

## **A hybrid solution for an abstract-level daily-volume problem and hourly-level multilayer network problem**

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JAPAN

*Abstract:* - In this paper, we propose a method for operational planning of water transportation systems, by mutually and independently solving problems at an abstract level, for daily volume planning, and at a detailed level, for planning hourly-based flow and storage volumes, by utilizing reservoir water level restoration as a constraint. At the abstract level, a daily-volume plan is formulated, in the form of an interactive, multi-objective planning problem, for flexible application to human solutions. In the detailed level plan, we propose a method for automatically generating the costs of a multilayer network model constrained by reservoir water level restoration, based on the results at the abstract level, and obtain a detailed plan through minimum cost flow calculation.

A network transportation problem having holders can be solved by reverting to a multilayer extended network model. In this case it is necessary to define the expense (cost) for arcs, taking into account economics and safety. However, as the scale of the network increases, the work of defining all the costs for arcs becomes difficult. In addition, costs are tuned by a trial-and-error while planning results are compared with an ideal design. Another problem is that it is difficult to flexibly correct or modify a solution to match the desires of designers.

As a way of satisfying the multiple aspirations of designers, one approach is to formulate them as a multi-objective planning problem having multiple objective functions. Even in this case, if we apply a scalarizing technique that suitably weights and combines vector-valued objective functions, it becomes essentially impossible to solve the problem as a single objective function problem. One proposed planning method to address this issue involves interactively extracting the preference information from decision makers utilizing the concept of “aspiration level”. However, if a water supply system is formulated as a multi-objective planning problem in a multilayer network model as it is, the scale of the problem becomes extremely large, thereby necessitating enormous computing resources.

In order to resolve the issues with these traditional methods, this paper discusses the validity of separating problems into an abstract level and a detailed level and proposes a method for automatically generating the costs in a multilayer network model based on the results of establishing a plan at the abstract level, where daily volumes are set. Furthermore, by separating the problem into two layers—an abstract level for handling daily volumes (abstract planning level) and a detailed level for handling hourly-based flow rates (time-series flow planning level)—the method enables the application of solutions that take advantage of the features of each level. The method utilizes an interactive multi-objective planning method as a user interface and produces a Pareto solution as an abstract plan, through a process of interactively obtaining aspiration levels. If an abstract plan that satisfies the designer can be created, a “cost generator” can produce the costs for the arc variables of a multilayer network model, as described above, based on the abstract plan. Then by determining the minimum cost flow corresponding to these dynamically generated costs, it is possible to arrive at a detailed plan that reflects the various requirements, without having to tune costs in advance by trial and error.

In order to validate this proposed method, we formulated a plan using data from an existing water supply network. The results of this show that through variation in the water level, any water reservoir can absorb demand fluctuations and that after 24 hours, the water level restores to its original value, while even for the smoothing ratio, the above objective is satisfied. Furthermore, our results showed that computational effort is greatly reduced and that computing time is shortened.

*Key-Words:* - electric vehicle, energy resurrection, limitation of battery capacity, maximization of traveling point, round cost, monotonic increase

## 1 Introduction

Water supply system management involves planning the daily water intake, purified water quantity, and reservoir storage quantities in a water supply network. This kind of network transportation problem, having holders, can be reduced to a minimum cost flow problem in a multilayer extended network model (hereinafter “multilayer network model”) <sup>(1)</sup>. Here, “cost” refers to a concept for the purpose of mathematical modeling; it does not need to be the actual cost of water transportation <sup>(2)-(4)</sup>. In a water supply project that is largely for public utility, it is necessary to take into account not just economic factors like actual cost, but also safety. In view of this, the work of defining costs for each arc on a graph, taking into account all the various factors, becomes difficult. In practice, therefore, best-guess values are assigned and an optimized solution of the model is computed, after which costs are tuned by trial and error as the planning results are compared with an ideal design. Another issue is that because a minimum cost flow problem only has a single solution algorithmically, it is difficult to modify the solution flexibly to tailor it to the requirements of the designer.

In optimization that is based on this kind of single objective function, the solution at the moment of formulation is determined implicitly, and skillful modeling results merely in the preconditions for obtaining a satisfactory solution. One approach to overcome this limitation is to formulate the problem as a multi-objective planning problem having multiple objective functions <sup>(5)</sup>. In this case too, if we apply a scalarizing method that appropriately weights and combines the various vector-valued objective functions, it becomes effectively impossible to solve the problem of the single objective function <sup>(6)-(8)</sup>. We suggest that this issue can be resolved by means of an interactive multi-objective planning method, in which information about the preferences of the decision makers is extracted interactively using the concept of “aspiration level” and used to determine the Pareto solution that most closely agrees with the aspiration levels.

However, if we formularize water supply as a multi-objective planning problem in a multilayer network model, as is, the scale of the problem becomes extremely large. As a result, it becomes

impossible to employ a basis factorization method <sup>(1)</sup> (the technique that is usually applied to solve such a problem) and the required computational effort becomes enormous. A further difficulty is that as the model of the operation rules becomes larger, the problem becomes more complex and difficult.

In this paper, we present a method for resolving the above issues. The method works by applying the constraint that the water level of water reservoirs must be restored, which is generally considered the main aim of water supply system management, and it utilizes the fact that it is possible to develop separate and independent solutions for the problem at an abstract level, for planning of daily volumes, and at a detailed level, for planning water flows and water storage quantities on an hourly basis. The detailed-level plan for achieving the planned volumes of the abstract level focuses on ensuring that the water reservoir level is restored. The proposed method produces a detailed plan by means of a minimum cost flow computation as usual, after automatically generating costs in a multilayer network model, based on the results of the plan at the abstract level.

