Study of Oblique Collision between a Ball and a Prism

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Abstract: - The collision occurring when a ball in free falling strikes an inclined metallic fixed surface is analysed in the present paper. The launching height ensures plastic deformations due to ball indentation of the immobile surface (body, plate). The prints are scanned using laser profilometry and next analysed. An interesting result arises from the study of the correlation connecting the angle between the impact velocity and the immobile surface and the symmetry axis of the permanent prints

Key-Words: - oblique impact, coefficients of restitution, laser profilometry, plastic indentation

1 Introduction

One of the most effective methods applied for dynamical analysis of a mechanical system, category including both robots and mechanisms, is the multibody dynamic analysis, [1], [2]. The systems regarding collision phenomena outline a particular class of multibody dynamic problems.

The impact phenomenon is characterised by the sudden variation of kinematical parameters, producing in the joints of the system particularly powerful forces, [3], [4]. The huge values of collision forces produce in most of cases, plastic, permanent deformations of the contact regions. In the case when the variation of kinematical parameters produces extremely fast, the value of the stresses can be as high as to generate irremediable damage of contacting surfaces. A comprehensive study concerning the collision behaviour of a system with complex geometry is especially intricate due to the multitude of parameters to be considered. Accordingly, most of the collisions studies refer to systems with simple geometry, requiring a number of parameters for dynamical behaviour model as small as possible. Taking into consideration that the impact phenomenon is in essence a mechanical contact accomplished in a short time period, the effect of the contact is strictly local. For this reason, when studying collision phenomena from complex systems, only the local geometry neighbouring potential impact points is established. As a result, is sufficient the study of collision behaviour of two simple bodies having the curvature radii identical to the real contacting surfaces. Complex phenomena, including sudden variation of characteristic parameters, are met in a wide range of engineering applications, from hydrodynamics to cutting edge robotic systems, [5], [6]. When modelling a system, similar to modelling any complex system, problems of optimization, algorithms and advanced computer software occur, [7], [8], [9]. Consequently, a large number of works study the collision phenomena between bodies delimited by plane, spherical or cylindrical surfaces between which take place motions, more or less complex. For this reason, the present paper studies the collision between a freely falling ball and the surface of an inclined plane.

2 Theoretical background

The aim of the paper is the study of collision between a free falling ball and a metallic prism. As the plastic deformations of the plane surface are intended, it was chosen a $\phi = 19\text{mm}$ ball from a ball-bearing and the prism material is OLC-45 carbon steel, as delivered. This pair of materials ensures plastic deformations only for the plane surface.

2.1 Study methods of impact phenomenon

Every impact phenomenon is characterized by existence of two distinctive phases: compression and restitution. The compression phase lasts since the moment the first points of the two bodies are contacting till the instance the normal approach between the two bodies reaches the maximum. The restitution phase begins from the moment corresponding to the maximum approach and lasts till the last points of the two bodies separate from contact. Two main directions in approaching the impact phenomena were outlined in technical literature. The first method considers the impact between any two bodies as an instantaneous event. The method aims to determine the global variations of kinematical and dynamical post-impact parameters of the system, with respect to the kinematical and dynamical parameters of the system before contact beginning. The method is straightforwardly applied, without requiring advanced mathematical knowledge, [10], [11], [12]. The variations of different parameters are expressed using the ratios of the same parameters corresponding to initiation and ending of impact. Among characteristic coefficients, the most important is undoubtedly the coefficient of restitution. A first defining of it is due to Newton, [11], who describes cinematically the coefficient of restitution e as the ratio with changed sign between the normal components of relative velocities of initial contact points.

$$e_{Newton} = -\frac{(\boldsymbol{v}''_1 - \boldsymbol{v}''_2) \cdot \boldsymbol{n}}{(\boldsymbol{v}'_1 - \boldsymbol{v}'_2) \cdot \boldsymbol{n}} \tag{1}$$

