Diagnosis and Sensor Placement for a Gas lift well

RUBEN LEAL[1], JOSE AGUILAR[2], LOUISE TRAVÉ-MASSUYÈS[3],
EDGAR CAMARGO[1], ADDISON. RÍOS[2]
[1]Automation Department, [2]CEMISID, [3]CNRS, LAAS
[1]PDVSA, [2]Universidad de los Andes, [3]Université de Toulouse
[1]La Salina, Av. Hollywood, Cabimas 4013, [2]Av. Alberto Carnevalli La Hechicera, Mérida 5101,
[3]7 avenue du colonel Roche, F-31400 Toulouse
[1,2]VENEZUELA, [3]FRANCE
lealrs@pdvsa.com, aguilar@ula.ve, louise@laas.fr, camargoea@pdvsa.com, ilich@ula.ve

Abstract: - A major contribution to supervision is to provide reliable diagnosis results reporting about the change of health status of the components of a system. An important related property, which allows one to provide guarantees about which faults are discriminable by the diagnoser, is diagnosability. Diagnosability analysis for production systems examines detectability (which faulty behaviors are discriminable from the normal behavior of the system) and isolability (which faulty behaviors are discriminable from which). Interestingly, diagnosability analysis provides the foundations for sensor placement, indicating parts of the system that suffer poor observability. In this paper, structural analysis is proposed as an efficient tool for diagnosability analysis and sensor placement, and applied to a real-world process of extraction of oil by gas injection.

Key-Words: Diagnosability, Detectability, Isolability, Sensor Placement, Structural Analysis, Extraction Of Oil.

1 Introduction

Monitoring and control systems are important tools for industry as they allow optimizing the use of infrastructure. In automation systems, supervisory functions are used to indicate undesirable or unplanned states of the processes, and take appropriate actions to resume operation, and avoid damage or accidents.

Most processes for the production of oil and gas require constant supervision and control of the associated facilities (wells, flow stations, multiple injections of gas lift, etc.). In particular, early detection and diagnosis of faults is mandatory to have enough time to cross out the fault propagations that could result in failures. Some of the possible operations to counteract the presence of a fault are reconfiguration, maintenance or repair actions. Early detection can be achieved by acquiring information on the process, then using mathematical models that provide analytical redundancy when evaluated with measured quantities. Moreover, for fault diagnosis it is important to use cause-effect relationships [1].

A typical diagnosis system must be able to detect and isolate a set of faults using the measurements acquired at several test points in the system.

In the oil extraction industry, it is essential to achieve maximal production, for which the diagnostic process must be improves and streamlined. This study considers one of the most used methods for the artificial extraction of oil, which relies on wells with gas injection [2]. Oil wells based on gas injection with highly oscillatory flow, are a major problem in the oil industry. The efforts to find low-cost solutions based on automatic control and fault diagnosis are carried out in both the academic and the industrial communities for a long time, [3,4,5,6]. In the literature, there are several diagnostic studies related to pipelines, storage tanks and wells, but previous works do not include diagnosability analysis in gas lift wells.

This paper addresses the problem of analyzing diagnosability in gas lift wells, as the first step towards analyzing the diagnosability of an overall oil production process. The approach that we propose is based on structural models of the process.

The study is carried out in two steps: first, we analyze the diagnosability property using structural analysis and sequential residual generation techniques for the process of oil extraction with the current measured variables, i.e. with the sensors that are actually installed in the process. Second, we draw the conclusions of the previous analysis to recommend new sensors, which are indicated as providing the required observability to detect and isolate specific faults that are just “invisible” in the actual process.

The main contribution of the paper is to provide the insights for applying advanced diagnosability analysis methods based on structural analysis [7], to a complex real-world process of the oil and gas industry, and to show their value for designing efficient diagnosers and the required instrumentation. We believe that this study can be of great value both for the academic community that worked out the theoretical foundations and for the gas industry that can benefit from the case study of the paper.

The paper is organized as follows. Section 2 reviews some important concepts and the methods used in structural analysis, residual generator and a sensor placement analysis. Section 3 is a diagnosability analysis for a gas lift well and some results based in current instrumentation, and then a sensor placement analysis to increase the observability to detect and isolate faults that are invisible today. Finally Section 4 is the conclusion and future works.

2 Model Based Diagnosis

A failure means any change in the behavior of any of the components of the system, so that it can no longer fulfill the function for which it was designed [8]. Notice that a fault may be present without the system failing.

Fault diagnosis can be viewed as a process to detect the presence of faults through abnormal behaviors in the process, and locate and identify the component(s) or cause that originated the abnormality. There are three objectives to reach [8].

- Fault detection
- Fault isolation.
- Identification and estimation of the fault.

The *Fault detectability* is the ability to detect the faults. The diagnosis must be able to decide if there is a fault or not, as well as to determine the instant of the apparition, from observations of the process. For that, model-based diagnosis proposes to compare the actual behavior with the expected behavior predicted with a model. *Fault isolation* is the ability to isolate a fault that has occurred, from the other faults. The *identification* of the fault is based on the consistency between the normal behavior and the current process behavior, obtained from measurements, in order to determine the depth and magnitude of the fault.

There are several methodologies for obtaining the relationship between faults and the diagnosis of the system. There are methodologies that are based on expert knowledge, or experience of the past erroneous behaviors. With this information, it is possible to obtain a representation based in diagnostic rules or structures.

Other methods are based on learning approaches and pattern recognition, which are a very useful tool when intended to diagnose systems for which it is very hard to obtain an analytical model. In this case, learning methods can identify the relations between the symptoms to the faults. With these techniques, the diagnosis is achieved by comparing the evolution of the system to diagnose with the classes that have been learned. The techniques used to make this diagnosis can be: vector support machines, fuzzy logic, among others [9,10].

