# A Frequency Domain Approach for Design of Stable Fuzzy Logic Systems with Parallel Distributed Compensation

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*Abstract:* - Most industrial processes are nonlinear, multivariable, with time delay, variable parameters and model uncertainty, subjected to disturbances. There exist methods that compensate the impact of these plant peculiarities but are developed for linear control systems. Most of them are based on the system frequency response since they give a compact and easy design solution. A novel frequency domain approach is suggested for the design of fuzzy logic nonlinear systems with parallel distributed compensation (PDC) from requirements for global nonlinear system stability, robustness and good performance. It uses modified transfer functions based Takagi-Sugeno-Kang (TSK) plant model and PDC and the merits of the linear control frequency design methods. The modified PDC-TSK system is represented by a number of equivalent linear systems for which equivalent frequency responses are defined and frequency domain design criteria applied to ensure stability, robustness and desired performance specifications. The approach is demonstrated for the design of a PDC for the real time control of the air temperature in a laboratory scale dryer and its effectiveness confirmed in the experiments.

*Key-Words:* - Frequency response, Fuzzy logic control system design, Nyquist plot, Parallel distributed compensation, Real time temperature control, Takagi-Sugeno-Kang plant model, Nonlinear system stability

### **1** Introduction and State-of-the-Arts

The parallel distributed compensation (PDC) opened a new era in the development of fuzzy logic controllers (FLCs). Their better response to plant nonlinearity, model uncertainty and inertia leads to improvement of the closed loop system performance and energy efficiency. Introduced first in [1, 2] the PDC establishes as a model-based controller. The necessary for its development Takagi-Sugeno-Kang (TSK) plant model can easily be derived out of an available nonlinear plant model usually of a mechanical system (robot, helicopter, etc.) or form experimental data from the operation of the plant (process) in an open loop or in a feedback system with a model-free Mamdani controller [1-5]. The PDC is appealing because it is simple in structure and the fuzzy logic nonlinear PDC system design and stability analysis are based on the well mastered and proven in the engineering practice linear control methods.

The PDC structure is determined by the TSK plant model and is designed to perform soft switching among local linear mainly state feedback controllers [1]. Further advanced PDCs with PIDbased local controllers are developed and the Lyapunov system stability conditions modified [3, 6].

Presently modified transfer function-based TSK plant models are introduced in [7] to facilitate both the TSK plant modelling and the PDC engineering design. More sophisticated but compact linear local plants models such as multivariable, with pure time delay, etc. are easily derived via advanced linear control methods [8-10]. On their basis simple PDCs are designed consisting of local controllers which can perform various sophisticated linear control algorithms such as Smith predictors, internal model controllers, multivariable decoupling controllers, adaptive controllers, etc. [8-10] to better conform with the real world nonlinear plant model uncertainty, inertia, time delay, multivariable character, disturbances impact, parameter variations etc.

The design of the PDC nonlinear system consists generally of two stages [1, 3]. First, the local linear systems are designed to ensure systems stability and desired performance using linear control systems methods. Then the global nonlinear PDC system stability is validated using the time domain Lyapunov approaches based on the TSK-PDC system description or on Lyapunov functions. The stability conditions, however, are sufficient and strong. If the validation fails the PDC is redesigned [3, 6]. In the more progressive approaches the global nonlinear PDC system design is bounded with the nonlinear system stability. In [1] a relationship is derived between the local linear state feedback controllers' gains and the nonlinear system stability. The relationship, however, is difficult to solve, needs numerical techniques and may have no solution.

Recently the frequency domain approaches mark a new progress in the advanced linear control system design. In [11] a stability boundary locus of the individual channels is suggested for the design of a stabilizing PI controller for a multi-input multi output (MIMO) coupled tank system. A MIMO system is passified and stabilized via shaping of the individual channels sensitivity using gain and phase margins in [12]. A MIMO controller is designed after approximation of the generalized Nyquist stability criterion by convex constraints with respect to the controller's parameters in [13]. The frequency domain methods which are simple, well developed and widely spread for linear control systems design out of stability and robustness considerations [3, 14-16] have found a limited application in the nonlinear PDC system design. They are employed mainly for the design of the local linear control systems. Their potential for direct design of the nonlinear PDC ensuring system stability, good performance and robustness especially for plants with time delay, model uncertainty, etc. stays unexplored. So, the investigation of the possibility to design the PDC the global nonlinear system stability from requirements in the frequency domain is a challenging task.