In Section 2, we describe the functions of a water supply system and the main principles of water supply system planning. In Section 3, we explain in detail the problems encountered when using traditional planning methods. That is, since methods for solving a multi-objective planning problem with multiple objective functions apply scalarizing techniques for appropriately weighting and combining vector-valued objective functions, they cannot effectively solve the problem of a single objective function. In order to formulate an effective plan, it becomes necessary to tune the costs for arcs using trial and error. Even assuming that these costs can be adjusted effectively, only one optimal solution can be determined, and if this solution is not satisfactory to the designers, it is difficult to make effective corrections or modifications to the solution. Another problem with existing solution methods is that because the scale of the problem in a multilayer network model becomes very large, the computational effort required becomes enormous and complex. In Section 4, we discuss the validity of separating problems into an abstract level and

detailed level, and we propose a method for automatically generating the costs in such a multilayer model based on the results of the plan formulated at the abstract level for daily quantities. Furthermore, by separating the problem into two layers, the abstract level (abstract planning level) for handling daily volumes and the detailed level (time-series flow planning level) for handling hourly flows, it is possible to apply solution methods that take advantage of the distinctive features of each particular layer. In Section 5, we formulate a plan using data from an existing water supply network in order to verify our proposed method. The results of this validation show that for any water reservoir, demand fluctuations can be absorbed by variation of the water level and that after 24 hours the water level is restored to its original value, while even for the smoothing ratio, the above objectives are satisfied. In addition, we demonstrate that the required computational effort is greatly reduced and that computing time is shortened.

## 2 Outline of water supply planning problems

A water supply system is made up of a network of pipeline or other conduits for distributing water to end users according to their demand by means of a series of connected processes involving the intake of raw water from water sources, transport of raw water to water purification plants, purification of raw water at the purification plants, transport of the purified water to water reservoirs, and distribution of water from reservoirs to end users on demand (see Fig. 1). In terms of the functions involved in delivering water to the end user, a water supply system is made up of three essential systems: water collection, water conveyance, and water distribution. The role of the water collection system is to collect the necessary volume of water from multiple intake points at rivers and dams and carry it to multiple water purification plants. The role of the water conveyance system is to transport the water that has been purified at the multiple purification plants to multiple water reservoirs, that is, to move water that has been purified to the reservoirs for temporary storage. The role of the water distribution system is to supply (distribute) water from the water reservoirs to the many ultimate end users of the water, such as homes, offices, and factories. In terms of total pipeline length, the distribution system is a far bigger network

than the conveyance system.

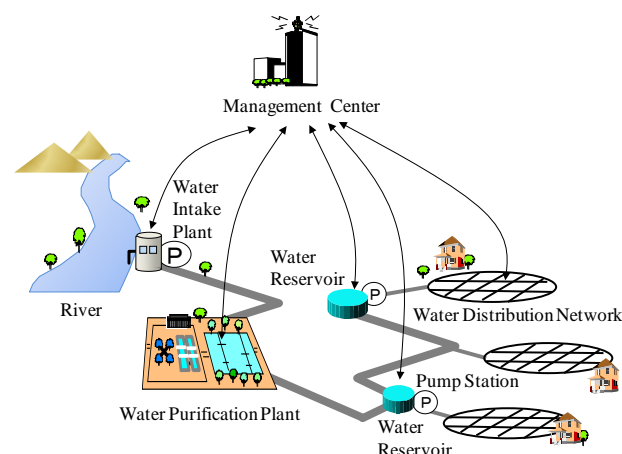


Fig.1 Overview of water works

This study focuses on the water collection and water conveyance systems, which we will collectively refer to as the “water transportation system”. Together they serve the function of transporting water from the water intake points to the water reservoirs for storage.

The flow of water in the system is controlled by pumps and valves, based on monitoring of flow rate and pressure. Although on one hand water demand varies throughout the day and day by day, according to the rhythms of daily life, temperature, and climate, from the point of view of facility capacity and water rights, it is desirable for the intake pumps and purification plant to transport a constant volume at all hours. The role of the buffer for adjusting between fluctuating demand and the constant flow of purified water produced is played by the water reservoirs. Accordingly, throughout the day, the operation of the water transportation system causes fluctuation in the water level of the water reservoirs <sup>(9)-(11)</sup>.

In order to ensure a plentiful supply of water in the event of an emergency, such as an earthquake disaster, it is also desirable that the rate of water storage at the reservoirs is maintained as high as possible at all times. Thus, it is also necessary for water volumes to be adjusted within the range of operational capacity; sometimes it may be necessary to connect and transfer water between water reservoirs. The more extensive and complex a water supply network is, the more difficult this problem is to solve. This is referred to as “water management”. In the event of a crisis situation, typified by drought, or when water supply is interrupted due to pump inspections or pipeline work, it is necessary to make

quick and flexible decisions about water collection fees, water distribution adjustments, and reservoir operation, in accordance with the conditions of facilities. The guidelines usually applied in dealing with the operation of this kind of water transportation system are outlined below.

- 1) Quickly restore the water level of each reservoir
- 2) Minimize flow fluctuations in pipelines used for flow smoothing

Below, we discuss a method for formulating a plan to satisfy these requirements.

### 3 Problems with Traditional Methods

The fundamental equations used in water management plans are based on conservation of flow. A water system can be expressed in the form of a network diagram, with nodes representing water purification plants, ( $n_R$ ) reservoirs, and branch points ( $n_j$ ), and arcs representing pipelines ( $n_K$ ). The flow conservation at time  $t$  is expressed in terms of  $v_i(t)$ , the storage volume at node  $i$ , as follows:

$$v_i(t) - v_i(t - 1) = \sum_{j \in N_i^I} x_j(t) - \sum_{j \in N_i^O} x_j(t) - d_i(t) \tag{1}$$

where  $x_j(t)$  is the flow in arc  $j$ ,  $N_i^I$  is an index set for arcs into node  $i$ ,  $N_i^O$  is an index set for arcs out of node  $i$ , and  $d_i(t)$  is the volume demand at node  $i$  at time  $t$ .