One of the advantages of machine learning techniques is that they avoid the problems associated with the development of an explicit model, but their main disadvantage is the need of a set of training data, usually large, which is often not available in real systems. So, very often it is necessary proceed to simulations. Another disadvantage of these systems applied to the diagnosis is the inability to recognize situations for which they have not been trained.

The other approach that is most advocated for dynamic systems is the model based diagnosis (MBD). In MBD, the fundamental aspects are the definition of a process model and the analysis of the behavior of the system components against the behavior predicted with the model [11]. The comparison is performed thanks to generation of residuals.

There are several technics for the generation of residuals, but all consist in the measurements, if the observed situation does not meet the estimate made by the model for a given situation, then it is concluded that there is a fault, and further analysis of the differences are carried out to identify the specific component of the fault. A way to generate residuals, and which is used in this paper, is based on analytical redundancy relations (ARR) [12]. It uses analytical mathematical models that characterize the system, to reproduce the behavior of the components and system under evaluation.

The approach for generating a residual is based on a finite sequence of calculations that ends with the evaluation of an analytically redundant equation. Similar approaches have been described and exploited in [8]. The ARR only contains measured or known variables, and is composed of a subset of equations from the model. ARRs allows us to check whether the measured variables are consistent with the model.

A residual is a signal ideally zero in the non-faulty case and non-zero otherwise. A residual generator takes measurement variables from the system as input, and produces a residual as output. The method for residual generation presented in this paper relies on structural analysis and sequential generation [1,12,13].

The residual generation approaches have in common that the sub-systems should be over-determined to include the required redundancy. Several algorithms for calculating ARRs from over-determined systems have been proposed in [1,13,14] proposes an algorithm that analyzes the structure of a system to detect all the redundancies. The causality establishing the sequence to generate the residuals is left for a post-processing unlike [12].

Diagnosability depends on the residuals that can be generated, as it depends on the redundancy embedded in the system. Decoupling of faults in a set of tests based on residuals, means that the residuals must be sensitive to, or respond to different subsets of faults. Thus, decoupling of faults is a fundamental problem in choosing the residuals that will compose the diagnose.

2.1 Structural Analysis

Structural analysis is a set of tools to explore the fundamental properties of a system using a structural model, either in the form of a graph or an incidence matrix. In our work, we rely on these tools to achieve residual generation and diagnosability analysis.

2.1.1 Structural model

A structural model is a representation of a system in which only couplings between variables and equations are retained [7]. The structural model contains only the information of which variable belongs to which equation, regardless of the value of the parameters and the detailed form of the mathematical expression [13].

A structural model can be represented by a bipartite graph or an incidence matrix. Let's call this model is *M(X,Z,E,F)*, where *E* is a set of equations $E = \{e_1,\ldots, e_m\}$, *X* is a set of unknown variables $X = \{x_1, \ldots, x_n\}$, *Z* is a set of known variables $Z=\{z_1,\ldots,z_p\}$. and $F=\{f_1,\ldots,f_o\}$ is a set of fault parameters which modify the normal behaviour of the system (they are considered as unknown variables). In the case of a differential model, it is necessary to add a fifth set, $D = \{\dot{x}_1, \ldots, \dot{x}_n\}$, which contains the derivatives of the variables of *X*.

We assume that the equations in the set *E* have the form.

$$e_i: h_i(\dot{x}, x, z, f) = 0,\ 1 \le i \le m \quad (1)$$

Where, $\dot{x}$, *x*, *F* and *z* are vectors of the sets *D*, *X*, *F* and *Z*, respectively.

Example 1: consider.

$$e_1: \dot{x}_1 = x_1 + u + f_1 \quad (2)$$
$$e_2: y_2 = x_1 \quad (3)$$
$$e_3: y_1 = \dot{x}_1 \quad (4)$$

The structural model of the system is as follows: $e = \{e_1, e_2, e_3\}$, $X = \{x_1, \dot{x}_1\}$, $Z = \{y_1, y_2, u\}$, $F = \{f_1\}$

$$e_1 = h_1(x_1, \dot{x}_1, u, f) \quad (5)$$
$$e_2 = h_2(y_2, x_1) \quad (6)$$
$$e_3 = h_3(y_1, \dot{x}_1) \quad (7)$$

The Fig.1, and Table 1, are the bipartite graph and incidence matrix representation for this example

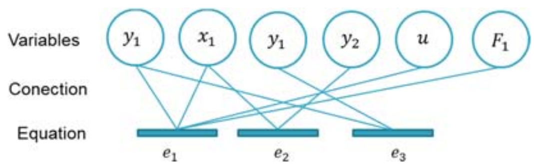

Fig.1.Bipartite Graphs of the example

Table 1, Incidence Matrix.

Parameter	***Unknown Variables***		***Faults***	***Know Variables***		
	$\dot{x}_1$	x_1	F_1	y_1	y_2	u
e_1	1	1	1	0	0	1
e_2	0	1	0	0	1	0
e_3	1	0	0	1	0	0

Now, we present some definitions to use the structural analysis for diagnosis purposes.

Definition 1: ARR for M(X,Z,E,F). Let M(X,Z,E,F) be a model, then an equation $\mathrm{r_i}: \mathrm{b}(\mathrm{z}, \dot{\mathrm{z}}, \ddot{\mathrm{z}}, \ldots) = 0$ is an ARR for M(E,X,Z,F), if for each z consistent with M(E,X,Z,F), the equation is fulfilled [1].

These relationships can be derived only if the model has more equations than unknown variables, i.e. if the system is structurally over-determined (SO) [13].

Example 2: According to example 1, an ARR would be:

$$Arr: y_1 - y_2 - u = 0 \quad (8)$$

An ARR can be used to check if the observed variables *z* are consistent with *M(E,X,Z,F),* and can be used as the basis of a residual generator.

