Observer For Nonlinear Systems Using Mean Value Theorem and Simulated Annealing Algorithm

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Abstract: In this note, we consider a new nonlinear unknown input observer design for large class nonlinear systems. The principal idea consist on using estimation error and mean value theorem parameters β in proposed observer structure, based on the feedback mechanism. This process is performed using mean value theorem and simulated annealing algorithm. A stability study was performed using classical Lyapunov function. Numerical examples are designed to show the effectiveness of the approach proposed for nonlinear dynamic systems concerned. Proposed observer can treat nonlinear systems without a linear term ($\dot{x} = f(x, u)$) and with a linear term ($\dot{x} = Ax + f(x, u)$).

Key–Words: Simulated annealing algorithm;Nonlinear observer; Nonlinear system; mean value theorem; State estimation

1 Introduction

Nonlinear state observation has been an active field of research. The goal is to reconstruct non-measurable state of nonlinear system. Several types of observers have been designed to solve this problem, and many outstanding results have been obtained. Despite significant progress, the main objective remains unresolved, who consist to find generalized observer for all nonlinear systems. We mention just a few: The nonlinear Luenberger observer approach [1-3], sliding mode observers [4-6], adaptive observers [6-8] and Local unknown input observer [17].
The design of nonlinear observer has been a field of great evolution in recent decades. It is used to different discipline in control theory field. For these reasons, nonlinear observers design has received considerable attention in literature. Nevertheless, several important problems remain unresolved. The goal is to look for a global method for all nonlinear systems. In some recent works [10-12], mean value theorem is used to convergence study of estimation error, it is considered that the system is linear with uncertain parameters, this kind of problem is solved thanks to tools used for linear systems. The use of mean value theorem provides a solution even for a large Lipschitz constant.
In this note, we consider the design of a new nonlinear observer structure. The principal idea is the estimation error and mean value theorem parameters (β) determination for state estimation correction, using a feedback mechanism in observer structure. This process is designed based on mean value theorem and simulated annealing algorithm. The stability study relies on use of a classical quadratic Lyapunov function. Two numerical examples are provided to show the performance of the proposed approach. the first studies a chaotic system without linear term ($\dot{x} = f(x, u)$) and the second deals with a nonlinear system with linear term ($\dot{x} = Ax+f(x, u)$).

2 Preliminaries

We present mean value theorem and simulated annealing algorithm.

2.1 Mean-Value Theorem

Lemma 1: Mean Value Theorem for a Vector Function [13-14]
Let $f : R^n \to R$ is differentiable at each point of the line segment $[a \quad b]$, then there exists on that line segment a point $c = b + \beta(a - b)$ between $[a \quad b]$ and

$\beta \in [0 \quad 1]$ such that

$$f(a) - f(b) = \nabla f(c)(a - b) \tag{1}$$

Note that β is a variable that changes continuously with the values of a and b. To use the Mean-Value Theorem, it is necessary to determine at each iteration the value of β. In this paper, we have used simulated annealing algorithm to solve this problem.

2.2 Particle Swarm Optimization method

The simulated annealing algorithm (SA), developed by Kirkpatrick and his collaborators [20]. The basic idea is to probabilistically accept worse quality candidate solution than the current solution in order to escape from local minima.
These steps can be represented schematically as shown in following Figure.

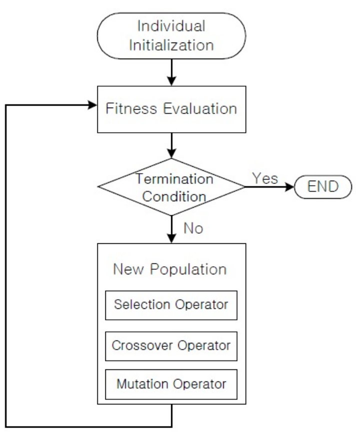

Figure 1: Diagram of simulated annealing algorithm

3 Problem formulation and main results

The nonlinear system can be described as following:

$$\begin{cases} \dot{x}(t) = f(x(t), u(t)) \\ y(t) = Cx(t) \end{cases} \tag{2}$$

where $u(t) \in R^k$ and $y(t) \in R^m$ are the input and output vectors. $f(x(t); u(t))$ is supposed to be continuously differentiable and $C \in R^{m\times n} = [I_m \quad 0]$ are known constant matrices of appropriate dimensions and I_m is an identity matrix. We assume that $rank(C) = m$.