In (1), $\boldsymbol{n}$ represents the versor of the normal to the bodies' surfaces. For the present work, the symbol $(\,'\,)$ is used for the parameters characteristic to initial collision time and the symbol $(\,''\,)$ is used for the parameters corresponding to the final moment. The definition (1) for the coefficient of restitution was useful till Kane, [13], analyzing the plane impact with friction for a double pendulum reaches the conclusion that accepting the definition (1) for the coefficient of restitution leads, when the geometry and initial kinematical state are conveniently chosen, to disobedience of the law of conservation of energy. To surpass this intricacy another definition for the coefficient of restitution is required. To this end, the hypothesis of finite continuous variation of impact force must be accepted. The new definition of the coefficient of restitution is due to Poisson, [11]. Thus:

$$e_{Poisson} = \frac{\int_{t_c}^{t_f} \boldsymbol{F} \cdot \boldsymbol{n}\, dt}{\int_{t_i}^{t_c} \boldsymbol{F} \cdot \boldsymbol{n}\, dt} \tag{2}$$

Considering that:

$$\int_0^t \boldsymbol{F} dt = \boldsymbol{P} \tag{3}$$

represents the percussion, Hibbler [14], the definition states that the coefficient of restitution is the ratio between normal percussions corresponding to the restitution and compression phases. There were denoted t_i, t_c t_f the moments corresponding to contact initiation, maximum approach and impact ending, respectively. The second method of collision study accepts from start that during impact, all kinematical and dynamical parameters have a continuous variation. This assumption allows for estimating the values of contact time, maximum impact force and maximum approach. For the simplest impact model, namely the central impact of two elastic spheres, Timoshenko, [15], accepts the force versus normal approach variation following the elastic contact theory:

$$F_{el} = -K\delta^{3/2} \tag{4}$$

where δ is the normal approach and K is a constant, depending on the bodies' geometry and elastic properties. Timoshenko gives the contact time, maximum impact force and normal approach. According to the fundamental law of dynamics, $F = m\ddot{\delta}$ (where the superscript point represents the derivative with respect to time), one can easily observe that even the simplest dynamic system with percussions leads to a nonlinear differential equation. Hunt and Crossley, [16], describe the impact with friction between two bodies using a Kelvin-Voigt model, expressed by a nonlinear differential equation. The main drawback of the model consists in the fact that the hysteresis loop is opened in origin, Fig. 1. According to this model, at the moment of impact ending, the interaction force between the bodies is not of rejection but of attraction. Hunt and Crossley abandoned the model and draw the conclusion that a closed in origin hysteresis loop requires that the force of internal friction is proportional to the elastic force and to the relative approaching velocity. The equation describing the impact with internal friction of two elastic spheres is:

$$m\ddot{\delta} + c\delta^{\frac{3}{2}}\dot{\delta} + K\delta^{\frac{3}{2}} = 0 \tag{5}$$

where c is a constant and K is the same constant as in eq. (4). Lankarani, [17], finds the constant c from energy considerations and applies eq. (6) in the study of multibody systems dynamics. The energy hypothesis used by Lankarani allow for model employment only for quasi elastic bodies. Recently, Flores, [18], amended the Lankarani model and thus it can be applied for any impact type.

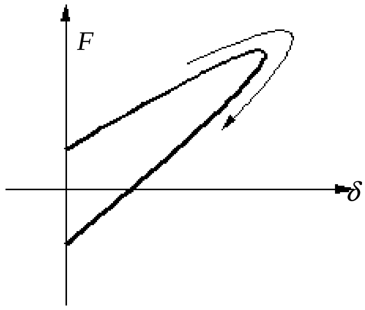

Fig.1 Hysteresis loop for Kelvin-Voigt model

Alves et al. makes a review of viscoelastic models existent in literature, [19]. To be noticed that nonlinear differential equations describe all the models presented in [19]. Another aspect to be mentioned is that the coefficient of restitution appears in the structure of equations describing the dynamic behaviour of a dynamic system with percussions. From here it results the necessity of establishing this parameter as accurate as possible; moreover, as Goldsmith shows, [20], the name of coefficient is inadequate because it depends on velocity and especially for low velocities there is a strong dependence.

