Decentralized Stability Analysis of Interconnected Systems with Time Varying Delays

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Abstract: - In this paper, we present a decentralized stability problem for a class of linear interconnected systems with time-varying delay in the state of each subsystems and in the interconnections. Based on the Lyapunov method, we characterize decentralized linear matrix inequalities (LMI) based delay-dependent stability conditions such that every local subsystem of the linear interconnected delay system is asymptotically stable. The solutions of the LMIs can be obtained easily using efficient convex optimization techniques. A practice example is given in order to show the efficiency of the obtained result.

Key-Words: - Interconnected systems; Time-delay systems; Delay-dependent stability; Lyapunov method; linear matrix inequality (LMI).

1 Introduction

In this paper, we study the stability analysis of a class of linear interconnected systems with time varying delays. The problem of stability analysis of time varying delay systems is important both in theory and in practice. Considerable attention has been devoted to this problem over the past years, and many research results have been reported in the literature [1-10]. The approach used to derive the stability condition of systems starts usually from the standard Lyapunov-Krasovskii functional [15-18].

This work is concerned with the design problem of decentralized stability for interconnected systems with Time Varying Delays. The delay parameter is assumed to be an unknown time-varying function for which the upper bound on the magnitude and the variation are given. The sufficient conditions for the stability of the interconnected systems is derived in terms of LMIs using the Lyapunov method.

The seat of this work is arranged as following: in section 2, an overview of system models have been provided. stability analysis of interconnected system with time varying delays has been introduced in section 3. In section 4, we present numerical example to show the usefulness of the proposed results. Finally, the paper is concluded by brief conclusion in section 5.

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Notation: In this paper, the notation $P \succ 0 (\prec 0)$ is used for positive (negative) definite matrices. I denotes the identity matrix with appropriate dimension. $*$ stands for the symmetric term of a square symmetric matrix.

2 Problem Formulation

Consider a class of linear large-scale systems with time-varying delays composed of N interconnected subsystems, where the ith subsystem is given by:

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + A_{di} x_i(t-\tau_i(t)) + \sum_{j=1,\, j\neq i}^{N} A_{ij} x_j(t-\eta_{ij}(t)), \\ z_i(t) &= C_i x_i(t) + C_{di} x_i(t-\tau_i(t)) \end{aligned} \tag{1}$$

where $i, j \in \{1, ..., N\}$

$x^T(t) = \left[x_1^T(t), ..., x_N^T(t) \right]$,

$z^T(t) = \left[z_1^T(t), ..., z_N^T(t) \right]$, $x_i(t) \in \Re^{n_i}$ and $x_j(t) \in \Re^{n_j}$ are the states of the ith and the jth subsystem, $z_i(t) \in \Re^{q_i}$ is the controlled output, the system matrices A_i, A_{di}, A_{ij}, C_i and C_{di} are of appropriate dimensions. τ_i, η_{ij} are unknown time delay factors satisfying the following conditions:

$$\begin{aligned} &0 \le \tau_i(t) \le \rho_i, \quad \dot{\tau}_i(t) \le \mu_i \\ &0 \le \eta_{ij}(t) \le \rho_{ij}, \ \dot{\eta}_{ij}(t) \le \mu_{ij} \end{aligned} \tag{2}$$

where the bounds ρ_i, ρ_{ij}, μ_i and μ_{ji} are known constants in order to guarantee smooth growth of the state trajectories.

The class of systems described by (1) subject to delay pattern (2) is frequently encountered in modeling several physical systems and engineering applications including large space structures, multi-machine power systems, transportation systems, water pollution management [19].

Proposition 1 For any $x, y \in \Re^n$ and positive definite matrix $P \in \Re^{n\times n}$, we have

$$2x^T y \le y^T P y + x^T P^{-1} x \tag{3}$$

Proposition 2 (Schur complement lemma [11]) Given constant matrices X, Y, Z with appropriate dimensions satisfying $X = X^T, Y = Y^T \succ 0$. Then $X + Z^T Y^{-1} Z \prec 0$ if and only if

$$\begin{pmatrix} X & Z^T \\ Z & -Y \end{pmatrix} \prec 0 \quad \text{or} \quad \begin{pmatrix} -Y & Z \\ Z^T & X \end{pmatrix} \prec 0 \tag{4}$$

Proposition 3 [10] For any constant matrix $Z = Z^T \succ 0$ and scalar $h \succ 0$, the following integrations are well defined

$$-\int_{t-h}^{t} x^T(s) Z x(s)\, ds \le -\frac{1}{h}\left(\int_{t-h}^{t} x(s) ds\right)^T Z \left(\int_{t-h}^{t} x(s) ds\right) \tag{5}$$

We end this section with the following technical well-known propositions, which will be used in the proof of the main results.