3.1 Nonlinear Observer Proposition

In this part, we will present a nonlinear observer structure. Considering the observer of following form

$$\dot{\hat{x}}(t) = f_{\hat{x},u} + \overline{G}_{\hat{x},u,e} e_1(t) + K(y(t) - \hat{y}(t)) \tag{3}$$

$$\dot{e}(t) = (D_x(f_{\hat{x},u,e})(c_i) - \overline{G}_{\hat{x},u,e})e(t) - K(y(t) - \hat{y}(t)) \tag{4}$$

The sufficient conditions are given in following theorem.

Theorem 1

If there exist positive constants $\eta \geq 1$ and $\gamma \geq 0$ such that:

$$D_x(f_{\hat{x},u,e})(c_i) - \overline{G}_{\hat{x},u,e} - \gamma C^T C < 0 \tag{5}$$

$K \in R^{n\times m}$ and $\overline{G}_{\hat{x},u,e} \in R^{n\times n}$ are matrices which have to be designed such that $\hat{x}$ asymptotically converges to x, $c_i = \hat{x} + \beta_i e$ with $\beta_i \in \begin{bmatrix} 0 & 1 \end{bmatrix}$. $\overline{G}_{\hat{x},u,e}$ is a diagonal matrix, which can be written as follows

$$\overline{G}_{\hat{x},u,e} = \eta \begin{bmatrix} \alpha_1(\hat{x}, u, e) & 0 & \cdots & 0 \\ 0 & \alpha_2(\hat{x}, u, e) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \alpha_n(\hat{x}, u, e) \end{bmatrix} \tag{6}$$

with

$$\alpha_k(\hat{x}, u, e) = \sum_{j=1}^{n} |g_{kj}| + \sum_{i=1}^{n} |g_{ik}| \tag{7}$$

with $k = \{1, 2, ..., n\}$, g_{ij} represent the coefficients of the matrix $D_x(f_{\hat{x},u,e})(c_i)$, η is positive constant $\eta \geq 1$, $e(t) \in R^n$ is the estimation error vector determined by observer, $y(t) - \hat{y}(t) \in R^m$ is the measurable estimation error and $e_1(t) \in R^n$ is vector that includes measurable estimation errors and the errors determined by observer such that $e_1(t) = [y_1(t) - \hat{y}_1(t), \ldots, y_m(t) - \hat{y}_m(t), e_{m+1}(t), \ldots, e_n(t)]^T$.

3.2 Proof of condition (3.2-3.3):

Our goal is to build an observer that ensures the reconstruction of states system (2). Consequently, we consider the estimation error $e(t)$:

$$\begin{aligned}\dot{e}(t) = \dot{x}(t) - \dot{\hat{x}}(t) = f_{x,u} - (f_{\hat{x},u} + \overline{G}_{\hat{x},u,e} e_1(t) \\ + K(y(t) - \hat{y}(t)))\end{aligned} \tag{8}$$

we note that $e_1(t) = e(t)$, the observer asymptotically reconstructs the state of the system (2), when $e(t)$ converges to zero as t tends to infinity. In this context,

$f(x,u)$ is expressed according to the known terms, such that $e(t)$ and $\hat{x}(t)$ will be determined by observer (4) and (3) respectively, c_i using simulated annealing algorithm (12-13) and the input system $u(t)$:
We can write:

$$f_{x,u} = f_{e+\hat{x},u} = f_{\hat{x},u} + D_x(f_{\hat{x},u,e})(c_i)e \tag{9}$$