2.2 The plane of percussions method (Routh)

The models mentioned above are applicable where the friction force presents a continuous variation with velocity. In the case of dry Coulomb friction, the condition is not obeyed due to the fact that the friction force is characterized via inequalities, the dry friction forces being unilateral constraints, [21]. A reference work in the impact with friction domain is owing to Wang and Mason, [11]. For the study of two-dimensional impact with dry friction, they apply the plane of percussions method, a method extremely intuitive, proposed by Routh, [22]. As a principle, for the two bodies denoted 1 and 2, contacting in point O , an axis system it is defined, with the axes directed along the normal and tangent to the surfaces of the bodies, and the centers of mass of the two bodies, C_1 and C_2 are established according to it. For both bodies, the Newton-Euler dynamic equations, [23], are written and the final kinematical parameters are found, as function of tangential P_t and normal P_n components of interaction percussion, Fig. 2.

Finding the relative velocity of impact points, in the plane of percussions, [22], the geometrical locus of the points where the normal component of the velocity $\boldsymbol{v}_{21}$ is zero is represented by the line of maximum compression (C), Fig.3, and the geometrical locus of the points from percussions plane where the tangential component of the velocity $\boldsymbol{v}_{21}$ is zero is represented by the stiction line (S); in addition to these two straight lines, the straight line of limit friction is traced, defined as:

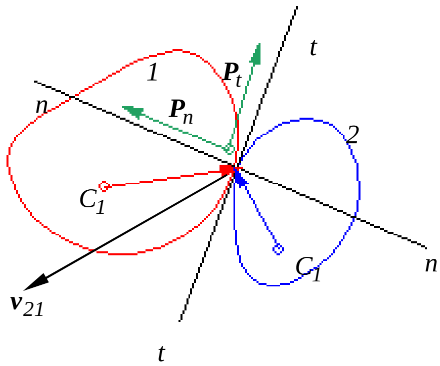

Fig.2 Plane impact with friction for two bodies

$$P_t = \mu P_n \tag{6}$$

and the ending straight line (T), parallel to (C), on which the impact finishes according to Newton's hypothesis. For the study of a plane impact with friction, the hypothesis that during the entire impact process the normal percussion increases monotonically, is made. Thus, at the impact beginning, the characteristic point starts from origin and moves along the limit friction line LF. When the compression line is reached, the approaching phase ends.

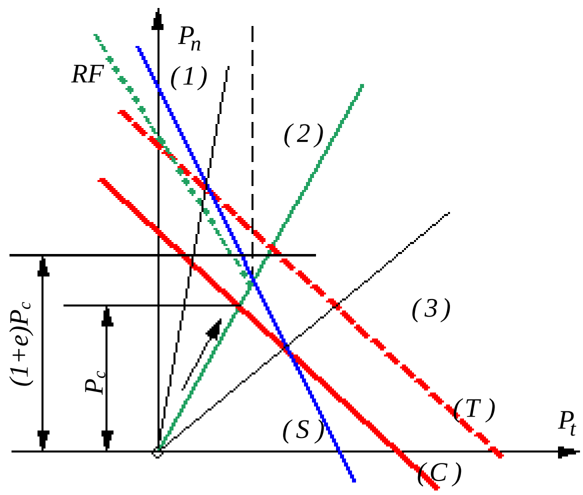

Fig.3 Plane of percussions, [6]

The impact ends when, according to eq. (2), the following relation is fulfilled:

$$P_f = (1+e)P_c \quad (7)$$

The motion state of the system modifies only once, when the characteristic point reaches the stiction line (S). The characteristic point will move along the steepest from the stiction lines (rolling relative motion) or along the reverse friction line (RF) when the sliding reverses its sign. At the intersections between the stiction line and compression line, and stiction line and terminal line, two points are defined, these points, together with the origin describe two half-lines crossing the plane in three regions, 1,2 and 3, Fig.3. Only in the domain 1 the same result is obtained with both definitions of coefficient of restitution.