Definition 2: **Residual Generator for *M(E,X,Z,F).*** A system taking a subset of the variables *z* as input, and generating a scalar signal r_i as output, is a residual generator for the model *M(E,X,Z,F)*, if for all *z* consistent with *M(E,X,Z,F)*, it hold that $\lim_{t\to\infty} r_i(t) = 0$ [1].

Example 3: According to example 1, a residue would be structurally formulated as:

$$e_1: b(e_3(y_1); e_2(y_2); u) = 0 \quad (9)$$

The structure of the system can be abstracted as a representation of which variables are involved in the different equations which make up the model of the system. The structural model of a system is an abstraction that allows one to study the diagnosability properties, independently of the linear or nonlinear nature of the systems. However, it must be kept in mind that results obtained with such a structural representation are a best case scenario. Causality considerations and the presence of algebraic and differential loops, determine which structural redundancies can be exploited for the design of residual generators.

Each r_i should be evaluated in order to decide if it can be used or not. Finally, the evaluation of each detection test constitutes the fault signature vector ($S = \{S_1, \ldots, S_n\}$), that is a set of vector in order to isolate the fault.

Given a set of vector $S = \{S_1, \ldots, S_n\}$ and a set of faults $F=\{f_1,\ldots,f_o\}$, the theoretical fault signature matrix can be defined codifying the effect of every fault in a residual [1]

Definition 3: **The fault signature matrix of *M*.** It is a table obtained by the concatenation of all possible signatures of faults. Each row corresponds to an *ARR* and each column to a failure mode. A "1" in position (*ij*), indicates that the fault *j* is detected by the *ARR i* [13].

F(M) is the set of faults that affect either equation in *M*, then the detectability is achieved if it complies with the following definition.

Definition 4: Detectability for *M(E,X,Z,F)*. A fault F_o, where $o = 1,\ldots,n$ which belongs to *F(M)* in the diagnosis system of *M,* is detectable if there is a residue different from zero in the residual generator, i.e. $r_i \neq 0$.

Definition 5: Isolability for *M(E,X,Z,F)*. When two signatures are identical, the related faults are considered non-decoupled, that mean they cannot be isolated [9]. Therefore, all signatures must be different from each other $S(f_o) \neq S(f_t), \forall o,t \in \{1,..,n\}, o \neq t$ The fault isolation will consist in looking for the theoretical fault signature in the fault matrix that matches with the observed signature, to distinguish between all the possible faults.

Example 4: Consider a diagnosis system containing a set of residuals $\{ARR_1, ARR_2, ARR_3\}$constructed to detect and isolate three faults $\{f_1, f_2, f_3\}$. The following fault signature matrix shows the sensitivity of ARRs to faults even in the system in normal behaviour *N*. Arr_1 is sensitive to faults f_2 and f_3, and so on. Each fault has a different signature, so we can isolate all considered faults

Table 2, Fault signature matrix.

	N	f_1	f_2	f_3
Arr_1	0	0	1	1
Arr_2	0	1	0	1
Arr_3	0	1	1	0

2.1.2 ARR Generation

In this section we present some concepts linked to the FDI techniques, and ARR paradigm. The procedure consists in the generation of residual [12]. They can be obtained by previously designed diagnostic models, which allow the fault detection and isolation [1].

We adopt the design method of minimal structurally over determined (MSO) sets based on ARR, [13]. Unobserved variables can be eliminated for the subset of equations.

Definition 6: Over-determined System (SO). *M* is an SO if the cardinality in *E* is greater that the cardinality in *X* i.e.$|E| > |X|$.

If the cardinality of M are equal $|E| = |X|$ is a Just-determined system and if smaller $|E| < |X|$is a Under-determined system. A condition that must be satisfied is that must be at least one more equation than unknown variables, which means that the system is over-determined.

Definition 7: Minimal Structurally Over-determined (MSO). A MSO contains only one equation more than unknown variables, and each MSO is equivalent to ARR [13].

In [1] provides an algorithm that identifies the MSO, enabling the construction of more efficient ARRs. Each ARR correspond to an MSO.

In [14] introduced an algorithm and the notion of TES (Test Equation Support) which are sets of equations which express redundancy specific to a set of considered faults. Each TES corresponds to a set of faults which influence the residual generator constructed from the TES. The corresponding quantities expressing minimal redundancies are denoted minimal TES (MTES), and the set of MTES can be seen as a subset of the set of MSOs for the set of faults of interest of the system.

Definition 8: Test Equation Support (TES): Given a model M and a set of faults $F(M)$, an equation set M is a test equation support if M is a SO set, and if $F(M')$ correspond a part of the model, being $F(M)\neq 0$ and for any $M' \subseteq M$, where M' is a SO set it holds that $F(M') \subseteq F(M)$.

Definition 9: Minimal Test Equation Support (MTES): A MTES of M is a minimal TES if it is the smaller subset sensitive to a fault, more equation than unknown variables.

Since there is a one-to-one correspondence between MTESs and ARR, we will only focus on MTESs in this paper, to generate residuals of the process. With that, we will be able build a signature matrix. A MTES set could be used to develop a consistency check for a part of the system, and a set of $\boldsymbol{F(M)}$ can be detected with this consistency check [14].

2.1.3 Sensors placement based on structural diagnosability analysis

The efficiency of a diagnosis system depends on the information it can retrieve from the diagnosed plant. Obviously, if the information is insufficient, the diagnosis system is not able to perform its task. Thus, the efficiency of a sensors system can be measured by the diagnosability degree it provides. However, the same diagnosability level may be obtained for different sensor configurations.

The sensors placement problem is solved here using a structural representation of the model as it is used in [15]. Without loss of generality, it is possible to assume that a single fault can only violate one equation. If a fault signal f appears in more than one equation, we simply replace f in the equations with a new variable f_x and add equation $f = f_x$ which will then be the only equation violated by this fault. Let $e_f \in M$ be the equation that might be violated by a fault $f_x \in F$. For the example 1, $f_1 = e_1$.