3 Stability Analysis

In this section, we investigate the decentralized stability analysis of interconnected systems with time-varying delays. Before introducing the main result, the following notations of several matrix variables are defined for simplicity

$$\pi_{i1} = P_i A_i + A_i^T P_i + (N-1)P_i + Q_i + \sum_{j=1,\, j\neq i}^{N} Z_{ji} - W_i$$

$$\pi_{i2} = -(1-\mu_i)Q_i$$

$$\vartheta_{i1} = P_i A_i + A_i^T P_i + (N-1)P_i + Q_i + \sum_{j=1,\, j\neq i}^{N} Z_{ji} + (N-1)\rho_i^2 A_i^T W_i A_i - W_i$$

$$\vartheta_{i2} = -(1-\mu_i)Q_i + (N-1)\rho_i^2 A_{di}^T W_i A_{di}$$

$$\vartheta_{i3} = -\sum_{j=1,\, j\neq i}^{N} (1-\mu_{ji})Z_{ji} + 2\sum_{j=1,\, j\neq i}^{N} \rho_j^2 A_{ji}^T W_j A_{ji} + \sum_{j=1,\, j\neq i}^{N} A_{ji}^T P_j A_{ji}$$

$$\Psi_{i1} = P_i A_i + A_i^T P_i + (N-1)P_i + Q_i + \sum_{j=1,\, j\neq i}^{N} Z_{ji} + N\rho_i^2 A_i^T W_i A_i - W_i$$

$$\Psi_{i2} = -(1-\mu_i)Q_i + N\rho_i^2 A_{di}^T W_i A_{di}$$

$$\Psi_{i3} = -\sum_{j=1,\, j\neq i}^{N} (1-\mu_{ji})Z_{ji} + 2\sum_{j=1,\, j\neq i}^{N} \rho_j^2 A_{ji}^T W_j A_{ji} + \left(\sum_{j=1,\, j\neq i}^{N} A_{ij}\right)^T \rho_j^2 W_j \sum_{j=1,\, j\neq i}^{N} A_{ij} + \sum_{j=1,\, j\neq i}^{N} A_{ji}^T P_j A_{ji}$$

(6)

The following is the main result of the paper, which gives sufficient conditions for the decentralized stability of interconnected systems with time-varying delays. Essentially, the proof is based on the construction of Lyapunov Krasovskii functions satisfying the Lyapunov stability theorem for a time delay system [11].

Theorem 1 Given $\rho_i \succ 0$, $\mu_i \succ 0$ and $\mu_{ji} \succ 0$, the system (1) is asymptotically stable if there exist symmetric positive definite matrices P_i, Q_i, Z_{ij} and W_i, $i, j = 1,\dots,N$, $i \neq j$, such that the following LMI holds:

$$
\begin{bmatrix}
\pi_{i1} & P_i\ A_{di} & 0 & W_i & 0 & \rho_i^2 A_i^T W_i & (N-1)\rho_i^2 A_i^T W_i & 0 \\
A_{di}^T P_i & \pi_{i2} & 0 & 0 & 0 & \rho_i^2 A_{di}^T W_i & 0 & (N-1)\rho_i^2 A_{di}^T W_i \\
0 & 0 & \vartheta_{i3} & 0 & \left(\sum_{j=1,\, j\neq i}^{N} A_{ij}\right)^T \rho_j^2 W_j & 0 & 0 & 0 \\
W_i & 0 & 0 & -W_i & 0 & 0 & 0 & 0 \\
0 & 0 & \rho_j^2 W_j \left(\sum_{j=1,\, j\neq i}^{N} A_{ij}\right) & 0 & -\rho_j^2 W_j & 0 & 0 & 0 \\
\rho_i^2 W_i\ A_i & \rho_i^2 W_i\ A_{di} & 0 & 0 & 0 & -\rho_i^2 W_i & 0 & 0 \\
(N-1)\rho_i^2 W_i\ A_i & 0 & 0 & 0 & 0 & 0 & -(N-1)\rho_i^2 W_i & 0 \\
0 & (N-1)\rho_i^2 W_i\ A_{di} & 0 & 0 & 0 & 0 & 0 & -(N-1)\rho_i^2 W_i
\end{bmatrix} \prec 0 \tag{7}
$$