3 Drawing the Percussions Plane for the Impact between a Dropping Ball and a Tilted Immobile Plane

The Newton-Euler dynamic equations for the collision between a dropping ball and a stationary inclined plane, Fig.4, have the form:

$$\begin{aligned} &m(\boldsymbol{v}_C - \boldsymbol{v}'_C) = \boldsymbol{P}_t + \boldsymbol{P}_n \\ &J_z(\omega - \omega') = \overline{CO} \times (\boldsymbol{P}_t + \boldsymbol{P}_n) \end{aligned} \quad (8)$$

For the first eq. (8), it results two scalar equations:

$$\begin{aligned} &m(v_t - v'_t) = P_t \\ &m(v_n - v'_n) = P_n \end{aligned} \quad (9)$$

The moment of momentum equation presents components only on $Oz_1 \equiv O_z$ axis. Because the components from the left side of second eq. (8) are expressed in the fix reference system and, additionally, $Oz \| Cz_1$, it is convenient to perform the moment of percussion in the fix system:

$$\begin{aligned} &\overline{CO} \times \boldsymbol{P} = -\overline{OC} \times \boldsymbol{P} = \\ &= -(x_c \boldsymbol{i} + y_c \boldsymbol{j}) \times (P_t \boldsymbol{i} + P_y \boldsymbol{j}) \end{aligned} \quad (10)$$

In (10) x_c and y_c are the coordinates of centre of mass of mobile body with respect to the fix system. Writing the moment of inertia of the body using the gyration radius, the moment of momentum theorem with respect to the centre of mass is written as:

$$m\rho^2(\omega - \omega') = y_c P_t - x_C P_n \quad (11)$$

The equations (9) and (11) form a system:

$$\begin{aligned} &m(v_t - v'_t) = P_t \\ &m(v_n - v'_n) = P_n \\ &m\rho^2(\omega - \omega') = y_c P_t - x_C P_n \end{aligned} \quad (12)$$

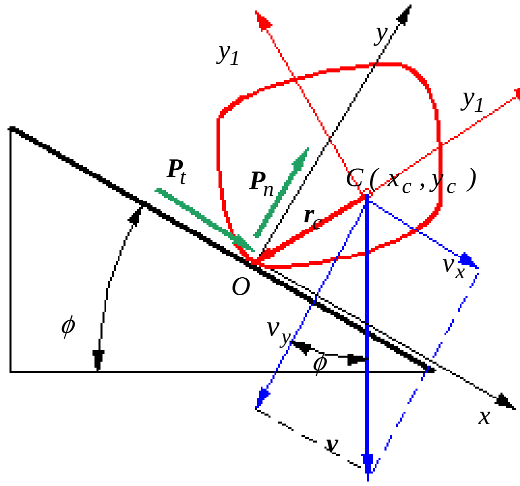

Fig.4 Plane impact between a body and an immobile tilted surface

having the solutions:

$$\begin{cases} v_t = v'_t + P_t / m \\ v_n = v'_n + P_n / m \\ \omega = \omega' + \dfrac{y_c P_t - x_c P_n}{m\rho^2} \end{cases} \quad (13)$$

The eq.(8) are valid for any instant of collision and therefore the relative velocity between the colliding points is:

$$\begin{aligned} &\boldsymbol{v}_{rel} = v_t \boldsymbol{i} + v_n \boldsymbol{j} + \overline{\omega} \times \overline{C0} = \\ &= v_t \boldsymbol{i} + v_n \boldsymbol{j} - \overline{\omega} \times \overline{OC} \end{aligned} \quad (14)$$

Imposing the condition of zero for velocity component along tangential direction and normal direction, respectively, the equation for stiction line is obtained:

$$(\rho^2 + y_C^2)P_t - x_c y_c P_n = m\rho^2(v'_t + \omega' y_c) \quad (15)$$

together to the maximum compression line equation:

$$\begin{aligned} &-x_c y_c P_n + (\rho^2 + x_C^2)P_n = \\ &= m\rho^2(v'_n - \omega' x_c) \end{aligned} \quad (16)$$

The relations (15) and (16) become simpler if the conditions for the particular case analyzed are applied (dropping ball):

$$\rho^2 = \frac{2}{5} r^2; \ \ x_c = 0; \ y_c = r \qquad (17)$$
$$\omega' = 0; \ v'_n = -v' \cos\phi; \ v'_t = v' \sin\phi$$