A fault f is detectable if there exist an observation that is consistent with fault mode f and inconsistent with the no-fault mode. This means that a detectable fault can violate a monitorable equation in the model. [8].

In a structural model of the system where there are faults that cannot be detected, we can add sensors to increase the observability of the system. We assume that a sensor y_{p+1} measuring x_n has been added to the process and included in the model by e_{m+1}: $y_{p+1} = x_n$.

In this way, the undetected faults can be detectable faults, and a residual is capable of detecting them because the redundancy of the system is increased.

A general assumption of the approach is that the model does not contain any underdetermined part. It is also assumed that a fault affects no more than one equation and that each possible sensor measures one unknown variable. Due to space limitations all theoretical aspect, theorems and the algorithm use to solve this problem based on structural analysis, can be found in [15].

A goal of the algorithm is to find minimal sensor sets that achieve structural detectability and isolability of faults in a just-determined set of equations. In this order, we can determine which part exactly of the model becomes over-determined when a sensor is added.

Let M be an exactly determined set of equations, F the corresponding set of faults, $P \subseteq X$ the set of possible sensor locations, and M_s the equation set describing the additional sensors set and P_s the additional sensors in M_s [16].

The above described can be summarized in an algorithm that, given a model M, faults F, and a set of possible sensor locations P, computes D the detectability and isolability, increasing the over-determined part of the model. In this algorithm, ϒ is the minimal additional sensor set such that the faults become detectable and isolable in $M_e = M' \cup M_s$, where M_e is the extended part of the model that includes the sensors. The main algorithm proposed in [16] is summarized as following.

function Υ= Sensorplacement (M, F, P)
1 $\Upsilon = \emptyset$;
2 M^0 = just-determined part of M;
3 F^0 = the set of faults F included in M^0;
4 D = Detectability (M^0, F^0, P);
5 Υ_d = minimalsensorset(D);
6 **for** $\Upsilon_i \in \Upsilon_d$
7 create the extended model $M_e = M \cup M_s$;
8 F_e = the faults included in M_e;
9 D = Isolability (M_e, F_e, P);
10 Υ_d = minimalsensorset (D);
11 $\Upsilon = \Upsilon \cup \{\Upsilon_i \uplus \Upsilon' \mid \Upsilon' \in \Upsilon_i\}$;
12 **end**
13 Delete nonmininal sensor sets in Υ;

In line 2, the algorithm checks de just-determined part of M, and in the line 3 it determines all the faults in M^0. Then, it checks the detectability of M^0, where Υ_d is the minimal sensors set to reach detectability in M^0. Lines 6 and 12 describe the procedure to add new sensors, creating M_e as the new model with the new sensors and F_e the faults included in this model. The algorithm determines D the isolability property and the required minimal sensor set. That is repeated for all the possible sensor placements in order to reach the isolability property of M_e. The idea is to obtain a minimal number of sensors to diagnose the faults F_e.

In [17] is introduced a sensor placement based in genetic algorithm and MTES theory to verify the functionality of it, and also is proven in a gas lift well, comparing both methodology to a complete analysis of diagnosability.

3 Analysis of diagnosability for a gas lift well

In this section we present the method of production of oil by gas injection, the structural analysis of the process, and then apply our approach for fault diagnosis.

3.1 Oil extraction by gas lift injection

According to [5], the method of extraction of oil by gas injection is a method using compressed gas as energy source, for carrying the reservoir fluids from downhole to the surface; thus, the main consideration to select a group of oil wells, is the availability of a cost-effective source of high pressure gas (Fig.2, shows the components of this method).

In [6] indicates that gas lift is one of the primary methods used in the production of fluids from a well, which consists in the continuous injection of high pressure gas, to lighten the oil column in the production tubing. In other words, this method involves injecting gas at high pressure (through the compressor plant) at a preset rate, to lighten the column of oil, and thus improving the production of wells with reservoir pressure lower than the head at different depths. It is considered by experts as the most similar to the natural flow [18].

Gas from the annulus starts to flow into the tubing, as the gas enters into the tubing the pressure in the tubing falls, accelerating the inflow of gas lift. The gas pushes the major part of the liquid out of the tubing, while the pressure in the annulus falls dramatically. The annulus is practically empty, and the gas flow into the tubing is blocked by liquid accumulating in the tubing. Due to the blockage, the tubing becomes filled with liquid and the annulus with gas. Eventually, the pressure in the annulus becomes high enough for gas to penetrate into the tubing, and a new cycle starts.

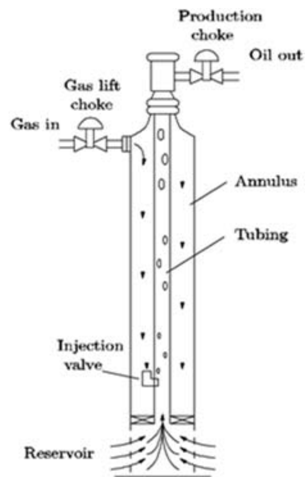

Fig.2, An Artificial lift of oil by gas injection

In general, gas lift has been used effectively to achieve the following objectives

- Starting wells producing by natural flow.
- Increase production from wells that have naturally declined, but still produced without using artificial methods.