With $c_i = \hat{x} + \beta_i(x - \hat{x}) = \hat{x} + \beta_i e$ such that $\beta \in [0 \quad 1]$ and $i = \{1, 2, ..., n\}$. D_x is the differential operator defined by

$$D_x(f_{\hat{x},u,e})(c_i) = \frac{\partial f_{x,u,e}(c_i)}{\partial x}|_{x=c_i} \tag{10}$$

Using mean value theorem then error dynamic equation becomes:

$$\begin{aligned}\dot{e}(t) = (D_x(f_{\hat{x},u,e})(c_i) - \overline{G}_{\hat{x},u,e})e(t) \\ -K(y(t) - \hat{y}(t))\end{aligned} \tag{11}$$

3.2.1 Determination of the parameters β_i

For the determination of the β_i parameters, we use simulated annealing algorithm and Simulink block (figure 1) "Interpreted MATLAB Function". based on the mean value theorem (1), we consider the following system of equations:

$$X(\beta_i) = f_{\hat{x}+e,u} - f_{\hat{x},u} - D_x(f_{\hat{x},u,e})(\hat{x} + \beta_i e)e = 0 \tag{12}$$

$X(\beta_i) = [X_1, \ X_2, \ldots, X_n]^T$, $f_{\hat{x}+e,u} = [f_{1\hat{x}+e,u}, f_{2\hat{x}+e,u}, \ldots,$
$f_{n\hat{x}+e,u}]^T$, $f_{\hat{x},u} = [f_{1\hat{x},u}, f_{2\hat{x},u}, \ldots, \ f_{n\hat{x},u}]^T$, $D_x(f_{\hat{x},u,e})(\hat{x} + \beta_i e) = [\nabla f_{1\hat{x},u,e}(\hat{x} + \beta_i e), \nabla f_{2\hat{x},u,e}\hat{x} + \beta_i e), \ldots, \nabla f_{n\hat{x},u,e}\hat{x} + \beta_i e]^T$ and $\beta_i = [\beta_1, \beta_2, \ldots, \beta_n]^T$.
The parameter estimation β_i is transformed into an optimization problem. A performance objective function is defined for the minimization; f_{obj} is used as objective function and is given by:

$$\min_{\beta_i \in [0 \quad 1]} f_{obj}(\beta_i) = \min_{\beta_i \in [0 \quad 1]} \left(\sqrt{\sum_{j=1}^{n} (X_j(\beta_i))^2} \right) \tag{13}$$

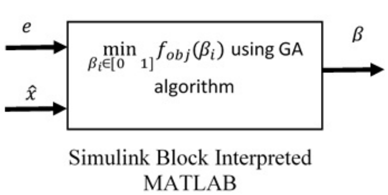

Figure 2: Determination of β_i parameters

3.3 Proof of theorem 1

The aim is to define the matrix $\overline{G}_{\hat{x},u,e}$ so that the error of estimation converges asymptotically to zero. Let

$$V(e) = \tfrac{1}{2}e^T e \tag{14}$$

The dynamic Lyapunov function can be writing as follows:

$$\dot{V}(e) = e^T(D_x(f_{\hat{x},u,e})(c_i) - \overline{G}_{\hat{x},u,e} - KC)e \tag{15}$$

To ensure the asymptotic convergence of e to zero, the derivative of V must be negative, to satisfy this condition. The term $e^T D_x(f_{\hat{x},u,e})(c_i)e$ must be increased as follows:

$$\begin{aligned} e^T D_x(f_{\hat{x},u,e})(c_i)e < |e^T D_x(f_{\hat{x},u,e})e| = \\ \sum_{i,j=1}^{n} |D_x(f_{\hat{x},u,e})||e_i e_j| \end{aligned} \tag{16}$$