Consequently, the stiction line has the equation of a vertical line:

$$P_t = -\frac{2}{7} mv' \cos\phi \qquad (18)$$

And the maximum compression line becomes an horizontal line of equation:

$$P_n = mv' \cos\phi \qquad (19)$$

Due to the horizontality of compression line, the terminal line equation has the form:

$$P_n = (1+e) mv' \cos\phi \qquad (20)$$

In order to draw the plane of percussions, one can consider the modulus of tangential percussion in eq. (18). Fig.5 presents the plane of percussions for the case of dropping ball on tilted plane impact. From Fig.5 one can notice that two domains occur, depending on the relation between the friction angle $\beta = atan\,\mu$, and the angle δ, namely smaller or larger than it. The angle δ is defined by:

$$\delta = atan \frac{2}{7} \frac{\tan\phi}{1+e} \qquad (20)$$

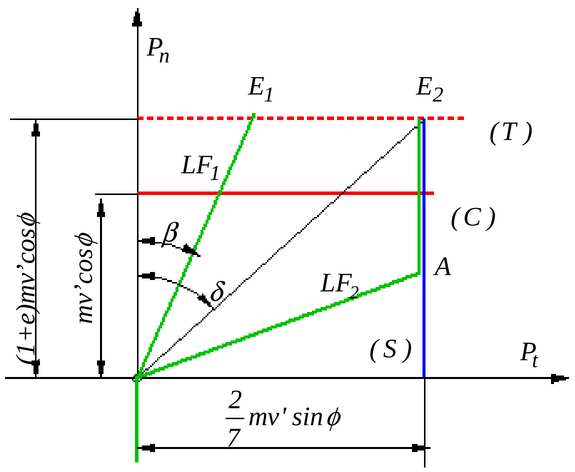

Fig.5 Plane of percussions for the impact of a dropping ball on tilted plane

If $\mu \le \frac{2}{7}\frac{\tan\phi}{1+e}$ the characteristic point moves only along the limit friction line $LF1$ (sliding impact) and the impact ends in the E_1 point having the coordinates:

$$P_{n_1} = (1+e) mv' \cos\phi$$
$$P_{t_1} = -\mu (1+e) mv' \cos\phi \qquad (21)$$

If $\mu > \frac{2}{7}\frac{\tan\phi}{1+e}$ the characteristic point moves along the limit friction line LF_2 (sliding) until it reaches the stiction line in A, after that it moves along the stiction line till the end of impact, in E_2 (rolling motion, reverse sliding being impossible for this case) where the percussions have the values:

$$P_{t_2} = -\frac{2}{7} mv' \sin\phi$$
$$P_{n_2} = (1+e) mv' \cos\phi \qquad (22)$$

Using the values (21) and (22) of final percussions, and the eq. (13), written for particular case (17) of geometrical and kinematical parameters, the final values for the kinematical parameters are found, for the first case:

$$v''_{1t} = [\sin\phi - \mu(1+e)\cos\phi] v'$$
$$v''_{1n} = e \cos\phi \, v' \qquad (23)$$
$$\omega''_1 = -\frac{5}{2}\mu(1+e)\cos\phi \frac{v'}{r}$$

and for the second case, respectively:

$$v''_{2_t} = (5/7) v' \sin\phi$$
$$v''_{2n} = e\cos\phi\, v' \qquad (24)$$
$$\omega''_2 = -(5/7)\sin\phi \frac{v'}{r}$$

4 Experimental Method for Post Impact Parameters Finding

Literature analysis [24], [25], show that for residual prints incidence, the height of free falling of a ball on to a plane surface is $0.0715\text{in} \cong 2\text{mm}$. Goldsmith, [20] and Johnson, [26] also presented this value. Thus is confirmed the fact that conditions of plastic deformation occurrence are quite easy to fulfil. Two aspects are intended:

- finding the velocity variation after impact;
- analysis of remnant indentation geometry.