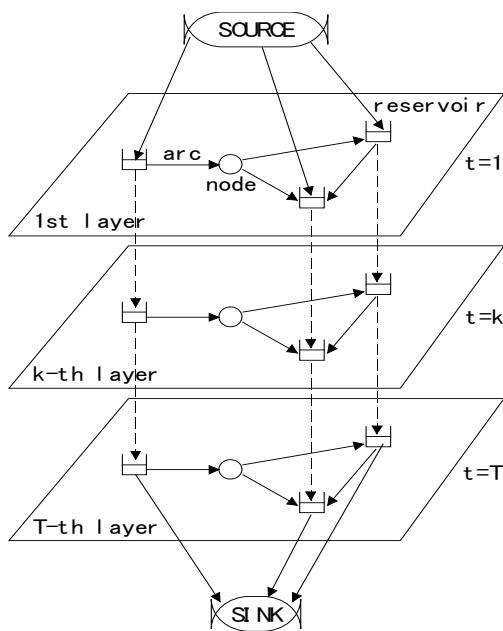


Fig.2 Multi-layered network(MLN) method

If we separate the network into layers for each time interval, as shown in Fig. 2, and express the storage volume in the arcs between the layers as  $v_i(t)$ , it becomes possible to treat the whole of the extended network, including time changes, as a closed network. Such a model is known as a multilayer network model.

In this case,  $v_i(t)$  is equivalent to the flow variable  $x_j(t)$  of the arc, and Eq. (1) can be rewritten as

$$\mathbf{A}_1 \mathbf{x}(t - 1) + \mathbf{A}_2 \mathbf{x}(t) = \mathbf{b}(t) \tag{2}$$

In addition, if  $m = n_R + n_j$ ,  $n = n_R + n_K$ , then we obtain the following results:

$$\mathbf{A}_1 = \begin{pmatrix} -I & 0 \\ 0 & 0 \end{pmatrix} \tag{3}$$

$$\mathbf{A}_2 = \begin{pmatrix} I & A_R \\ 0 & A_K \end{pmatrix} \tag{4}$$

$$\mathbf{x}(t) = (x_1(t), x_2(t), \dots, x_n(t))^T \tag{5}$$

$$\mathbf{b}(t) = (-d_1(t), -d_2(t), \dots, -d_m(t))^T \tag{6}$$

where  $I$  is an  $n_R$ -dimensional identity matrix and represents the arcs between layers that connect reservoirs,  $A_R$  is an incidence matrix representing the pipelines flowing in and out of the reservoirs, and  $A_K$  is an incidence matrix representing the pipelines flowing in and out of non-reservoir branch points.

If we repeatedly apply Eq. (2) from time 1 to the final time  $T$ , we obtain the following

$$\mathbf{A} = \begin{pmatrix} \mathbf{A}_2 & & & & \\ \mathbf{A}_1 & \mathbf{A}_2 & & & \\ & \mathbf{A}_1 & \mathbf{A}_2 & & \\ & & \dots & \dots & \\ & & & \mathbf{A}_1 & \mathbf{A}_2 \end{pmatrix} \tag{7}$$

$$\mathbf{x} = (x(1), x(2), \dots, x(T))^T \tag{8}$$

$$\mathbf{b} = (b(1) + x(0), b(2), \dots, b(T))^T \tag{9}$$

If we combine these, we obtain the equation

$$\mathbf{Ax} = \mathbf{b} \tag{10}$$

The above formulates the relationship of flow conservation. If we now define a cost  $c_i$  for each arc variable  $x_i(t)$ , water management can be formulated

as a minimum cost flow problem.

$$\min. \quad z = \sum_{t=1}^T \sum_{i=1}^n c_i x_i(t) \quad (11)$$

$$\begin{aligned} s.t. \quad & \mathbf{Ax} = \mathbf{b} \\ & \mathbf{1} \leq \mathbf{x} \leq \mathbf{u} \end{aligned}$$

Here,  $\mathbf{1}$  is the lower limit vector of operation for  $\mathbf{x}$ , and  $\mathbf{u}$  is the upper limit vector of operation. In this case, to ensure that a feasible solution exists, we recommend that when defining multi-stage costs in the simplex method ( $-c_\infty \ll c_1 < c_2 < \dots < c_s \ll c_\infty$ ), as shown in Table 1, and entering non-basic variables into basic variables, calculation should be done by selecting intervals that make the relative cost coefficients small.

Table 1 Multi-stage cost coefficients in simplex method

| Stage Number | Lower Bound | Upper Bound | Cost coefficient $c_i$ |
|--------------|-------------|-------------|------------------------|
| 0            | $-\infty$   | 1           | $-c_i$                 |
| 1            | 1           | $u_1$       | $c_1$                  |
| 2            | $u_1$       | $u_1$       | $c_2$                  |
| .            | .           | .           | .                      |
| .            | .           | .           | .                      |
| $s$          | $u_{s-1}$   | $u$         | $c_s$                  |
| $s+1$        | $u$         | $+\infty$   | $+c_\infty$            |

As with problem (11), for a linear planning problem, having constraint equations where small matrixes are arranged in a tiered manner, a solution can be determined very quickly by utilizing the basic factorization method. In addition, in the case of a minimum cost flow problem, the following applies for an arbitrary square sub-matrix of the coefficient matrix  $\mathbf{A}$ .

$$\det C = 0, 1 \text{ or } -1 \quad (12)$$

It is possible to show total unimodularity and derive an integral solution by constraining all the coefficients used to integer values. If this property is utilized when solving problem (11) using the simplex method, all computations can be performed using only addition/subtraction operations <sup>(1)</sup>. Hereafter, this technique will be referred to as the multi-stage primal method.