In the present research a novel approach is suggested based on the definition of a frequency response of the PDC-TSK system and its application for the design of stable global nonlinear closed loop PDC systems with desired performance. The frequency domain approach considers single-input single-output (SISO) systems and needs no approximation of the plant pure time delay. It is especially effective for building more sophisticated PDCs.

The paper is further organized as follows. In Section 2 the theoretical background is presented. Section 3 is devoted to the novel frequency approach for the design of modified PDCs from nonlinear fuzzy logic closed loop system stability and robustness requirements defined in the frequency domain. The approach is demonstrated for the design of a PDC for the control of the air temperature in a laboratory scale dryer in Section 4. The designed PDC is tested in Section 5 in a closed loop system with the TSK plant model via simulations and in real time control of the dryer's air temperature. There the advantages of the PDC control compared to a Mamdani fuzzy logic control (FLC) in real time are discussed. The conclusion and the future work are outlined in Section 6.

## 2 Theoretical Background

The PDC-based FLC is designed on the basis of an existing TSK plant model. The standard TSK plant model is described by the following fuzzy rules [1, 2]:

$$\mathbf{R}_{k}: \mathbf{IF} z_{1}(t) \text{ is } \mathbf{L}z_{k1} \mathbf{AND...AND} z_{p}(t) \text{ is } \mathbf{L}z_{kp}$$
$$\mathbf{THEN.} \begin{cases} \dot{x}(t) = \mathbf{A}_{k} x(t) + \mathbf{B}_{k} u(t) \\ y(t) = \mathbf{C}_{k} x(t) \end{cases}, \qquad (1)$$

where  $z(t)=[z_j(t)]$ ,  $j=1\div p$  is the vector of the premise variables  $z_j$  that measure or estimate the system current state,  $z_j$  take linguistic values  $Lz_{kj}$ , represented by fuzzy sets,  $x(t)\in \mathbf{R}^n$  is the vector of *n* state space variables,  $u(t)\in \mathbf{R}^m$  is the vector of *m* plant inputs (control actions),  $y(t)\in \mathbf{R}^m$  is the vector of *m* plant outputs and  $\mathbf{A}_i \in \mathbf{R}^{n\times n}$ ,  $\mathbf{B}_i \in \mathbf{R}^{n\times m}$ ,  $\mathbf{C}_i \in \mathbf{R}^{m\times n}$ are the corresponding state, control and output matrices,  $k=1\div r$ . The number of the rules *r* corresponds to the number of the operation zones where the plant is described by a linear model.

Each current measured or estimated by z(t) plant state matches the defined by  $Lz_{kj}$  overlapping linear operation zones to different degrees. Thus each rule condition is fulfilled to a certain degree. The rules conclusions with the local linear models outputs are qualified to the degrees of activation of the rules. Then the scaled outputs in all rules are united. The final plant output is computed after a weighted average defuzzyfication.

As a result of the fuzzy inference mechanism and the defuzzyfication the model output is obtained as a soft blending of the individual rules conclusions (weighted average of the outputs of the local linear models) in the form:

$$\dot{x}(t) = \frac{\sum_{k=1}^{r} w_k(z(t)) [\mathbf{A}_k x(t) + \mathbf{B}_k u(t)]}{\sum_{k=1}^{r} w_k(z(t))},$$
(2)

where  $w_k(z(t)) = \prod_{j=1}^p \mu_{kj}(z_j(t))$  is the degree of fulfillment of the compound condition in the fuzzy rule premise and  $\mu_{kj}(z_j(t))$  is the degree of matching of  $z_j(t)$  with  $Lz_{kj}$ .

TSK models are developed via linearization of existing nonlinear models in [1, 2]. In [3-5] neuro-fuzzy structures that represent first order Sugeno models (TSK models) are trained on available experimental and expert data using MATLAB<sup>TM</sup> toolbox Adaptive Neuro-Fuzzy Inference System (ANFIS) [17]. First the number of the membership functions (MFs) is computed by partitioning of the input-output space via fuzzy clustering, then the fuzzy rules are automatically generated and finally the parameters of the MFs and the gains in the conclusions are optimized. The obtained TSK model

conclusions

$$\begin{cases} \dot{x}(t) = \mathbf{A}_{k} x(t) + \mathbf{B}_{k} u(t) + \mathbf{D}_{k} \\ y(t) = \mathbf{C}_{k} x(t) \end{cases}$$

contain free terms  $\mathbf{D}_k$  which approximate the nonlinear terms in the state space linearization. The training sample should be representative and preprocessed by normalization or standardization, noise filtering, correlation elimination, etc. The training, however, can be slow and validation may turn out unsuccessful.