Wang and Mason, [27], in the study of frictional plane impact from robotics applications, show that for usual forces the presence of Coulomb friction leads to inconsistency of d`ynamics relations, but introducing into analysis the percussions characteristic to collision problems, this drawback is eliminated. The percussions problem occurs in the ball couplings, [28] and also in the design of linkages extra care must be accorded to percussions, when exterior forces are applied by shock, [28], [29], [30]. The scheme from Fig.6 is used to establish the velocity variation subsequent to collision. The ball, $R = 9.5mm$ radius, is launched on vertical direction and at the moment of reaching the surface of inclined plane, its centre have been travelled the distance h. At the instant of impact between the ball and the plane, the centre of the ball is in the point O_1. After impact, the centre of the ball has the velocity v_0'' and its direction makes an angle α with the horizontal line. After impact, the centre of the ball will follow a parabolic trajectory To study the motion of the ball, a reference system is attached, the origin in coincidence to the centre of the ball in the moment it reaches the inclined plane, the axes being oriented to the horizontal and vertical direction. The two purposed parameters, the v_0'' velocity and the α angle, can be found only if, as it will be revealed subsequently, the magnitude and orientation of vector $\boldsymbol{l} = \overrightarrow{O_1O_2}$ in Oxy system can be précised. To this end, as experimental data is registered, beside the flight period, the distance $L_1 = |\overline{O_1O_2}|$, by fixing the tips of a pair of compasses in the points A_1 and B_1 respectively.

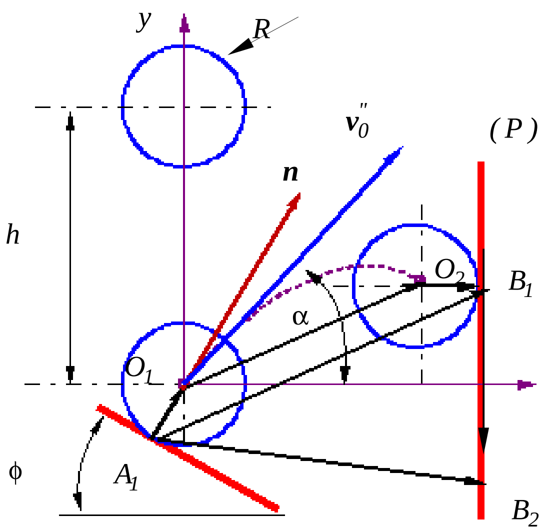

Fig.6 Diagram for finding post-collision velocity and angle

Additionally, on the vertical line of the B_1 point is set a B_2 point, at an imposed distance:

$$H = |\overline{B_1B_2}| \tag{25}$$

Alaci et al, [31] applies for positional analysis the Hartenberg-Denavit method for a spatial linkage, obtaining analytical results. The same method could be applied here, but for avoiding cumbersome calculus, a particular method, specific to planar linkages is adopted. The length of line segment B_1B_2 is measured using the pair of compasses. In the coordinate system, a vector $\boldsymbol{v}$ is précised by its magnitude v and the angle it makes with the positive Ox semiaxis. The equation:

$$\overline{A_1B_1} + \overline{B_1B_2} = \overline{A_1B_2} \tag{26}$$

have to be solved, and the method of vector contour analysis is applied, [32]. The vector equation (26) projected on the axes leads to the following system:

$$\begin{cases} L_1 \cos\varphi_1 + H\cos\psi = L_2\cos\varphi_2 \\ L_1 \sin\varphi_1 + H\sin\psi = L_2\sin\varphi_1 \end{cases} \tag{27}$$

where L_1, L_2, H and $\varphi_1, \varphi_2, \psi$ are size and orientation angles of vectors $\overline{A_1B_1}$, $\overline{A_1B_2}$ and $\overline{B_1B_2}$ respectively. The ψ angle takes only two values:

$$\psi = \pm\frac{\pi}{2} \tag{28}$$

depending on the position of the point B_1, above or bellow B_2 point B_2, respectively. By solving system (27), the values of the angles φ_1 and φ_2 are found; from the multiple solutions, the ones corresponding to actual situation ought to be chosen. Applying the vector contour analysis for the $A_1 - O_1 - O_2 - B_1 - A_1$ contour, the following system of scalar equations is obtained:

$$\begin{cases} R\cos\left(\dfrac{\pi}{2} - \phi\right) + \ell_2\cos\varphi_2 + R - L_1\cos\varphi_1 = 0 \\ R\sin\left(\dfrac{\pi}{2} - \phi\right) + \ell_2\sin\varphi_2 + R - L_1\sin\varphi_1 = 0 \end{cases} \tag{29}$$