Now, since the above model expresses its goals implicitly in the form of cost coefficients, the problem of formulating a good plan reduces to that of tuning  $c_i$  and interval variables  $u_i$  by trial and error.

However, even if  $c_i$  and  $u_i$  can be adjusted favorably, only one optimal solution can be obtained, and so if the solution does not satisfy the system designer, it is difficult to correct or modify the plan.

To get around this limitation, another approach is to mathematically model the explicit operating principles of the system and formulate it as a multi-objective planning problem to optimize the solution under  $p$  objective functions

$$f_i(\mathbf{x}) (1 \leq i \leq p) \text{ [6][7][8].}$$

$$\mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), \dots, f_p(\mathbf{x})) \quad (13)$$

For example, if we assume as an operating principle that the storage volume  $x_i(t_1)$  of reservoir  $i$  at time  $t_1$  is restored to the target storage volume  $g_i$ , the problem can be modeled as the minimization of  $f_1(\mathbf{x}) = g_i - x_i(t_1)$ . As shown in Eq. (18) below, it also possible to model for flow smoothing by minimizing the deviation from average flow. Here, we can introduce the concept of aspiration level, as a goal for defining optimality. The aspiration level quantitatively expresses the level to which a target or goal  $f_i$  must be achieved to result in satisfaction. The Pareto solution closest to the aspiration level is derived as the satisfactory solution. For scalarizing functions, an extended Chebyshev scalarization function can be used to formulate the mathematical planning problem, as follows:

$$\begin{aligned} \max_{1 \leq i \leq p} w_i(f_i(\mathbf{x}) - \bar{f}_i) + \alpha \sum_{i=1}^p w_i(f_i(\mathbf{x}) - \bar{f}_i) \\ \rightarrow \min. \end{aligned} \quad (14)$$

$$s.t. \quad \mathbf{Ax} = \mathbf{b}$$

$$\mathbf{1} \leq \mathbf{x} \leq \mathbf{u}$$

where  $\alpha$  is a sufficiently small scalar,  $w_i$  is a factor for normalization of  $f_i - \bar{f}_i$ , and  $\bar{f}_i$  is the aspiration level for  $f_i$ . In this case, problem (14) is equivalent to the following auxiliary min-max problem:

$$\begin{aligned} z + \alpha \sum_{i=1}^p w_i(f_i(x) - \bar{f}_i) \rightarrow \min. \\ s.t. \quad w_i(f_i(x) - \bar{f}_i) \leq z (1 \leq i \leq p) \end{aligned} \quad (15)$$

$$\mathbf{Ax} = \mathbf{b}$$

$$\mathbf{1} \leq \mathbf{x} \leq \mathbf{u}$$

the term “node” will be used in referring to destinations in the following related discussions.

By solving problem (15), a solution that satisfies the aspiration can be obtained. In this case, it is also a simple matter to correct or modify the solution by interactively changing the aspiration level. However, in the expanded constraint matrix of Eq. (15), even if  $f_i(\mathbf{x})$  is a linear function of integer coefficients and all  $w_i$  are integers, total unimodularity of Eq. (12) is not established, and the integrality of the solution is not guaranteed. Because it is not possible to use the multi-stage primal method, the quick technique specialized to problem (11), in order to solve problem (15), we must rely on a general solution method such as the revised simplex method, which necessitates massive computational effort.

Modeling the water system management rules described in Section 2 becomes very complex under a large-scale model such as a multilayer network model. For example, if modeling flow smoothing, the average flow  $avg_i$  for flow  $x_i(t)$  at time  $t$  is defined by the following equation:

$$avg_i = \frac{1}{T} \sum_{t=1}^T x_i(t) \quad (16)$$

Ideally, we assume smoothing is done relative to the average flow, which means minimizing the deviation from the average flow rate,

$$\|X_i(t)\| = \|x_i(t) - avg_i\| \quad (17)$$

In engineering applications, a formulation that minimizes the value of the  $l_2$  norm, as in the square error method, is widely used, but non-linear optimization generally presents many difficulties. Thus, when linearization is possible, it is simpler to use the  $l_1$  norm or  $l_\infty$  norm rather than the  $l_2$  norm. If we consider the  $l_\infty$  norm in this case, smoothing can be achieved by minimizing the following expression:

$$\begin{aligned} \|X_i(t)\|_\infty &= \|x_i(t) - avg_i\|_\infty \\ &= \max_{1 \leq t \leq T} |x_i(t) - avg_i| \\ &= \max_{1 \leq t \leq T} \left| x_i(t) - \frac{1}{T} \sum_{t=1}^T x_i(t) \right| \quad (18) \end{aligned}$$

However, formulating the problem here as a linear planning problem, by inserting the second term of

the objective function of the auxiliary min-max problem (15) into Eq. (18), is difficult. Leaving out the second term, it is possible to formulate under an ordinary Chebyshev scalarization function, but in that case the only obtainable solution is a weak Pareto solution, and we are faced with the difficulty of unnecessary trade-offs and zero sensitivity to changing the aspiration level.

In the next section, we propose a method for formulating plans that enables the above-described difficulties to be effectively avoided.

## 4 Proposed method

### 4.1 Multilayering a water system planning problem

If the condition for restoring the water reservoir storage volume is expressed such that the unit of time  $t$  in Eq. (1) is taken to be 1 day, then  $v_i(t-1) = v_i(t)$ . Equation (1) then becomes the following:

$$\sum_{j \in N_i^I} x_j(t) = \sum_{j \in N_i^O} x_j(t) + d_i(t) \quad (19)$$

When formulating the daily volume plan, treating the water reservoirs the same as other branch points enables the condition of reservoir water level restoration to be embedded in the constraint formula. However, because the supply of water from the water intake points must satisfy the demand for water, nodes that are considered to be large reservoirs connect between source and sink. Thus, the daily volume plan can be formulated under a multilayer network model with only a single layer, allowing the number of design variables to be greatly reduced.