In [3, 6, 7] a modified transfer function based TSK plant model is derived based on plant inputoutput data via model parameters optimization for minimization of the modelling error using genetic algorithms (GA). An example of a modified TSK plant model is shown in Fig.1. It consists of a Sugeno model for defining of r plant linearization zones, r=3, and for computing of the degrees of matching of the current plant output to each of the zones, and r parallel branches with input u - the real world plant input, and dynamics, described by transfer function  $P_k(s)$  in each zone. The Sugeno model has a single input - the measured plant output when deriving the TSK plant model or the TSK model output after the derived TSK model is validated. In Fig.1 three input Gaussian MFs of the Sugeno model define three linearization zones. The Sugeno model has a separate output for each k-th linearization zone. Each k-th output has singletons for MFs, the k-th singleton is located at 1 and the rest are placed at 0. The fuzzy rules, the fuzzy



Fig.1. Modified transfer functions based TSK plant model

inference and the defuzzyfication in the Sugeno model aim each of its outputs to yield the MFs values  $\mu_k$  of matching of the current measured plant output to each k-th zone. Thus the output of each local linear plant model  $y_k$  is scaled by the corresponding match  $\mu_k$  to the *k*-th zone and then all scaled outputs of the local plant models are summed to result in the final nonlinear TSK plant model output  $y_{\text{TSK}}$ . The TSK plant model parameters – the parameters of the transfer functions of the local dynamic models and the parameters in the input MFs of the Sugeno model if not assigned by experts are computed to minimize the integral squared relative modelling error usually using GA [6, 7, 10]. In Fig.1 the input MFs of the Sugeno model are GA optimized using experimental input-output data about the plant.

The transfer functions based description of the local plants dynamics makes representation compact as transfer functions and matrices represent plant input-output relationships instead of detailed state space variables. Thus the nonlinear plant is represented by soft switching between linear plants, which dynamics experts can easily define by the use of simple transfer functions that also ease the design of the corresponding local linear controllers even the sophisticated ones by engineering methods.

The standard PDC, based on the TSK plant model, is described by the following fuzzy rules [1,2]:

**R<sub>k</sub>: IF** 
$$z_1(t)$$
 is  $Lz_{k1}$  **AND...AND**  $z_p(t)$  is  $Lz_{kp}$   
**THEN**.  $u(t) = -F_k x(t)$ , (3)

where the state feedback controller  $F_k$  is designed to compensate the local plant in the corresponding

TSK plant model rule using linear control systems methods.

As a result of the fuzzy inference mechanism and the weighted average defuzzyfication the control action u(t) is computed by soft blending of the individual rules conclusions in the form:

$$u(t) = -\frac{\sum_{k=1}^{r} w_{k}(z(t))F_{k}x(t)}{\sum_{k=1}^{r} w_{k}(z(t))}$$
(4)

In case of PI local linear controllers, the incremental PI algorithm is transformed into state space representation in the fuzzy rules conclusions [6]:

$$\mathbf{R_{k}: IF } z_{1}(t) \text{ is } Lz_{k1} \text{ AND...AND } z_{p}(t) \text{ is } Lz_{kp} \\ \mathbf{THEN} \begin{vmatrix} \dot{u}(t) = -\mathbf{F}_{k}x(t) + \mathbf{G}_{k}x_{r} \\ \text{or } \dot{u}(t) = K_{pk}\dot{e}(t) + (K_{pk}/T_{ik})e(t) \end{vmatrix}$$
(5)

where  $\mathbf{F}_{k} = \begin{bmatrix} K_{pk} / T_{ik} & K_{pk} \end{bmatrix}$  and  $\mathbf{G}_{k} = \begin{bmatrix} K_{pk} / T_{ik} & 0 \end{bmatrix}$ with  $K_{pk}$  and  $T_{ik}$  - the gain and the integral action time of the local PI controller in the *k*-th zone, and  $x_{r}$ - the vector of references for the state variables. The necessary integration of the control rate  $\dot{u}(t)$  is included as an extension to the plant input thus making the local plants critically stable and increasing their order by one.

The global PDC-TSK nonlinear closed loop system stability analysis is based on the Lyapunov stability sufficient condition - the existence of a common positive determined matrix  $\mathbf{P}$  for all local linear systems that satisfies the matrix inequalities [1, 3, 6]:

 $\begin{aligned} \mathbf{G}_{mm}^{\mathrm{T}} \mathbf{P} + \mathbf{P} \mathbf{G}_{mm} &< 0\\ 0.5(\mathbf{G}_{mn} + \mathbf{G}_{nm})^{\mathrm{T}} \mathbf{P} + \mathbf{P}.0.5(\mathbf{G}_{mn} + \mathbf{G}_{nm}) &\leq 0, \end{aligned} \qquad (6)\\ \mathbf{G}_{mn} &= \mathbf{A}_{m} - \mathbf{B}_{m} \mathbf{F}_{n} \end{aligned}$ 

$$m, n=1 \div r, n > m.$$

The solution of (6) is searched by the numerical linear matrix inequalities (LMIs) technique [1, 6].