The unknowns from system (29) are the characteristics of $\overline{O_1O_2}$ vector, ℓ_2 and φ_2, that connects the centres of the R radius ball at the instants of beginning and ending of flight.

$$\begin{cases} \ell_2 = \sqrt{[R(1+\sin\phi) - L_1\cos\varphi_1]^2 + (R\cos\phi - L_1\sin\varphi_1)^2} \\ \varphi_2 = -\operatorname{asin}\dfrac{R\cos\phi - L_1\sin\varphi_1}{\ell_2}, \end{cases} \tag{30}$$

Using the values (30), the Cartesian coordinates of O_2 point at the impact with vertical plate result:

$$x_0 = \ell_2\cos\varphi_2;\ \ y_0 = \ell_2\sin\varphi_2 \tag{31}$$

By imposing the condition that the point $O_2(x_0, y_0)$ should verify the equation of the trajectory of a point launched with v_0'' velocity in gravitational field under α'' angle with horizontal, [33], the following expression is obtained:

$$y_0 = x_0\tan\alpha_0'' - \frac{g}{2v_0''^2\cos^2\alpha_0''}x_0^2 \tag{32}$$

Adding the following relation:

$$x_0 = v_0'' t_z \cos\alpha_0'' \tag{33}$$

that describes the horizontal displacement of the centre of the ball, a system of equations is obtained, that allows founding the post impact velocity and launching angle:

$$\begin{cases} v_0'' = \dfrac{1}{t_z}\sqrt{x_0^2 + \left(y_0 + \dfrac{1}{2}gt_z^2\right)^2} \\ \alpha_0'' = \operatorname{atan}\dfrac{2y_0 + gt_z^2}{2x_0} \end{cases} \tag{34}$$

The experimental device from the laboratory is simple, consisting in a launcher that ensures tests repeatability, a system for fixing the prism and an electronic precision bench level, Fig.7, with precision of 0.1 degrees, for establishing the prism's tilt. The prisms should fulfil two conditions: the active surface must be fine polished and the active face and its opposite have to be strictly parallel, required by the laser scanning process. For finding the v_0'' velocity and the α angle, a vertical plane is placed in the path of the ball. Indigo paper is arranged both onto the inclined plane and the vertical one, for precise identification of points A_1 and B_1. The flight time between the instants of contact with the inclined plane and the vertical plane respectively is measured for finding the v_0'' velocity and the α angle.

(a)

(b)

Fig.7 Adjusting the tilt angle ϕ of the prism (a) and the launching height of the ball, h, (b)

The precise flight period is obtained by recording the acoustic signals of impact and converting them to image, using specialised software, Fig.8.

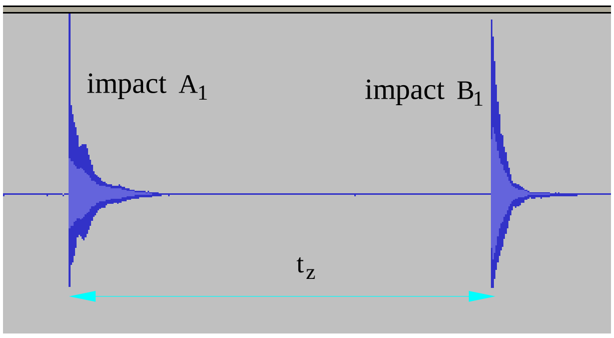

Fig.8 Finding the flight time of the ball between the collisions with the two planes

The impact instants can be clearly evidenced as shock waves, [34] and [35]. Experiments were carried out for different tilt angles and the data are synthesized in Table 1. These experimental data are used in applying the relations presented above, considering that the impact velocity v_0' is given by:

$$v_0' = \sqrt{2gh} \tag{35}$$

As a result, the $\alpha_0^{''}$ angle and the post-impact velocity are found.

Table 1.