By setting the intake volume as an objective here, the objective can be expressed in simple form as in the following equation:

$$\sum_{i=1}^p \sum_{t=1}^T x_i(t) = \sum_{t=1}^T \sum_{j=1}^m d_j(t) = A \text{ constant} \quad (20)$$

If one of the intake volumes is varied, at least one other intake volume changes. Hence, this setup has the advantage of making trade-off analysis relatively simple.

However, if we do not take into account this kind of change over time, it becomes impossible to plan the flow smoothing, such as to minimize change over time in reservoir management (transfer of storage

volume between reservoirs) and in pipeline flow. Therefore, we propose here a method for automatically generating the costs in a multilayer network model based on the results of a plan formulated at the abstract level that sets the daily volumes.

Figure 3 shows the overall structure of the proposed system. The lower part, which determines the minimum cost flow in the multilayer network model, is solved using a traditional method. In this case, however, it is necessary to prepare the cost coefficients for each arc in advance by forming a database, and it becomes difficult to tune costs effectively. In view of this, our proposed method utilizes an interactive multi-objective planning method as a user interface, and as aspiration levels are obtained interactively, Pareto solutions to the problem are presented in the form of abstract plans. If an abstract plan that satisfies the designers can be created, a cost generator is then employed to produce costs for the arc variables of the multilayer network model, as described above, based on the abstract plan. Then by determining the minimum cost flow under these dynamically generated costs, it is possible to obtain a detailed plan that reflects the various requirements of the system without having to tune costs in advance by trial and error.

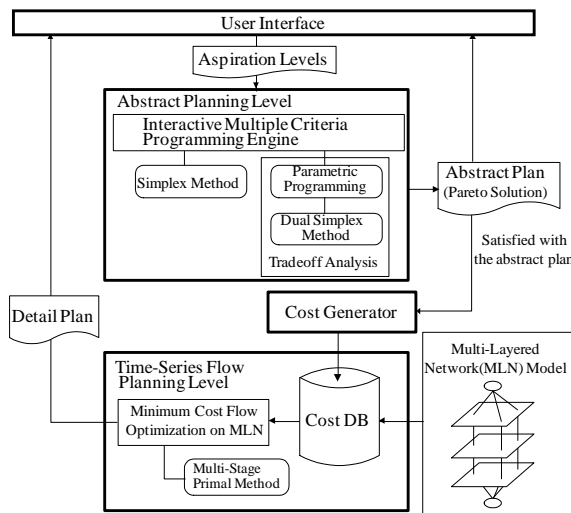


Fig.3 Overall structure of the proposed system

As outlined above, separating the problem into separate layers—an abstract level (abstract planning level), which deals with daily volumes, and a detailed level (time-series flow planning level), which deals with hourly flow—enables the

application of solution methods that take advantage of the characteristics of each particular layer. At the same time, it allows a substantial reduction in the computational effort required to solve the problem.

### 4.2 Multi-objective planning method at the abstract level

The daily pipeline flow volume  $\mathbf{X}$  and demand volume  $\mathbf{D}$  are defined as follows.

$$\mathbf{X} = \sum_{t=1}^T \mathbf{x}(t) \tag{21}$$

$$\mathbf{D} = \mathbf{x}(0) + \sum_{t=1}^T \mathbf{b}(t) \tag{22}$$

Based on these, if we apply the conservation of flow rule on a daily basis, Eq. (2) can be simplified further to obtain the following:

$$\mathbf{A}_2 \mathbf{X} = \mathbf{D} \tag{23}$$

Now, if we make the intake volume from the water source an objective at the abstract level, then the objective function in terms of the daily intake volume  $X_i$  from a given intake pipeline  $i$  can be simply expressed by the following equation:

$$f_i(\mathbf{X}) = X_i \tag{24}$$

Here, representing the factor  $w_i$ , for normalizing the difference between the objective function and aspiration level as the equation below, in terms of the ideal point  $f_i^*$  and the worst point  $f_{i*}$  becomes second nature.

$$w_i = \frac{1}{f_{i*} - f_i^*} \tag{25}$$

In some cases,  $f_{i*}$  and  $f_i^*$  can be determined by an optimization calculation, but because this is inefficient in practice, the ideal point and worst point can be considered heuristically, resulting in the following:

$$f_i^* = 0 \tag{26}$$

$$f_{i*} = U_i - L_i \tag{27}$$

Note that  $U_i$  and  $L_i$  are the upper and lower limit values for daily operation of pipeline  $i$ , respectively. That is,  $L_i = l_i T$ ,  $U_i = u_i T$ .

Thus, the auxiliary min-max problem (15) in the multi-objective planning method becomes the following:

$$z + \alpha \sum_{i \in N_{obj}} (X_i - \bar{X}_i) / (U_i - L_i) \rightarrow \min.$$

$$s.t. \quad X_i - (U_i - L_i)z \leq \bar{X}_i \quad (\forall i \in N_{obj}) \quad (28)$$

$$A_2 X = D$$

$$L \leq X \leq U$$

where  $N_{obj}$  is the index set for pipelines directly connected to the intake points and  $\bar{X}$  is the aspiration level vector for  $f_i(x)$ .