Proof

We consider the following Lyapunov-Krasovskii functional for system (1):

$$V(t) = \sum_{i=1}^{N} V_i(t) = \sum_{i=1}^{N} \left[V_{ai}(t) + V_{bi}(t) + V_{ci}(t) + V_{di}(t)\right] \tag{8}$$

where

$$
\begin{aligned}
V_{ai}(t) &= x_i^T(t) P_i\ x_i(t) \\
V_{bi}(t) &= \int_{t-\tau_i(t)}^{t} x_i^T(s) Q_i\ x_i(s)\, ds \\
V_{ci}(t) &= \sum_{j=1,\, j\neq i}^{N} \int_{t-\eta_{ij}(t)}^{t} x_j^T(s) Z_{ij} x_j(s)\, ds \\
V_{di}(t) &= \rho_i \int_{-\rho_i}^{0} \int_{t+s}^{t} \dot{x}_i^T(\alpha) W_i\ \dot{x}_i(\alpha)\, d\alpha\, ds
\end{aligned} \tag{9}
$$

Taking the derivative of V in t along the solution of system (1), we have

$$
\begin{aligned}
\dot{V}_{ai}(t) &= 2x_i^T(t) P_i\ \dot{x}_i(t) = 2x_i^T(t) P_i\ \Big[A_i x_i(t) \\
&\quad + A_{di} x_i(t-\tau_i(t)) + \sum_{j=1,\, j\neq i}^{N} A_{ij} x_j(t-\eta_{ij}(t))\Big]
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
\dot{V}_{bi}(t) &\le x_i^T(t) Q_i\ x_i(t) \\
&\quad - (1-\mu_i) x_i^T(t-\tau_i(t)) Q_i\ x_i(t-\tau_i(t))
\end{aligned} \tag{11}
$$

$$
\begin{aligned}
\dot{V}_{ci}(t) &= \sum_{j=1,\, j\neq i}^{N} \Big[x_j^T(t) Z_{ij} x_j(t) \\
&\quad -(1-\dot{\eta}_{ij}(t)) x_j^T(t-\eta_{ij}(t)) Z_{ij} x_j(t-\eta_{ij}(t))\Big] \\
&\le \sum_{j=1,\, j\neq i}^{N} \Big[x_j^T(t) Z_{ij} x_j(t) \\
&\quad - (1-\mu_{ij}) x_j^T(t-\eta_{ij}(t)) Z_{ij} x_j(t-\eta_{ij}(t))\Big]
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
\dot{V}_{di}(t) &= \rho_i \int_{-\rho_i}^{0} \Big[\dot{x}_i^T(t) W_i\ \dot{x}_i(t) - \dot{x}_i^T(t+s) W_i\ \dot{x}_i(t+s)\Big] ds \\
&= \rho_i^2\ \dot{x}_i^T(t) W_i\ \dot{x}_i(t) - \rho_i \int_{-\rho_i}^{0} \dot{x}_i^T(t+s) W_i\ \dot{x}_i(t+s) ds \\
&= \rho_i^2\ \dot{x}_i^T(t) W_i\ \dot{x}_i(t) - \rho_i \int_{t-\rho_i}^{t} \dot{x}_i^T(s) W_i\ \dot{x}_i(s) ds \\
&\le x_i^T(t) \rho_i^2 A_i^T W_i\ A_i x_i(t) \\
&\quad + x_i^T(t) \rho_i^2 A_i^T W_i\ A_{di} x_i(t-\tau_i(t)) \\
&\quad + 2x_i^T(t) \rho_i^2 A_i^T W_i \sum_{j=1,\, j\neq i}^{N} A_{ij} x_j(t-\eta_{ij}(t)) \\
&\quad + x_i^T(t-\tau_i(t)) \rho_i^2 A_{di}^T W_i\ A_i x_i(t)
\end{aligned}
$$