We assume that $G_{\hat{x},u,e} = D_x(f_{\hat{x},u,e})$ and $|e_i e_j| \leq \frac{1}{2}(e_i^2 + e_j^2)$, consequently the inequality (16) becomes

$$e^T G_{\hat{x},u,e} e \leq \sum_{i,j=1}^{n} |g_{ij}||e_i e_j| \tag{17}$$

with g_{ij} represent the coefficients of the matrix $G_{\hat{x},u,e}$.Therefore, we obtain

$$e^T G_{\hat{x},u,e} e < \sum_{k=1}^{n} \left(\sum_{j=1}^{n} |g_{kj}| + \sum_{i=1}^{n} |g_{ik}| \right) e_k^2 \tag{18}$$

Following this latest development, we can conclude that for all $(e, \hat{x}) \in R^n$, we find:

$$e^T G_{\hat{x},u,e} e < e^T \overline{G}_{\hat{x},u,e} e \tag{19}$$

with $\overline{G}_{\hat{x},u,e}$ is a diagonal matrix, which can be written as follows:

$$\overline{G}_{\hat{x},u,e} = \eta \begin{bmatrix} \alpha_1(\hat{x},u,e) & 0 & \cdots & 0 \\ 0 & \alpha_2(\hat{x},u,e) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \alpha_n(\hat{x},u,e) \end{bmatrix} \tag{20}$$

where

$$\alpha_k(\hat{x},u,e) = \sum_{j=1}^{n} |g_{kj}| + \sum_{i=1}^{n} |g_{ik}| \tag{21}$$

Such that $k = \{1, 2, ..., n\}$.
Now, considered $e(t) \in R^n$ and research the matrix K satisfying the condition of the convergence of $e(t)$

to zero. The gain K must be chosen such that the $\dot{V}$ must be negative. A structure of K is:

$$K = \gamma C^T \tag{22}$$

We obtain the following expression:

$$\dot{V}(e) = e^T(D_x(f_{\hat{x},u})(c_i) - \overline{G}_{\hat{x},u,e} - \gamma C^T C)e \tag{23}$$

The previous expression becomes negative if $\eta \geq 1$ and $\gamma \geq 0$.

4 Illustrative example

- Example 1 : Nonlinear system without linear term

The numerical simulation example provided to verify the effectiveness of the proposed approach is represented by ordinary differential equations. We consider the chaotic system without linear term [18]:

$$\begin{aligned} \dot{x}(t) &= \begin{pmatrix} ln(0.1 + exp(x_2 - x_1)) \\ x_1 x_3 \\ 0.2 - x_1 x_2 \end{pmatrix} \\ y(t) &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} x(t) \end{aligned} \tag{24}$$

with $u(t) = 0$.
We consider the initial conditions given by:
$x_0 = [-2 \quad -1 \quad -1]^T, \hat{x}_0 = [-1 \quad -0.5 \quad -1.5]^T$ and $e_0 = x_0 - \hat{x}_0 = [-1 \quad -0.5 \quad 0.5]^T$.
Two case studies are considered:
In the first, we assume that the proposed observer has the initial estimation error of (4) equal at $e_0 = x_0 - \hat{x}_0 = [1 \quad -0.5 \quad 0.5]^T$ and $e_0 = e_{01} = [1 \quad -0.5 \quad 0.2]^T$ for the second case and we choose $\eta = 1$ and $\gamma = 1$.

4.0.1 Determination of β_i parameters

Based on mean value theorem, simulated annealing algorithm and equation (13). We note that the functions $X(\beta_i) \simeq 0$ (figure 3) at each moment of the simulation for the β_i (figure 2) parameters that evolve between 0 and 1, such that $i = 1, 2, 3$.