The modified PDC is based on a modified TSK plant model of the type, demonstrated in Fig.1. It uses the same Sugeno model of the TSK plant model to recognize the degrees of belonging of the current plant state to the defined operation zones where the plant model is assumed linear. An example of a PDC structure is shown in Fig.2, where  $C_k(s)$  are the local linear controllers, each designed for its corresponding local linear plant



Modified transfer functions based

Fig.2. Modified transfer functions based PDC with three linearization zones

model in the TSK plant model. In Fig.2 the presented Sugeno model is designed by experts.

# **3** A Frequency Domain Approach for PDC Design from Nonlinear Closed Loop System Stability and Robustness Requirements

The frequency domain methods for the design of SISO linear control systems are well developed for transfer function based plant and controller description. They are suitable for plants with time delay as the transcendental time delay element  $e^{-ts}$  is represented in the frequency domain without approximation by the Euler's expression

 $e^{-j\tau\omega} = \cos(\tau\omega) - j \sin(\tau\omega)$ . The main frequency domain performance indicators such as degree of oscillations m, gain C and phase G margins, oscillation index M, etc. are closely related with the location of the dominating roots of the closed loop system characteristic polynomial or of the open loop system Nyquist plot. Thus they are related with the closed loop system stability and robustness [14-16]. These indicators allow also to estimate important time domain performance measures of the closed loop system such as settling time  $t_s$ , overshoot  $\sigma$ , etc. [16]. The design of the linear controllers from the requirement to ensure desired frequency domain performance indices of the closed loop system can also ensure system stability and robustness and desired time domain specifications.

The nonlinear PDC-TSK closed loop system (1), (3) for each measured current values for  $z(t_i)$ ,  $u(t_i)$ ,  $x(t_i)$  and  $y(t_i)$  computes the next moment  $t_{i+1}$ necessary control and plant output according to (2), (4). The closed loop system with the modified transfer functions based TSK plant model and PDC is presented in Fig.3, where z(t) = y(t) and hence in (2), (4)  $w_k = \mu_k$ . For each different combination of  $\mu_1 \div \mu_r$  as a result of the current measurement of  $v(t_i)$ the nonlinear closed loop system is represented by a different linear system which consists of an equivalent linear plant  $P^{eqi}(s)$  and a corresponding equivalent linear controller  $C^{eqi}(s)$ , both computed as current for this measured  $y(t_i)$  as weighted average of the local plants and controllers. So, the nonlinear PDC-TSK system can be viewed upon as a family of linear systems of the possible equivalent linear plants and controllers computed for all combinations of  $\mu_1 \div \mu_r$ . The number of the linear systems that represent the PDC-TSK nonlinear system is theoretically infinite since  $\mu_1 \div \mu_r$  can take continuous values in the range [0, 1]. Usually the Sugeno model is designed by an expert to have an overlapping of two adjacent fuzzy sets for the input and orthogonal MFs -  $\sum_{k=1}^{r} \mu_k(y(t_i)) = 1$ .

The Laplace representation of the equivalent linear plants and controllers are defined considering (2) and (4) and Fig. 3 as follows:

$$P^{\text{eqi}}(s) = \frac{\sum_{k=1}^{r} \mu_{k}(Y^{i}(s))P_{k}(s)}{\sum_{k=1}^{r} \mu_{k}(Y^{i}(s))},$$

$$C^{\text{eqi}}(s) = \frac{\sum_{k=1}^{r} \mu_{k}(Y^{i}(s))C_{k}(s)}{\sum_{k=1}^{r} \mu_{k}(Y^{i}(s))}.$$
(8)

The equivalent linear plant and controller description (8) enables to define equivalent open loop system Nyquist plots  $W_{OL}^{eqi}(j\omega)$  and equivalent closed loop system spectrums  $A_{CL}^{eqi}(\omega)$ . The worst Nyquist plot of all with respect to stability, i.e. the



**Fig.3.** Modified transfer functions based TSK plant model and PDC with three linearization zones

closest to the Nyquist point (-1, j0), computed for some measured  $y(t_i)$  or combination of  $\mu_1 \div \mu_r$ , defines the equivalent gain  $C^{eq} = \min_i (C^{eqi})$  and phase  $G^{eq} = \min_i (G^{eqi})$  margins of the nonlinear PDC-TSK system. From the oscillation indices  $M^{eqi} = A_{CLmax}^{eqi}(\omega_{peak}^{i})$  of all equivalent closed loop systems the highest determines the equivalent oscillation index of the PDC-TSK system  $M^{eq} = \max(M^{eqi})$ .