$$+ x_i^T(t-\tau_i(t))\rho_i^2 A_{di}^T W_i \ A_{di} x_i(t-\tau_i(t))$$
$$+ 2\, x_i^T(t-\tau_i(t))\rho_i^2 A_{di}^T W_i \sum_{j=1,\, j\neq i}^{N} A_{ij} x_j(t-\eta_{ij}(t))$$
$$+ \sum_{j=1,\, j\neq i}^{N} x_j^T(t-\eta_{ij}(t))\rho_i^2 A_{ij}^T W_i \sum_{j=1,\, j\neq i}^{N} A_{ij} x_j(t-\eta_{ij}(t))$$
$$-\rho_i \int_{t-\rho_i}^{t} \dot{x}_i^T(s) W_i \ \dot{x}_i(s) ds \tag{13}$$

Applying Proposition 3 and the Leibniz-Newton formula

$$\int_{t-h}^{t} \dot{x}_i(s)\, ds = x_i(t) - x_i(t-h) \tag{14}$$

we obtain

$$-\rho_i \int_{t-\rho_i}^{t} \dot{x}_i^T(s) W_i \ \dot{x}_i(s) ds \leq$$
$$-\left(\int_{t-\rho_i}^{t} \dot{x}_i^T(s)\, ds\right)^T W_i \left(\int_{t-\rho_i}^{t} \dot{x}_i(s) ds\right)$$
$$= -\left(x_i(t) - x_i(t-\rho_i)\right)^T W_i \left(x_i(t) - x_i(t-\rho_i)\right)$$
$$\leq -\left(x_i^T(t) W_i \ x_i(t) - 2\, x_i^T(t) W_i \ x_i(t-\rho_i)\right)$$
$$\left. + x_i^T(t-\rho_i) W_i \ x_i(t-\rho_i)\right) \tag{15}$$

Using Proposition 1 gives

$$2 x_i^T(t) P_i \sum_{j=1,\, j\neq i}^{N} A_{ij} x_j(t-\eta_{ij}(t))$$
$$= \sum_{j=1,\, j\neq i}^{N} 2\left[P_i \ x_i(t)\right]^T \left[A_{ij} x_j(t-\eta_{ij}(t))\right]$$
$$\leq (N-1) x_i^T(t) P_i \ x_i(t)$$
$$+ \sum_{j=1,\, j\neq i}^{N} x_j^T(t-\eta_{ij}(t)) A_{ij}^T P_i \ A_{ij} x_j(t-\eta_{ij}(t)) \tag{16}$$

$$2 x_i^T(t) A_i^T W_i \sum_{j=1,\, j\neq i}^{N} A_{ij} x_j(t-\eta_{ij}(t))$$
$$= \sum_{j=1,\, j\neq i}^{N} 2\left[W_i \ A_i x_i(t)\right]^T \left[A_{ij} x_j(t-\eta_{ij}(t))\right]$$
$$\leq (N-1) x_i^T(t) A_i^T W_i \ A_i x_i(t)$$
$$+ \sum_{j=1,\, j\neq i}^{N} x_j^T(t-\eta_{ij}(t)) A_{ij}^T W_i \ A_{ij} x_j(t-\eta_{ij}(t)) \tag{17}$$

$$2 x_i^T(t-\tau_i(t)) A_{di}^T W_i \sum_{j=1,\, j\neq i}^{N} A_{ij} x_j(t-\eta_{ij}(t))$$
$$= \sum_{j=1,\, j\neq i}^{N} 2\left[W_i \ A_{di} x_i(t-\tau_i(t))\right]^T \left[A_{ij} x_j(t-\eta_{ij}(t))\right]$$
$$\leq (N-1) x_i^T(t-\tau_i(t)) A_{di}^T W_i \ A_{di} x_i(t-\tau_i(t))$$
$$+ \sum_{j=1,\, j\neq i}^{N} x_j^T(t-\eta_{ij}(t)) A_{ij}^T W_i \ A_{ij} x_j(t-\eta_{ij}(t))$$
$$\leq (N-1) x_i^T(t-\tau_i(t)) A_{di}^T W_i \ A_{di} x_i(t-\tau_i(t))$$
$$+ \sum_{j=1,\, j\neq i}^{N} \left(A_{ij} x_j(t-\eta_{ij}(t))\right)^T W_i \sum_{j=1,\, j\neq i}^{N} A_{ij} x_j(t-\eta_{ij}(t)) \tag{18}$$