Experiment	1	2	3
ϕ	26^o	34.1^o	45.6^o
t_z [sec]	0.110	0.083	0.054
A_1B_1 [m]	0.130	0.128	0.105
A_1B_2 [m]	0.180	0.159	0.123
B_1B_2 [m]	0.100	0.100	0.100
ψ	90^o	90^o	90^o
h	0.290	0.287	0.280

From Fig.9 it can be noticed that the coefficient of restitution, normal, e_n and tangential, e_t, respectively, can be determined:

$$e_n = \frac{v_0^{''} cos(\pi/2-\phi-\alpha_0^{''})}{v_0^{'} cos\phi} = \frac{v_0^{''} sin(\phi+\alpha_0^{''})}{v_0^{'} cos\phi} \quad (36)$$

$$e_t = \frac{v_0^{''} sin(\pi/2-\phi-\alpha_0^{''})}{v_0^{'} sin\phi} = \frac{v_0^{''} cos(\phi+\alpha_0^{''})}{v_0^{'} sin\phi} \quad (37)$$

The values found for the parameters $v_0^{'}$, $v_0^{''}$, $\alpha_0^{''}$, $\pi/2-\phi-\alpha_0^{'}$, e_n, e_t are presented in Table 2.

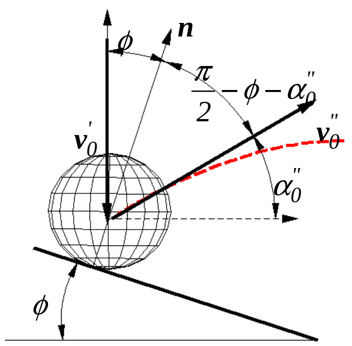

Fig.9 Finding the coefficients of restitution

Table 2.

Experiment	1	2	3
ϕ	26^o	34.1^o	45.6^o
$v_0^{'}$ [m/s]	2.408	2.396	2.366
$v_0^{''}$ [m/s]	1.259	1.386	1.615
$\alpha_0^{''}$	35.021^o	10.554^o	-14.307^o
$\pi/2-\phi-\alpha_0^{'}$	28.997^o	45.346^o	58.707^o
e_n	0.509	0.491	0.507
e_t	0.578	0.734	0.816

As it can be observed, after impact, the reflex angle of the ball is greater than the incident angle. An interesting aspect can be remarked analyzing the plastic print of the impact. To this end, the plastic indentation of a collision corresponding to the angle $\phi = 26^o$ was analysed using NANOFOCUS laser scan profilometer. The longitudinal profile of a plastic indent is presented in Fig.10.

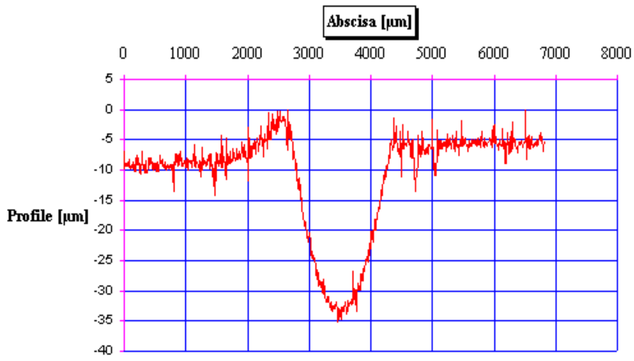

Fig.10 Profile of plastic print on logitudinal direction for $\phi = 26^o$

5 Conclusions

The paper presents a method for finding the velocity of the centre of a metallic ball falling on an inclined fixed plane. The main features of the methodology are straightforwardness and promptness. The proposed method was applied for three values of surface tilt angle and the coefficients of restitution, normal and tangential, respectively, are found. The studied collision is a plane impact with friction and the coefficients of restitution found using the pre and post impact velocities might be altered, requiring instead the Poisson definition. The laser scan analysis of the plastic indentation profile shows the existence of a symmetry axis normal to the impact plane and not associated to the velocity direction, as would be expected. The paper aims to be a beginning point for the comprehensive analysis of collision with friction between a ball and an inclined plane, with the following objectives: finding ball angular velocity, plastic deformation work and internal friction work.

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