Of the objective functions  $f^{(k)}$  with respect to the  $k^{\text{th}}$  solution  $X^{(k)}$ ,  $f_q^{(k)}$  represents a function that the designer wishes to improve. Therefore, at this point, the designer is asked to input a new aspiration level  $\bar{f}_q$ , and a parametric linear planning problem for  $\Delta f_q^{(k)} = \bar{f}_q - f_q^{(k)}$  is defined as follows:

$$z \rightarrow \min.$$

$$s.t. \quad X_q \leq X_q^{(k)} + \theta \Delta f_q^{(k)}$$

$$X_i - (U_i - L_i)z \leq X_i^{(k)} \quad (i \in N_{obj} \setminus \{q\}) \quad (29)$$

$$A_2 X = D$$

$$L \leq X \leq U$$

In problem (29), up to  $\theta = 1$  the solution  $X^{(k+1)}$  is presented by following a Pareto surface, and the optimal basis inverse matrix when  $\theta = 0$  can be obtained by applying the theory of sensitivity analysis to the final tableau in problem (28). From the obtained optimal basis inverse matrix, we can directly determine the upper limit  $\theta_{max}$  of the optimal value  $\theta$ ; if  $\theta_{max} < 1$ , then the Pareto curve bends until reaching the point of satisfying the new aspiration level, so at the point  $\theta = \theta_{max}$ , the dual simplex method is applied to form a new optimal basis inverse matrix. This same calculation process is repeated until  $\theta_{max} \geq 1$ . If  $\theta_{max} \geq 1$ ,  $\theta = 1$

represents the point of aspiration level attainment. Applying the above process, it is possible to plan the flow of the whole network interactively, based on daily intake volume.

### 4.3 Multi-stage Primal Method at the Detailed Level

If a daily flow of  $X_i$  is planned at the abstract level for pipeline  $i$ , which is subject to smoothing, we can designate  $T_i^e$  as a period of time within a day during which the pipeline is usable. Here, "pipeline up time" is the sum of all the periods of time that water flows freely within the range defined by the upper and lower limits of water system operation. Conversely, "pipeline down time" is the sum of all the periods of time that water flow is interrupted for reasons such as inspections and pump stoppages. From these definitions, we obtain the following equation:

$$X_i = \sum_{t=1}^T x_i(t) \quad (30)$$

If flow smoothing is conducted ideally, the pipeline flow is maintained constant whenever the pipeline is in up time, and this constant value  $x_e$  can be defined as follows:

$$x_e = X_i / T_i^e \quad (31)$$

Thus, the desired flow volume per hour can be set as follows:

$$goal_i(t) = \begin{cases} x_e & (t: \text{pipeline up time}) \\ 0 & (t: \text{pipeline down time}) \end{cases} \quad (32)$$

In this way, it is even possible to achieve flow smoothing while maintaining the abstract level plan. Hence, by creating a cost function, as shown in Fig. 4, so that a flow that satisfies Eq. (32) is a minimum cost flow, it is possible to flexibly determine an operation plan that satisfies the system operation requirements using a traditional method. By formulating problem (11) in accordance with this cost, as described previously, the ideal minimum cost flow is defined as the flow value that restores the water levels of reservoirs to their original value after time  $T$ , where the flow volume is constant in each of the pipelines subject to smoothing. If we restrict the variables used here to integer values, it is possible to



determine a solution quickly, using a basic factorization method and a multi-stage primal method that utilizes integrality of the solution.

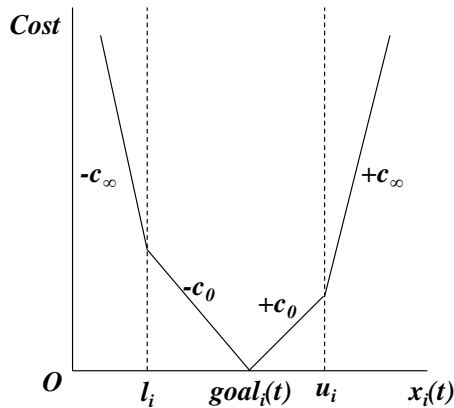


Fig.4 An example of auto-generated cost function

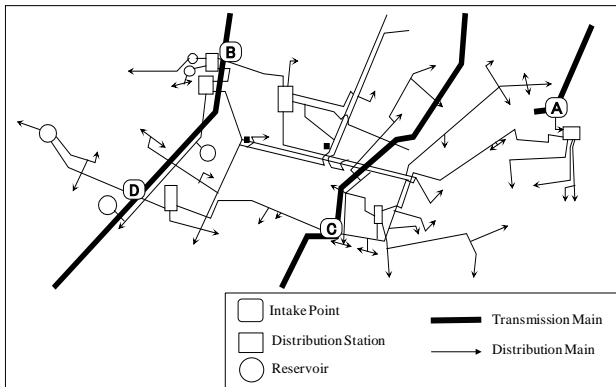


Fig.5 A water supply network used for evaluation

## 5 Numerical experiment

### 5.1 Assumptions

To verify the proposed method, we formulated a plan using data from an existing water supply system, as shown in Fig. 5 (79 arcs, 48 nodes, 11 reservoirs, and 4 intake points per layer, over a planning period of 24 hours) We assumed the most typical weekday pattern for water demand. We compared the following three methods of computation.

#### Multi-Stage Integer Programming (MSIP)

This established method combines a multi-stage primal method with a smoothing process performed as a post-processing step.

#### Multi-Objective Programming (MOP) only

This is a multi-objective planning method applied under a multi-stage network model that defines objectives for water level restoration for 6 reservoirs having sufficient effective storage

volumes and for flow smoothing for 12 pipelines.

### Proposed

This proposed method separates the system modeling into two layers, an abstract level and a detailed level, and applies a multi-stage primal method of solution after automatically generating costs in the multilayer network model

### 5.2 Computation time

Table 2 shows the results of a comparison of computing times. The CPUs used were an x58 processor rated at SPECint\_base95=4 and an x86 processor rated at SPECfp\_base95=2. The computation time for the proposed method was calculated as the sum of the time needed to formulate the initial abstract plan (time until solution for  $k=1$  was obtained) and the time needed to formulate the detailed plan (same as for MSIP); it does not include the time for the repeated trade-off analysis done by the system designers when formulating the abstract plan to obtain a solution for  $k \geq 2$ . As made clear by Table 2, solving a large-scale problem such as that of a multilayer network model using the versatile revised simplex method necessitates a huge amount of computational effort. On top of this, when a large-scale problem is formulated using a multi-objective planning method, the objective functions become complex and large, which would appear to further increase the required computational effort. On the other hand, because with our proposed method the computation needed for the multi-objective planning method is greatly reduced and it is possible to utilize the multi-stage primal method, which requires little computation, the end result is a very substantial savings in computing time.