Now Nothing that

$$\sum_{i=1}^{N} \sum_{j=1,\, j\neq i}^{N} x_j^T(t) Z_{ij} x_j(t) = \sum_{i=1}^{N} \sum_{j=1,\, j\neq i}^{N} x_i^T(t) Z_{ji} x_i(t) \tag{19}$$

$$\sum_{i=1}^{N} \sum_{j=1,\, j\neq i}^{N} (1-\mu_{ij}(t)) x_j^T(t-\eta_{ij}(t)) Z_{ij} x_j(t-\eta_{ij}(t))$$
$$= \sum_{i=1}^{N} \sum_{j=1,\, j\neq i}^{N} (1-\mu_{ji}(t)) x_i^T(t-\eta_{ji}(t)) Z_{ji} x_i(t-\eta_{ji}(t)) \tag{20}$$

$$\sum_{i=1}^{N} \sum_{j=1,\, j\neq i}^{N} \rho_i^2 x_j^T(t-\eta_{ij}(t)) A_{ij}^T W_i \ A_{ij} x_j(t-\eta_{ij}(t))$$
$$= \sum_{i=1}^{N} \sum_{j=1,\, j\neq i}^{N} \rho_j^2 x_i^T(t-\eta_{ji}(t)) A_{ji}^T W_j \ A_{ji} x_i(t-\eta_{ji}(t)) \tag{21}$$

$$\sum_{i=1}^{N} \sum_{j=1,\, j\neq i}^{N} x_j^T(t-\eta_{ij}(t)) A_{ij}^T P_i \ A_{ij} x_j(t-\eta_{ij}(t))$$
$$= \sum_{i=1}^{N} \sum_{j=1,\, j\neq i}^{N} x_i^T(t-\eta_{ji}(t)) A_{ji}^T P_j \ A_{ji} x_i(t-\eta_{ji}(t)) \tag{22}$$

$$\sum_{i=1}^{N}\left[\sum_{j=1,\, j\neq i}^{N} \left(A_{ij} x_j(t-\eta_{ij}(t))\right)^T \rho_i^2 W_i \sum_{j=1,\, j\neq i}^{N} A_{ij} x_j(t-\eta_{ij}(t))\right] =$$
$$\sum_{i=1}^{N}\left[x_i^T(t-\eta_{ji}(t)) \left(\sum_{j=1,\, j\neq i}^{N} A_{ij}\right)^T \rho_j^2 W_j \sum_{j=1,\, j\neq i}^{N} A_{ij} \ x_i(t-\eta_{ji}(t))\right] \tag{23}$$

Therefore, we have

$$\begin{aligned}
\dot{V}(t) \le \sum_{i=1}^{N} \Big[& 2x_i^T(t)P_i\ A_i x_i(t) + 2x_i^T(t)P_i\ A_{di}x_i(t-\tau_i(t)) \\
& + (N-1)x_i^T(t)P_i\ x_i(t) \\
& + \sum_{j=1,\, j\neq i}^{N} x_i^T(t-\eta_{ji}(t))A_{ji}^T P_j\ A_{ji}x_i(t-\eta_{ji}(t)) \\
& + x_i^T(t)Q_i\ x_i(t) - (1-\mu_i)x_i^T(t-\tau_i(t))Q_i\ x_i(t-\tau_i(t)) \\
& + \sum_{j=1,\, j\neq i}^{N} x_i^T(t)Z_{ji}x_i(t) \\
& - \sum_{j=1,\, j\neq i}^{N} (1-\mu_{ji})x_i^T(t-\eta_{ji}(t))Z_{ji}x_i(t-\eta_{ji}(t)) \\
& + x_i^T(t)\rho_i^2 A_i^T W_i\ A_i x_i(t) + x_i^T(t)\rho_i^2 A_i^T W_i\ A_{di}x_i(t-\tau_i(t)) \\
& + (N-1)\rho_i^2 x_i^T(t)A_i^T W_i\ A_i x_i(t) \\
& + \sum_{j=1,\, j\neq i}^{N} \rho_j^2 x_i^T(t-\eta_{ji}(t))A_{ji}^T W_j\ A_{ji}x_i(t-\eta_{ji}(t)) \\
& + x_i^T(t-\tau_i(t))\rho_i^2 A_{di}^T W_i\ A_i x_i(t) \\
& + x_i^T(t-\tau_i(t))\rho_i^2 A_{di}^T W_i\ A_{di}x_i(t-\tau_i(t)) \\
& + (N-1)\rho_i^2 x_i^T(t-\tau_i(t))A_{di}^T W_i\ A_{di}x_i(t-\tau_i(t)) \\
& + \sum_{j=1,\, j\neq i}^{N} \rho_j^2 x_i^T(t-\eta_{ji}(t))A_{ji}^T W_j\ A_{ji}x_i(t-\eta_{ji}(t)) \\
& + x_i^T(t-\eta_{ji}(t))\left(\sum_{j=1,\, j\neq i}^{N} A_{ij}\right)^T \rho_j^2 W_j \sum_{j=1,\, j\neq i}^{N} A_{ij}\ x_i(t-\eta_{ji}(t)) \\
& - \left(x_i^T(t)W_i\ x_i(t) - 2x_i^T(t)W_i\ x_i(t-\rho_i)\right) \\
& - x_i^T(t-\rho_i)W_i\ x_i(t-\rho_i) \Big] \\
= \sum_{i=1}^{N} & \xi_i^T(t)\,\Xi_i\,\xi_i(t)
\end{aligned} \tag{24}$$