In our approach, the diagnosability is developed based in residuals generation schemes derived from ARRs. Firstly, with the mathematical analysis described in this section, we identify equations governing the process defined in previous works

[4,5]. They are based on three state variables: x_1 is the mass of gas in the annulus, x_2 is the mass of gas in the tubing, and x_3 is the mass of oil in the tubing. With them, we can define the first equations set which represents the dynamics of the flow of each variable defined previously.

$$\dot{x_1} = w_{gc} - w_{iv} \quad (10)$$
$$\dot{x_2} = w_{iv} - w_{pg} \quad (11)$$
$$\dot{x_3} = w_r - w_{po} \quad (12)$$

Where w_{gc} is the mass flow rate of gas lift into the annulus, w_{iv} is the mass flow rate of gas lift from the annulus into the tubing, w_{pg} is the mass flow rate of gas through the production choke, w_r is the oil mass flow rate from the reservoir into the tubing, w_{po} is the mass flow rate of gas through the production choke, and w_{pc} is a mixed mass flow rate produced through the production choke, The flows are modelled by.

$$w_{gc} = C_{iv}\sqrt{(P_{plp} + 14.7).P_{gldp}} + f_1 \quad (13)$$
$$w_{iv} = C\sqrt{p_{a,inj}.(P_a - P_{t,inj})} \quad (14)$$
$$w_{pc} = C\sqrt{p_m.(P_t - P_s)} + f_2 \quad (15)$$
$$w_{pc} = \frac{x_2}{x_2+x_3}.w_{pc} \quad (16)$$
$$w_{pc} = \frac{x_3}{x_2+x_3}.w_{pc} \quad (17)$$
$$w_{pc} = C_r.(P_r - P_{t,b} \quad (18)$$

Unlike the model presented in [4,5], w_{gc} is not considered as constant but rather calculated through the equation of flow by orifice plate denote by *e4*. This way, is checking the consumption of gas in each well. Also we consider two more variables: P_{glp} is the pressure in the system distribution of gas, and P_{gldp} is the differential pressure of gas through an orifice plate.

To comply with the technique described in Section 3 and in [15], to detect and isolate faults that are really relevant for us, is necessary associate the faults in the equation where occurs. For this reason, f_1 and f_2 are added to the equations of the model, f_1 is added in equation 13*,* which is the failure in the flow of gas injected into the annular, and f_2 is added in equation 15 which is the failure in the mixed flow to the separator in the production line.

C_{iv}, C and C_r are constants, $\rho_{a,inj}$ is the density of gas in the annulus at the injection point, ρ_m is the density of the oil/gas mixture at the top of the tubing, P_a is the pressure in the annulus at the injection point, $P_{t,inj}$ is the pressure in the tubing at the gas injection point, P_t is the pressure at the top of the tubing, P_s is the pressure at the separator, P_r is pressure in the reservoir, and $P_{t,b}$ is the pressure at the bottom of the tubing. The reservoir pressure, P_r, is assumed to be slowly varying, and therefore treated as constant. Note that flow rates through the valves are restricted to be positive. The densities are modeled as follows:

$$p_{a,inj} = \frac{M}{RT_a}.P_a \quad (19)$$
$$p_m = \frac{x_2+x_3-\rho_O L_r A_r}{L_t A_t} \quad (20)$$
$$P_a = \left(\frac{RT_a}{M_o} + \frac{gL_a}{V_a}\right) \quad (21)$$
$$P_t = \frac{M}{RT_a}.\frac{x_2}{L_r A_r + L_t A_t - x_3 v_o} \quad (22)$$
$$P_{t,inj} = P_t + \frac{g}{A_r}.(x_2 + x_3 \quad (23)$$
$$P_{t,b} = P_{t,inj} + \rho_o g L_r \quad (24)$$

M_o is the molar weight of the gas, R is the gas constant, T_a is the temperature in the annulus, T_t is the temperature in the tubing, V_a is the volume of the annulus, L_a is the length of the annulus, L_t is the length of the tubing, A_t is the cross-sectional area of the tubing above the injection point, L_r is the length from the reservoir to the gas injection point, A_r is the cross-sectional area of the tubing below the injection point, g is the gravity constant, ρ_o is the density of the oil, and, v_o is the specific volume of the oil.

In practice, measurements of downhole variables are not available or are unreliable by nature. So, in this work to make an analysis of diagnosability we assumed that measurements are taken only at the top of the well, in the annular and the tubing, in order to determine the two failures. The major challenge is how to deal with multiphase flow in the tubing, while the single-phase flow in the annulus is measured accurately based on the pressure and temperature measured.

We also know that x_1 is calculated. The remaining two variables, the pressure at the top of the tubing and the production flow through the throttle are measured. We may also obtain pressure in the production tubing or separator because a control system maintains the pressure at the inlet of the separator in the flow station. The known variables, measured and calculated are therefore the subset *F(y).*The set of equations from 25 to 33 are the relationships between the measures variables in the model, and their behaviour is used as inputs for the residual generator to obtain the ARRs.

$$y_1 = x_1 \quad (25)$$

$$y_2 = P_{ai} \quad (26)$$
$$y_3 = P_t \quad (27)$$
$$y_4 = P_s \quad (28)$$
$$y_5 = P_{glp} \quad (29)$$
$$y_6 = P_{gldp} \quad (30)$$
$$y_7 = P_r \quad (31)$$
$$y_8 = P_m \quad (32)$$
$$y_9 = w_c \quad (33)$$

In order to evaluate the results, we used the previous equations that govern the system [18] and made a model of the well in Simulink. In this model are also included elements to simulate the faults that we wish to be detect, The Fig.3, shows the normal behaviour of the well. The red line with the highest amplitude is the amount of oil in the tubing, the blue line is the amount of gas in the annulus, and finally, the green line is the amount of gas in the tubing, Table 3, shows the values of the coefficients involved in the process.

Table 3, Numeral Coefficient's.