Table 2 Comparison on mean computational time(sec)

| MSIP | MOP only | Proposed |
|------|----------|----------|
| 2.4  | 393.4    | 2.7      |

### 5.3 Abstract Level Planning Results

As described above, the established MSIP method requires only a small amount of computational effort, but it is necessary to prepare appropriate cost values in advance. To improve on this point, our proposed method applies an interactive multi-objective planning method at the abstract level. Table 3 shows

an example of an abstract level plan. The objective functions represent the water intake volumes from four intake points (A through D). The top stage is the specified aspiration level, and the bottom stage is the plan values calculated under the top stage values. In the table,  $k$  represents the iteration of the solution produced. Now, let us assume a formulated plan capable of applying water intake restrictions at intake point D as a countermeasure in the event of a drought. Aspiration levels are input to specify how much water intake is desired at each of the intake points. Here, it is not necessary to pay attention to Eq. (20). Because there is a desire to restrict intake from intake point D, the aspiration level here is set to 0. The first obtained solution ( $k=1$ ) is the best approximation based on calculation of the extended Chebyshev distance for the given aspiration level vector. Or in other words, since the initial aspiration levels were all too difficult with respect to the prevailing demand, the optimum system solution is obtained by minimizing the maximum value of “objective non-achievement sensitivity” as normalized for each objective.

Table 3 An example of the tradeoff analysis on abstract level(Quantity of water intaken : ton)

|                 |            | Quantity of water intaken (ton) |          |          |          |
|-----------------|------------|---------------------------------|----------|----------|----------|
| Solution Number |            | Intake A                        | Intake B | Intake C | Intake D |
| k=1             | Aspiration | 24,000                          | 200,000  | 15,000   | 0        |
|                 | Plan       | 27,785                          | 230,202  | 22,335   | 9,320    |
| k=2             | Aspiration |                                 |          |          | 0        |
|                 | Plan       | 27,785                          | 237,701  | 24,156   | 0        |
| k=3             | Aspiration | 20,000                          |          |          | 0        |
|                 | Plan       | 20,000                          | 243,965  | 25,677   | 0        |

The designers find the initial solution unsatisfactory because the intake volume from point D could not be set to 0, so they once again set the aspiration level for the intake volume from point D to 0. The differences relative to the initial solution are that we now enter trade-off mode (as of  $k>1$ ), and that it is not necessary to set aspiration levels for all objectives. The points where no aspiration level is set are sacrificed in return for achieving the specified aspiration levels. As a result, in the second solution ( $k=2$ ), the flow values do satisfy the requirement that the intake volume from D be 0, but at the price that the intake volumes from intake point B and C increase.

It is even quite simple to change items initially

treated as objectives into constraints in the middle of the plan formulation by setting aspiration levels  $\bar{f}_q = X_q^{(k)}$  such that  $\Delta f_q^{(k)} = 0$  in Eq. (29). In the  $k=3$  solution in Table 3, we can see that by continuing to constrain the intake volume from D to 0, entering an aspiration level to constrain the intake volume from A, and conducting a trade-off analysis, a satisfactory plan can be obtained.

### 5.4 Detailed Level Planning Results

The time-series data of the detailed level plan results are shown in Figs. 6 and 7. As an example, the results of the proposed method are shown. Figure 6 shows the planning results over 24 hours for a particular pipeline subjected to flow smoothing. The horizontal axis indicates time, and the vertical axis indicates the hourly flow rate (ton/hr). We can see that smoothing was achieved within a variation range of  $\pm 300$  tons relative to the average flow rate.

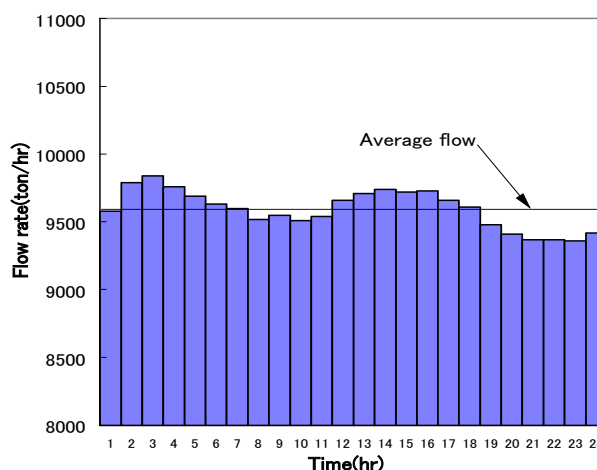


Fig.6 An example of time-series flow plan

Figure 7 shows the variation in water level of the main reservoirs of the system. The horizontal axis indicates time, and the vertical axis indicates the water level after normalization relative to the initial level (=1). This confirms that any reservoir absorbs variations in demand by changing the water level and that after 24 hours the water level is restored to its original value.

