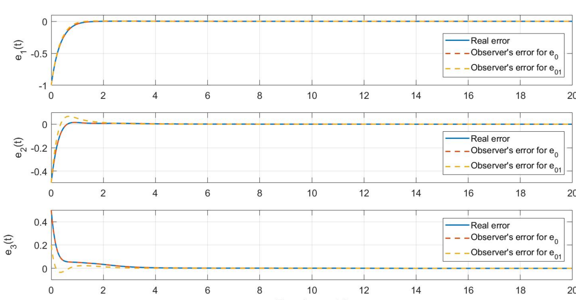

Figure 4: Evolution of observer's error and real error

4.0.3 Comparison between two case e_0 and e_{01} for the proposed observer

Figure 6 show satisfactory of proposed observer performance in dealing with a nonlinear system without linear term.

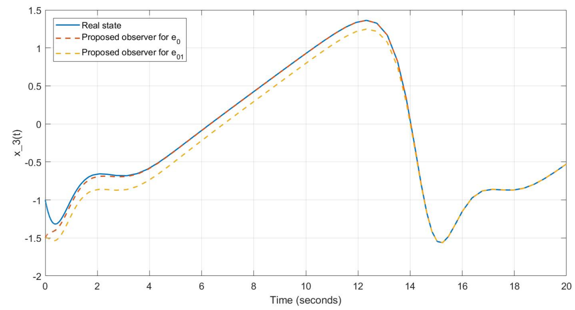

Figure 5: Evolution of the state x_3

- Example 2 Nonlinear system with linear part
  In this example, we will compare the proposed observer with two recent nonlinear observers design [17] and [16]. Consider a single-link flexible robotic [19]:

$$f(x,u) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -48.6 & -1.25 & 48.6 & 0 \\ 0 & 0 & 0 & 1 \\ 19.5 & 0 & -19.5 & 3.33 \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ 21.6 \\ 0 \\ 0 \end{pmatrix} u(t) + \begin{pmatrix} 0 \\ 0 \\ 0 \\ -3.33sin(x3) \end{pmatrix},$$

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix},\ u(t) = sin(t).$$

We consider the initial conditions given by: $x_0 = [3 \quad 3 \quad 3 \quad 3]$; $\hat{x}_0 = [-1 \quad -2 \quad -0.5 \quad -1]$ and $e_0 = x_0 - \hat{x}_0$.

  - Proposed observer

First case
We chosen that $\eta = 1$ and $e_0 = x_0 - \hat{x}_0 = [2 \quad 1 \quad 2.5 \quad 2]^T$.
Second case
We chosen that $\eta = 1$ and $e_0 = e_{01} = [2 \quad 1 \quad 2.5 \quad 2.5]^T$.

  - [17] observer

We keep the same condition considered in this paper, for more details see [17].

  - [16] observer

We consider the following matrices:

$$L = \begin{pmatrix} 0.5409 & -0.6315 & 0.2392 \\ -10.3506 & 21.2355 & 22.8782 \\ 1.8370 & 3.4444 & 10.6638 \\ 15.5282 & 8.49.67 & 62.7550 \end{pmatrix}.$$

For more details see [16].

4.0.4 Determination of β_i parameters

We note that the functions $X(\beta_i) \simeq 0$ (figure 8) at each moment of the simulation for the β_i (figure 7) parameters that evolve between 0 and 1, such that $i = 1, 2, 3, 4$.

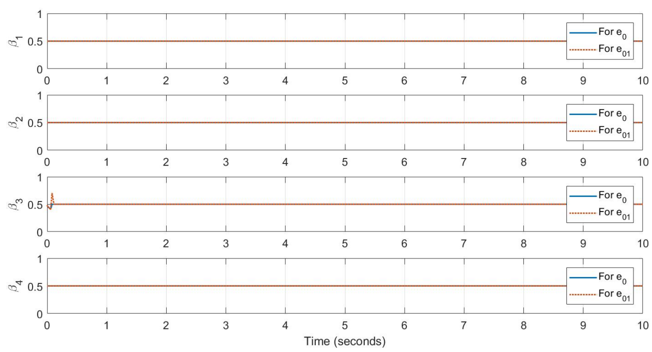

Figure 6: Evolution of β_i parameters

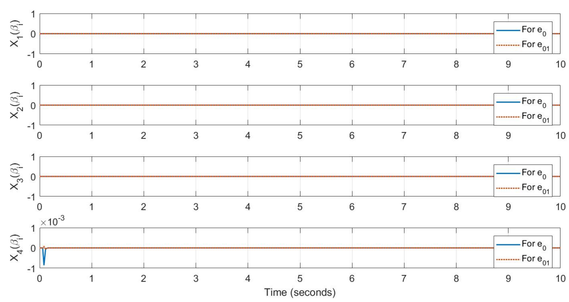

Figure 7: Evolution of $X(\beta_i)$

4.0.5 Comparison between evolution observer's error and real error

As shown fig.9, we notice that both observer's error (for e_0 and e_{01}) follows the real errors.

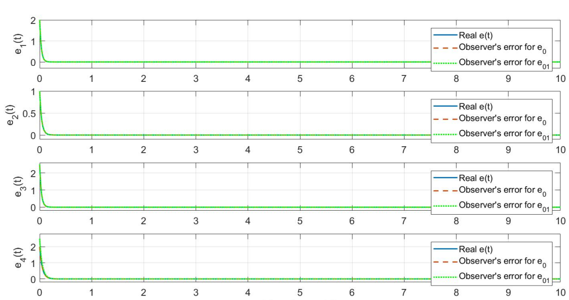

Figure 8: Evolution of observer's error and real error

4.0.6 Comparison between [16], [17] and proposed observer

The following figure show satisfactory of proposed observer performance in dealing with a nonlinear system with linear term.

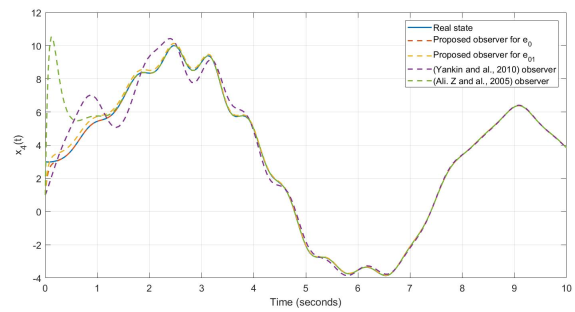

Figure 9: Evolution of the state x_4

5 Conclusion

A full order nonlinear observer was proposed for a large class of nonlinear systems with unknown inputs. Mean value theorem and lsqnonlin algorithm are the tools to use for the design of this observer. Numerical example is provided to show high performances of the proposed approach and the large class of nonlinear dynamical systems that are concerned.

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