Parameter	Value	Unit
M_o	0.028	Kg/mol
R	8.31	J/Kmol
G	9.81	m/s^2
T_a	293	K
L_a	0.907	M
V_a	22.3×10^{-3}	m^3
ρ_o	1,000	Kg/m^3
p_r	2.9×10^5	Pa
L_r	14	M
A_r	0.314×10^{-3}	m^2
C_{iv}	1.60×10^{-6}	m^2
C_{pc}	0.156×10^{-3}	m^2
C_r	12×10^{-6}	m^2
C_r	12×10^{-6}	m^2

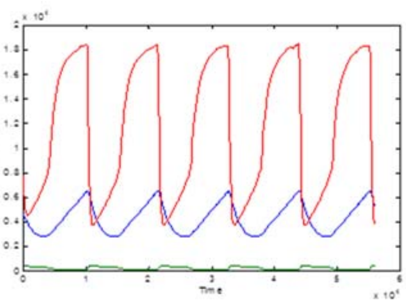

Fig.3, Well behaviour in the normal state

3.2 Analysis of diagnosability in extractions of oil by gas lift injection

When searching for residuals, it is good to know which properties make to a residual generator good. There are a number of factors that can be taken in to consideration, particularly the detectability and fault isolation.

According to section 2 detection is achieved if there are residues sensitive to the considered faults of the system. We calculate the MTES using the algorithm proposed in (Krysander, Åslund and Frisk, 2010), based on actual observable variables.

3.1.1 Diagnosis analysis for f1 and f2 based on current instrumentation

As in [17] Fig.4, shows the original incidence matrix of the structural analysis for gas lift well. It indicates the relationship between the variables and equations that characterize the system. The algorithm proposed in [13] returns as result two MTES's, each sensitive to one of the two faults. Table 4 indicates the signature of each fault.

In this case, to fulfill *definition 9*, the minimum subset of equations where the analytical relation can be found characterizing each fault, are shown below

$MTES_1$ =[1, 4, 5, 10, 11, 12, 13, 14, 16, 17, 18, 20, 21, 23, 24]
$MTES_2$=[1, 2, 5, 6, 7, 10, 11, 12,13, 14, 16, 17, 18, 19, 23, 24]

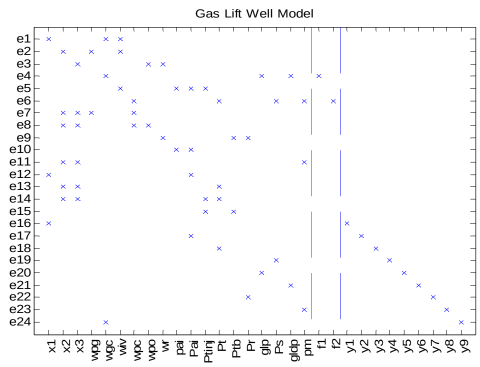

Fig.4, Incidence matrix

We have used a sequential residual generation to obtain ARRs from MTES proposed in [6]. The unknown variables in each subset of the model can be computed by solving equations, one at time, in a sequence. Then, the residuals are obtained.

$$ARR_1 = x_1 + c_{iv}\sqrt{\frac{M}{RT_a}.P_a(P_a + gL_tP_m)} - w_{gc} \quad (34)$$

$$ARR_2 = -\frac{\left[\frac{d}{dt}(x_2)-(w_{gc}-x_1)\right].(P_mL_tA_t+L_rA_r\rho_o)}{x_2} - c_{pc}\sqrt{P_m(P_t - P_s)} \quad (35)$$

Where

$$x_2 = \frac{((1-P_mV_o)L_tA_t+(1-V_o\rho_o)L_rA_r)}{RT_t-MV_oP_t} \quad (36)$$

The importance of our proposition to reach diagnosability in this case study is that we used only the surface variables of the well, without the need to place sensors on the bottom, those being very expensive and highly sensitive to damage.

Table 4, Fault signature matrix of the system under study.

	N	f_1	f_2
Arr_1	0	1	0
Arr_2	0	0	1

We have tested the behavior of the system when the fault occur and in normal state (see Fig. 5 to 8). Fig. 5 shows the residue analysis in normal state of the system; we can see that the ARRs can be evaluated zero.

Fig. 6 shows the behavior of ARR_1 versus ARR_2 when f_1 occurs, which is a malfunction in the flow rate of gas lift into the annulus, f_1 is introduced into the space time of 19000 until 23000 seconds, the blue line is the fault simulated introduced to the system while the red line is the behavior of the residue generated by such fault. We can see that ARR_1 is activated during this interval of time and its behavior is similar to the fault.

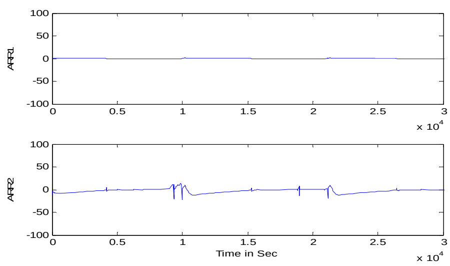

Fig.5, Residue Analysis in normal state

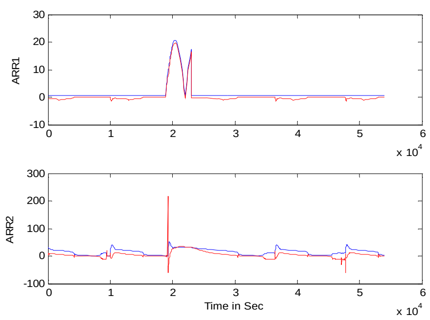

Fig.6, Residue Analysis of f_1

We simulate a malfunction in the output of the mixed flow of the system (f_2), to see how the ARRs are affected in the interval of time between 5000 and 10000 seconds. Fig. 7 shows the ARR_1 versus ARR_2 for this failure; ARR_2 is affected while the ARR_1 is zero

Finally, we simulate f_1 and f_2 on different time periods. f_1 occurs between 7.000 and 12.000 seconds and f_2 occurs between 22.000 and 27.000 seconds. In this case, both ARR_1 and ARR_2 are affected see (Fig. 8) that is, both residues are activated. Hence the two faults are diagnosable.