Next, we make a quantitative comparison of the detailed level plan results for the different methods explained above. In order to enable comparison with the MSIP method too, detailed plans were formulated without any trade-off analysis. Firstly, we compared the reservoir water level recovery ratio. The reservoir recovery ratio is defined as shown below as the average value of the ratio  $r_i$  of the initial to final

water level of the 6 of the 11 reservoirs having sufficient effective water storage volumes.

$$r_i = \frac{x_i(T)}{x_i(0)} \tag{33}$$

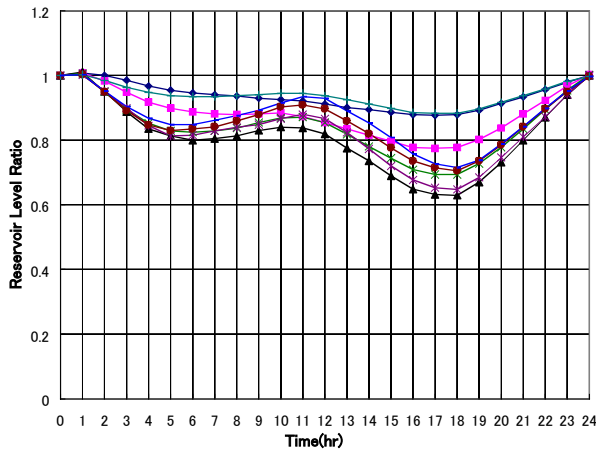


Fig.7 An example of reservoir level plan

The results, shown in Table 4, indicate that satisfactory water level restoration can be achieved with the new proposed method, without the use of trial and error or explicit objective functions.

To assess the flow smoothing ratio, we define the average smoothing ratio, in Eq. (34), in terms of the variance  $x_i(t)$  from the average flow rate  $avg_i$  at each point in time  $t$ . We can then define the maximum smoothing ratio as per the second equation below.

$$avg. = \frac{1}{T} \sum_{t=1}^T \frac{|x_i(t)|}{avg_i} \tag{34}$$

$$max. = \max_{1 \leq t \leq T} \frac{|x_i(t)|}{avg_i} \tag{35}$$

Of the average smoothing ratios and maximum smoothing ratios for each of the total of 12 pipelines subject to smoothing, the worst values are shown in Table 5. In the case of the MOP only method, which was set up in the multilayer network model with the explicit goal of flow smoothing, we assume that with sufficient expenditure of computing time an optimal solution would be reached. In the case of MSIP, there was a series of successive smoothing processes,[2] in order of priority, so the pipelines of low priority showed a slightly poor smoothing ratio. In the case of our proposed method, an optimal solution could not be reached, in part because of the adverse effect of

constraining the water level recovery in reservoirs. Nonetheless, it succeeded in obtaining a feasible solution equal to or better than that obtained by MSIP.

On the basis of the above empirical results, our proposed method seems capable of quickly and flexibly creating system plans that reflect designers' wishes.

Table 4 Comparison on reservoir recovery ratio(%)

| MSIP | MOP only | Proposed |
|------|----------|----------|
| 100  | 103      | 100      |

Table 5 Comparison on flow smoothing ratio(%)

| MSIP |      | MOP only |      | Proposed |      |
|------|------|----------|------|----------|------|
| avg. | max. | avg.     | max. | avg.     | max. |
| 1.22 | 8.54 | 0.00     | 0.00 | 1.22     | 2.58 |

## 6 Conclusion

In the mathematical modeling of transportation planning problems in a network, achieving a good design has until now depended on setting model parameters skillfully. However, setting appropriate model parameters required trial and error, and the simple rigid application of parameters made it difficult to formulate plans flexibly in accordance with the intentions of designers.

To overcome these difficulties, we examined the application of an interactive multi-objective planning method to a large-scale water transportation system planning problem. By constraining the system to restore reservoir water level, one of the objectives of system operation, we took advantage of the fact that a multilayer approach is possible, wherein the problem is separated into an abstract level, dealing with daily volumes, and a detailed level, dealing with hourly quantities. Furthermore, we described a method that conserves flow at the abstract level and automatically generates the costs in a multilayer network model, such that a flow that also satisfies the flow smoothing conditions is a minimum cost flow.

In this paper, we explained the validity of separating the plan into an abstract level and detailed level, and we proposed a method for automatically generating the costs in a multilayer network model

based on the results of a plan formulation at the abstract level, which sets daily volumes. In addition, by separating the problem into an abstract level that deals with daily quantities and a detailed level that deals with hourly flow rates, we could make use of solution techniques that take advantage of the characteristics of each layer. To flexibly adjust for the aspiration levels of the water system designers, we utilized an interactive multi-objective planning method as a user interface. That is, as the aspiration levels are interactively obtained, a Pareto solution can be determined to serve as the abstract level plan. If an abstract level plan that satisfies the water system designers can be created, a cost generator can then generate the costs of the arc variables of the multilayer network model, based on the abstract plan. Then by determining the minimum cost flow under these dynamically generated costs, it is possible to derive a detailed plan that reflects the various requirements without the need to tune costs in advance by trial and error.

Using data from an existing water supply network, we performed a verification of the plan formulation method. The results confirmed that all reservoirs absorb demand fluctuations by varying their water level, that after 24 hours the water level is restored to its original value, and that the above design goals above are even satisfied in terms of flow smoothing. It was also possible to greatly reduce computational effort and shorten computing time.

Use of the proposed method was confirmed to offer all of the following improvements:

- 1) The possibility of dynamically generating parameters matching the preferences of designers without the need to tune and provide rigid model parameters in advance.
- 2) Even if a produced plan is unsatisfactory, the method allows for corrections or modifications to make the design satisfactory by means of trade-off analysis.
- 3) By separation into an abstract level and detailed level and formulating each independently, computational effort is reduced greatly and computing time is shortened relative to the case where a multi-objective planning method is applied directly without such separation.
- 4) The minimum cost flow under the automatically generated costs satisfactorily meets the goals of practical system operation, such as appropriate reservoir water level recovery and pipeline flow smoothing.

Looking ahead, we assume that further research will focus on developing improvements to devise methods applicable to more realistic operation control to ensure that reservoir water levels can be restored not just to their original levels, but also to a target level, independently of initial conditions.

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