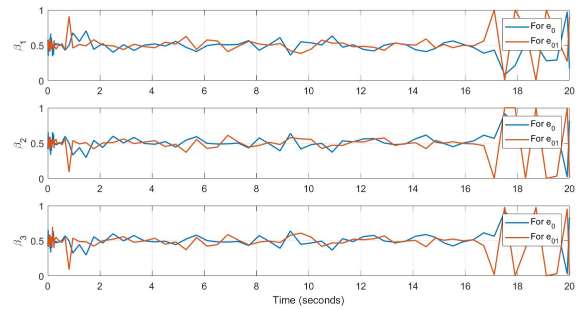

Figure 3: Evolution of β_i parameters

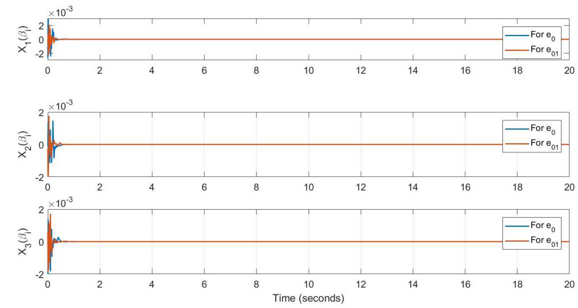

Figure 4: Evolution of $X(\beta_i)$

4.0.2 Comparison between observer's error for e_0, e_{01} and real error

As shown in fig.4, we notice that both observer's errors for e_0 and real errors are confused, the observer's error for e_{01} converge to real errors. Which validates the proposed development in the paper.

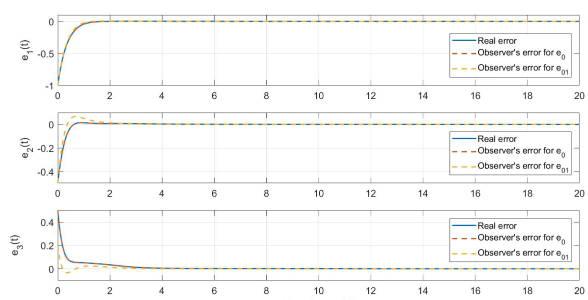

Figure 5: Evolution of observer's error and real error

4.0.3 Comparison between two case e_0 and e_{01} for proposed observer

Figure 5 show satisfactory of proposed observer performance in dealing with a nonlinear system without linear term.

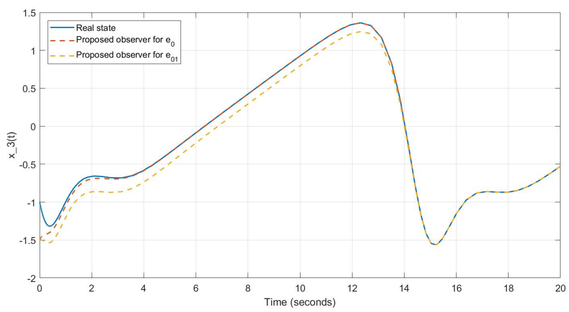

Figure 6: Evolution of the state x_3

- Example 2 Nonlinear system with linear part
In this example, we will compare the proposed observer with two recent nonlinear observers design [15] and [16]. Consider a single-link flexible robotic [19]: $f(x,u) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -48.6 & -1.25 & 48.6 & 0 \\ 0 & 0 & 0 & 1 \\ 19.5 & 0 & -19.5 & 3.33 \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ 21.6 \\ 0 \\ 0 \end{pmatrix} u(t) + \begin{pmatrix} 0 \\ 0 \\ 0 \\ -3.33sin(x3) \end{pmatrix}$,
$C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$, $u(t) = sin(t)$.
We consider the initial conditions given by: $x_0 = [3 \quad 3 \quad 3 \quad 3]$; $\hat{x}_0 = [-1 \quad -2 \quad -0.5 \quad -1]$ and $e_0 = x_0 - \hat{x}_0$.

  - Proposed observer

First case
We chosen that $\eta = 1, \gamma = 1$ and $e_0 = x_0 - \hat{x}_0 = [2 \quad 1 \quad 2.5 \quad 2]^T$.
Second case
We chosen that $\eta = 1$, $\gamma = 1$ and $e_0 = e_{01} = [2 \quad 1 \quad 2.5 \quad 2.5]^T$.