The definitions addressed in section 3 consider that if there is a fault in the system, it must be associated with an *ARR* to fulfil the *definition 4*. To fulfil diagnosability, the *definition 5* about isolability must be achieved, each signature of the residue must be different from other. Particularly, in our analysis the ARRs are useful for detection and isolation of the normal behaviours.

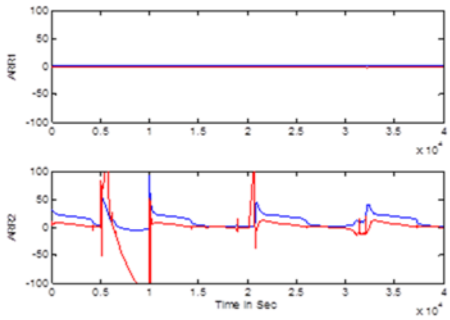

Fig.7, Residue Analysis of f_2

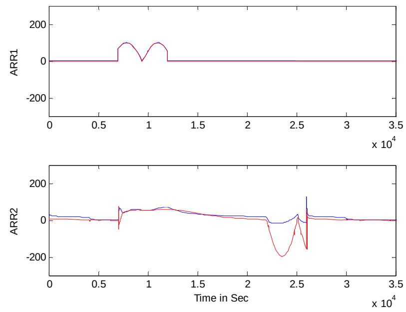

Fig.8, Residue Analysis, for f_1 and f_2 not simultaneously

3.1.1 Diagnosis analysis for five faults based on the current instrumentation

We have shown how isolability is achieved for f_1 and f_2 faults. Next, we extend the solution to models with faults that are not structurally detectable in the original model.

Particularly, we add three more faults to f_1 and f_2: f_3 (failure in the mixed fluid into the tubing), f_4 (failure in pressure at the bottom of the tubing) and f_5 (failure in the tubing at the gas injection point). The algorithm proposed in [13] shows in the Fig.9, the incidence matrix with five selected faults.

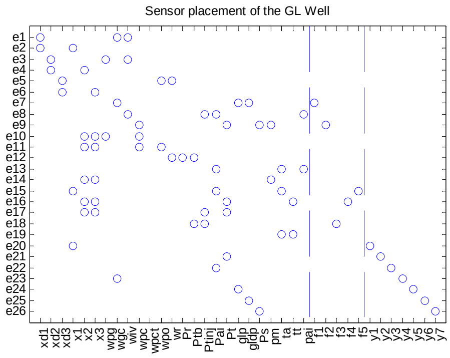

Fig.9, Incidence matrix for
the analysis of five faults

The fault signature matrix of Table 5, resulting from the algorithm shows that the only two MTES are not sufficient to achieve full detectability and isolability.

Table 5, Fault signature matrix of the system under study.

	N	f_1	f_2	f_3	f_4	f_5
Arr_1	0	1	0	0	0	0
Arr_2	0	0	1	1	0	0

3.2 Sensor placement for increased diagnosability

Given that five considered faults are not detectable we want to find the minimal sensor set that achieve fault detectability and isolability. For that, we run the algorithm proposed in [16]. Interestingly, a sensor in the reservoir pressure is sufficient to achieve complete diagnosability. Fig.10, shows the incidence matrix, and Table 6, the fault signature matrix. Additionally, the algorithm proposes adding a sixth fault, which is the fault in the sensor itself because the added sensor may fail.

Table 6, Fault signature matrix of the system under study.

	N	f_1	f_2	f_3	f_4	f_5	f_6
Arr_1	0	0	1	0	1	1	0
Arr_2	0	1	0	0	0	0	0
Arr_3	0	0	1	1	1	0	1
Arr_4	0	0	1	1	0	1	1
Arr_5	0	0	1	1	1	0	0
Arr_6	0	0	1	1	0	1	0
Arr_7	0	0	0	1	1	1	0
Arr_8	0	0	0	0	0	0	1

In table 6, it can be seen how detectability is fulfilled, because the fault signatures are different from each other, this is possible thanks to the new set of ARRs

$MTES_1$: [1, 2, 3, 4, 8, 9, 10, 13, 14, 15, 16, 17, 19, 20, 21, 22, 23, 26].
$MTES_2$: [7, 23, 24, 25].
$MTES_3$: [1, 2, 3, 4, 5, 6, 8, 9, 10, 11,12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 26, 27]
$MTES_4$: [1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 20, 21, 22, 23, 26, 27]
$MTES_5$: [1, 2, 3, 4, 5, 6, 8, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 27]
$MTES_6$: [1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 26, 28]
$MTES_7$: [1, 2, 3, 4, 5, 6, 8, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 28]
$MTES_8$: [27, 28]

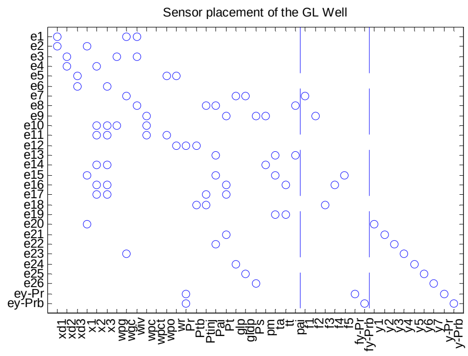

Fig.10, Incidence matrix proposed

4 Conclusion

The capability to detect a fault on time provides security, availability and reliability. The fault diagnosis mechanisms used in this paper are based on the principles of redundancy. This paper developed the technique of structural analysis on a real-world gas lift well case study. We have obtained a structural model based on the dynamic process model of the gas lift well, represented as an incidence matrix, and applied the algorithm of [16] to find MTES. We have carried out an analysis of detectability and isolability considering two faults and the current instrumentation, i.e. only surface variables, and showed the relationship between these variables and the redundant subsystems.