where

$$\xi_i^T(t) = \begin{bmatrix} x_i^T(t) & x_i^T(t-\tau_i(t) & x_i^T(t-\eta_{ji}(t) & x_i^T(t-\rho_i) \end{bmatrix}$$

$$\Xi_i = \begin{bmatrix} \Psi_{i1} & P_i A_{di} + \rho_i^2 A_i^T W_i A_{di} & 0 & W_i \\ A_{di}^T P_i + \rho_i^2 A_{di}^T W_i A_i & \Psi_{i2} & 0 & 0 \\ 0 & 0 & \Psi_{i3} & 0 \\ W_i & 0 & 0 & -W_i \end{bmatrix}$$

we readily see that $\dot{V}(t) \prec 0$ holds if

$$\Xi_i \prec 0 \tag{25}$$

using proposition 2 (Schur complements), inequality (25) is equivalent to

$$\begin{bmatrix} \Psi_{i1} & P_i\ A_{di} + \rho_i^2 A_i^T W_i\ A_{di} & 0 & W_i & 0 \\ A_{di}^T P_i\ + \rho_i^2 A_{di}^T W_i\ A_i & \Psi_{i2} & 0 & 0 & 0 \\ 0 & 0 & \vartheta_{i3} & 0 & \left(\sum_{j=1,\, j\neq i}^{N} A_{ij}\right)^T \rho_j^2 W_j \\ W_i & 0 & 0 & -W_i & 0 \\ 0 & 0 & \rho_j^2 W_j \left(\sum_{j=1,\, j\neq i}^{N} A_{ij}\right) & 0 & -\rho_j^2 W_j \end{bmatrix} \prec 0 \tag{26}$$

Also, inequality (26) is equivalent to

$$\begin{bmatrix} \vartheta_{i1} & P_i\,A_{di} & 0 & W_i & 0 \\ A_{di}^T P_i & \vartheta_{i2} & 0 & 0 & 0 \\ 0 & 0 & \vartheta_{i3} & 0 & \left(\sum_{j=1,\,j\neq i}^{N} A_{ij}\right)^T \rho_j^2 W_j \\ W_i & 0 & 0 & -W_i & 0 \\ 0 & 0 & \rho_j^2 W_j \left(\sum_{j=1,\,j\neq i}^{N} A_{ij}\right) & 0 & -\rho_j^2 W_j \end{bmatrix} + \begin{bmatrix} \rho_i^2 A_i^T W_i \\ \rho_i^2 A_{di}^T W_i \\ 0 \\ 0 \\ 0 \end{bmatrix} \left(\rho_i^2 W_i\right)^{-1} \begin{bmatrix} \rho_i^2 W_i\,A_i & \rho_i^2 W_i\,A_{di} & 0 & 0 & 0 \end{bmatrix} \prec 0 \quad (27)$$