  - [15] observer

We keep same condition considered in this paper, for more details see [15].

  - [16] observer

We consider the following matrices:

$$L = \begin{pmatrix} 0.5409 & -0.6315 & 0.2392 \\ -10.3506 & 21.2355 & 22.8782 \\ 1.8370 & 3.4444 & 10.6638 \\ 15.5282 & 8.49.67 & 62.7550 \end{pmatrix}.$$

For more details see [16].

4.0.4 Determination of β_i parameters

We note that the functions $X(\beta_i) \simeq 0$ (figure 7) at each moment of simulation for β_i (figure 6) parameters that evolve between 0 and 1, such that $i = 1, 2, 3, 4$.

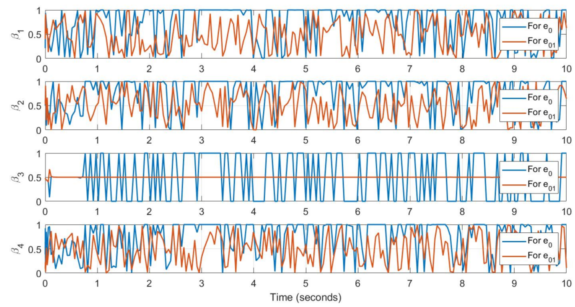

Figure 7: Evolution of β_i parameters

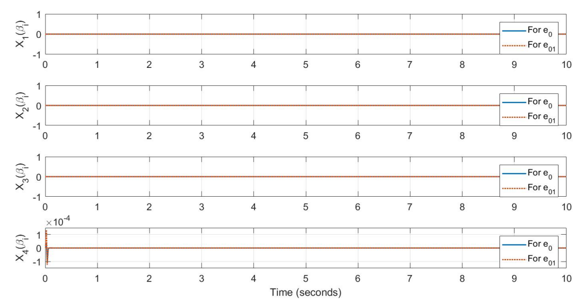

Figure 8: Evolution of $X(\beta_i)$

4.0.5 Comparison between evolution observer's error and real error

As shown in fig.8, we notice that both observer's error (for e_0 and e_{01}) follows the real errors.

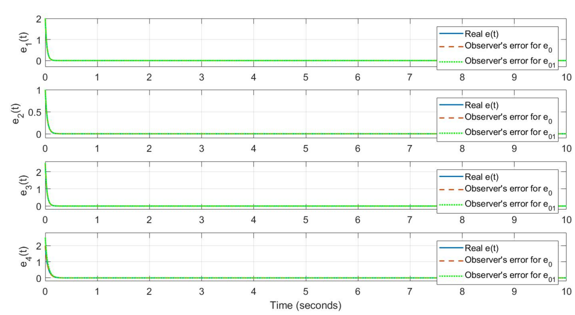

Figure 9: Evolution of observer's error and real error

4.0.6 Comparison between [15], [16] and proposed observer

Figure 9 show satisfactory of proposed observer performance in dealing with a nonlinear system with linear term.

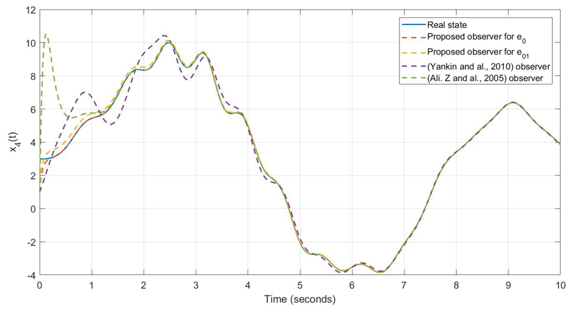

Figure 10: Evolution of the state x_4

5 Conclusion

A full order nonlinear observer was proposed for a large class of nonlinear systems without unknown inputs. Mean value theorem and simulated annealing algorithm are the tools to use for the design of this observer. Numerical example is provided to show high performances of the proposed approach and the large class of nonlinear dynamical systems that are concerned.

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