Our design objective was to create the residual generators for the gas lift wells in such situation. The two faults have been proved detectable and isolable in such situation.

The same analysis has then been performed extending the set of faults to five faults, in which case detectability and isolability cannot be achieved with the current instrumentation. We have then relied on a sensor placement algorithm to select the minimal sensor set to be added to achieve these properties. Interestingly, one single sensor is enough. Unfortunately, this sensor is located at the bottom of the well and it is very expensive and difficult to maintain.

Future work will include implementing the ARR-based alarms on the real monitoring system of the gas lift well, hence allowing to respond on time to problems that can turn critical. At the moment, only alarms reporting sensor faults are available.

From the methodological point of view, future work will consider to extend this approach with a data-based diagnosis method able to account for environmental features, like vibrations, noises, temperatures, that are difficult to manage with a model-based approach like the one used in this paper.

ACKNOWLEDGEMENTS

This work has been supported by Postgraduate Cooperation Program (PCP) between Venezuela and France entitled "Supervision and maintenance task in a shared organizational environment", contract No. 2010000307, between the University of Los Andes, Venezuela with the cooperation of LAAS-CNRS, University of Toulouse, France.

References:

[1] M. Krysander, J. Åslund, and M. Nyberg. An efficient algorithm for finding minimal over-constrained subsystems for model-based diagnosis, IEEE Transactions on Systems, Man, and Cybernetics – Part A: Systems and Humans, 38(1), 2008, pp 197-206.

[2] G. Hernández, E. Alí, T. Sharon, *Manual del curso de levantamiento artificial por gas avanzado*. 2da ed. Los Teques, Venezuela: PDVSA Intevep, 2001.

[3] B. Jansen, M. Dalsmo, L. Nskleberg, K. Havre, V. Knstiansen, and P. Lemetayer, (1999). Automatic control of unstable gas lifted wells: SPE paper no. 56832.

[4] L. Imsland, (2002). Topics in nonlinear control: Output feedback stabilization and control of positive systems. Ph.D. Thesis, NTNU, Norway.

[5] G.O. Eikrem, O.M. Aamo, H. Siahaan, B. Foss, "Anti-slug control of gas-lift wells - experimental results" *In Proceedings of the 6th IFAC Symposium on Nonlinear Control Systems*: Stuttgart, Germany, 2004.

[6] R. Leal, J. Aguilar, E. Camargo, A. Ríos, L. Travé, "Analysis of diagnosability for gas lift Wells", *Ingeniería y Ciencias Aplicadas: Modelos Matemáticos y computacionales (Ed. E. Dávila, J. Del Río, M. Cerrolaza, R. Chacón), Sociedad Venezolana de Métodos Numéricos en Ingeniería*: Venezuela, 2014, pp. EC91-EC97.

[7] J. Aslund and E. Frisk, "Structural analysis for fault diagnosis of models with constraints", *In Proceedings of the 7th IFAC Symposium on Fault Detection Supervision and Safety of Technical Processes*: Barcelona, Spain, 2009, pp. 384-389.

[8] M. Blanke, M. Kinnaert, J. Lunze, and M. Staroswiecki, *Diagnosis and fault tolerant control*, 2nd ed. Berlin, Germany: Springer, 2003.

[9] V. Puig, J. Quevedo, T. Escobet, B. Morcego y C. Ocampo, "Control tolerante a fallos (Parte I): Fundamentos y diagnóstico de fallos". *Revista Iberoamericana de Automática e Informática Industrial*: España, vol. 1(1), 2004, pp. 15-31.

[10] P.M. Frank, "Fault diagnosis in dynamic system using analytical and knowledge based redundancy a survey": *Automatica*: vol. 26, 1990, pp. 459-474.

[11] R. Isermann and P. Ballé, "Trends in the application of model-based fault detection and diagnosis of technical processes". *Control Engineering Practice*: vol. 5, 1997, pp. 709-719.

[12] M. Staroswiecki, G. Comtet-Varga. "Analytical redundancy relations for fault detection and isolation Analytical redundancy relations for fault detection and isolation", *International Journal Automatica*, vol 37, 2001, pp. 687-699.

[13] Armengol, A. Bregon, T. Escobet, E. Gelso, M. Krysander, M. Nyberg, X. Olive, B. Pulido, and L. Travé-Massuyès. "Minimal structurally overdetermined sets for residual generation: A comparison of alternative approaches", *In Proceedings of the 7th IFAC Symposium on Fault*: Barcelona, Spain, vol 2, pp. 2009, 1480-1485.

[14] M. Krysander, J. Åslund, and M. Nyberg, "An efficient algorithm for finding minimal over-constrained subsystems for model-based diagnosis", *IEEE Transactions on Systems, Man, and Cybernetics Part A*: vol. 38(1), 2008, pp. 197-206.

[15] M. Krysander, E. Frisk "Sensor Placement for Fault Diagnosis" *IEEE Transactions on Systems, Man, and Cybernetics Part A*: vol. 38(6), 2010, pp 106-113.

[16] E. Frisk, E., M. Krysander, "Sensor placement for maximum fault isolability". *In Proceedings of 18th international workshop on principles of diagnosis DX-07*: Nashville, TN, 2007, pp. 106-113.

[17] R. Leal, E. Camargo, J. Aguilar, A. Ríos, L. Travé-Massuyès, "Análisis de Diagnosticabilidad y Localización de Sensores en un Pozo de Extracción de Petróleo por Inyección de Gas", *Revista Politécnica – EPN Journal*: vol. 36(1), 2015, pp. 1-11.

[18] O.G. Santos, S.N. Bordalo, F.J. Alhanati: "Study of the dynamics, optimization and selection of intermittent gas-lift methods a comprehensive model", *Journal of Petroleum Science and Engineering*: London, vol. 32, 2001, pp. 231- 248.