we obtain

$$\begin{bmatrix} \vartheta_{i1} & P_i\,A_{di} & 0 & W_i & 0 & \rho_i^2 A_i^T W_i \\ A_{di}^T P_i & \vartheta_{i2} & 0 & 0 & 0 & \rho_i^2 A_{di}^T W_i \\ 0 & 0 & \vartheta_{i3} & 0 & \left(\sum_{j=1,\,j\neq i}^{N} A_{ij}\right)^T \rho_j^2 W_j & 0 \\ W_i & 0 & 0 & -W_i & 0 & 0 \\ 0 & 0 & \rho_j^2 W_j \left(\sum_{j=1,\,j\neq i}^{N} A_{ij}\right) & 0 & -\rho_j^2 W_j & 0 \\ \rho_i^2 W_i\,A_i & \rho_i^2 W_i\,A_{di} & 0 & 0 & 0 & -\rho_i^2 W_i \end{bmatrix} \prec 0 \quad (28)$$

it can readily verified that the condition of (28) is equivalent to the LMI (7), this establishes the internal asymptotic stability.

Remark 1. Theorem 1 presents a new stability criterion for system (1) with time-varying delay. It is worth noting that condition (7) is an LMI, which can be readily checked by using the standard numerical software.

To illustrate the application of the proposed method, we present the following example.

4 Numerical Example

Consider a large-scale system, which is composed of three subsystems, each is of the type (1) with:

$$A_1 = \begin{bmatrix} -2 & 0 \\ -2 & -1 \end{bmatrix},\ A_{d1} = \begin{bmatrix} -1 & 0 \\ -1 & 0 \end{bmatrix},$$

$$A_{12} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix},\ A_{13} = \begin{bmatrix} 0 & -1 \\ 0 & -1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -1 & 0 \\ -2 & -2 \end{bmatrix},\ A_{d2} = \begin{bmatrix} 1 & 0 \\ -2 & -1 \end{bmatrix},$$

$$A_{21} = \begin{bmatrix} -1 & -2 \\ 3 & 6 \end{bmatrix},\ A_{23} = \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -1 & 0 \\ -1 & -2 \end{bmatrix},\ A_{d3} = \begin{bmatrix} 0 & 0 \\ -1 & -2 \end{bmatrix},$$

$$A_{31} = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix},\ A_{32} = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}$$

By applying Theorem 1 and solving the corresponding optimization problem (7), we obtain

$\rho_1 = 3$; $\mu_1 = 1.5$; $\rho_{21} = 2$; $\mu_{21} = 0.8$;

$\rho_{31} = 2$; $\mu_{31} = 0.8$; $\rho_2 = 32.5$; $\mu_2 = 1.3$;

$\rho_{12} = 1.5$; $\mu_{12} = 0.9$; $\rho_{32} = 1.5$; $\mu_{32} = 0.9$;
$\rho_3 = 3$; $\mu_3 = 1.1$; $\rho_{13} = 1.8$; $\mu_{13} = 0.75$;
$\rho_{23} = 1.8$; $\mu_{23} = 0.75$;

$$P_1 = \begin{bmatrix} 2.2511 & 0.0020 \\ * & 0.0362 \end{bmatrix}; \; P_2 = \begin{bmatrix} 0.1386 & -0.0152 \\ * & 0.0724 \end{bmatrix};$$

$$P_3 = \begin{bmatrix} 1.8296 & 0.1004 \\ * & 0.5422 \end{bmatrix}; \; Q_1 = \begin{bmatrix} 1.39501 & 0.0576 \\ * & 20.0181 \end{bmatrix};$$

$$Q_2 = \begin{bmatrix} 5.9043 & 0.3326 \\ * & 5.4613 \end{bmatrix}; \; Q_3 = \begin{bmatrix} 21.3065 & -2.5036 \\ * & 28.6509 \end{bmatrix};$$

Since $P_i, Q_i, \succ 0$, $i = 1,2,3$
Then the conditions required by Theorem 1 are satisfied.

5 Conclusion

In this paper, the problem of the decentralized stability for large-scale time varying delay systems has been studied. The time delay is assumed to be a function belonging to a given interval. By effectively combining an appropriate Lyapunov functional with the Newton-Leibniz formula, this paper has derived new delay-dependent conditions for the stability in terms of linear matrix inequalities (LMIs). Numerical examples are given to show the effectiveness of the obtained result.

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