These definitions enable the simultaneous design of all local linear controllers from the requirement for desired equivalent stability margins ( $C^{eq}_{d}$ ,  $G^{eq}_{d}$ ) or a desired equivalent oscillation index  $M^{eq}_{d}$  thus ensuring nonlinear system stability, robustness and good time domain performance.

The most commonly used desired values for the frequency domain performance indicators are shown in Table 1, where the degree of oscillations  $m = \alpha/\beta$ , determined by the dominating roots of the characteristic equation of the closed loop system  $\rho_{1,2}=-\alpha+i\beta$ , and the oscillation index M shape underdamped closed loop system step responses. The gain C and the phase G margins are related to the system robustness. The corresponding values of the time domain indicators damping ratio  $\Psi$  and overshoot  $\sigma$  are also given in Table 1. They are computed on the basis of a relationship between the frequency domain and the time domain indicators for a second order system. For a higher order system relationship between the indicators the is approximate and refers to the dominating pair of roots. For overdamped closed loop system step responses M=1 and  $\sigma=0\%$ .

A PDC design procedure is suggested that consists of the following steps.

**Input data:** number of linearization zones *r*; derived modified TSK plant model of a Sugeno model and local linear dynamic plants, described by transfer functions; type of the local linear controllers; range  $\mathbf{D}_{\omega} = [\omega_{o}, \omega_{f}]$  of significant for the system frequencies, determined by the greatest time constant of the local plants; desired performance indicators of the tuned equivalent linear systems.

1. Computation of a finite number of combinations  $\mu = [\mu_1 \dots \mu_r]$  by discretizing the interval

Table 1 Most commonly used performance indices

Ψ	σ, %	т	M	С	G, deg
0.75	50	0.222	2.38	0.30	24
0.90	33	0.366	1.55	0.39	38
0.96	20	0.511	1.25	0.50	48

[0, 1] for each  $\mu_k$  with a fixed step  $\Delta \mu_k$  and considering equally spaced discrete values for  $\mu_k$ . The combinations of the discrete values for  $\mu_1 \div \mu_r$  are of specific type for orthogonal MFs, e.g. for *r*=3 and  $\Delta \mu_k$ =0.2 the eleven combinations are:

2. Estimation of the ranges for the tuning parameters with respect to the acceptable tolerance for the system performance.

3. Initial tuning of the parameters of the local linear controllers using engineering methods for ensuring local linear systems stability and performance.

4. Computation of the equivalent open loop system Nyquist plots  $W_{OL}^{eqi}$  (j $\omega$ ) and closed loop system spectrums  $A_{CL}^{eqi}$  ( $\omega$ ) for each combination  $\mu = [\mu_1 \dots \mu_r], i=1 \div 11.$ 

5. Estimation of the current performance indicator from the worst equivalent system. If the current performance indicator is close to the desired, the tuning is over and the design procedure ends. Else the procedure continues with step 6.

6. Correction of the tuning parameters of the local linear controllers within the established ranges and repeating of the design procedure from step 4.

# 4 Application of the Frequency Domain Procedure for the Design of PDC for the Control of Temperature

The suggested frequency domain procedure for the design of a PDC accounting for the global nonlinear system stability and performance is applied to design a PDC for the real time control of the air temperature in laboratory-scale dryer [3, 18]. The temperature is controlled in MATLAB<sup>TM</sup> real time by the help of an interfacing board between the plant and the Simulink model of the PDC. The PDC computes the control u on the basis of the measured temperature by the temperature sensor and transmitter and its reference. The control is then passed to a pulse-width modulator (PWM) that connects via a solid state relay an electrical heater and a fan to the power supply during the duration of the pulses. The control action u is limited in the range [0, 10]V for the proper operation of the PWM.

The TSK plant model derived and validated from plant input-output experimental data is shown in Fig. 4. It distinguishes three linearization zones



**Fig.4.** Modified TSK plant model for air temperature in three linearization zones

which are defined by experts in the Sugeno model. The TSK plant model parameters  $\mathbf{q}_{\text{TSK}}$  are computed to minimize the integral squared relative modelling error using GA –  $\mathbf{q}_{\text{TSK}} = [K_1 = 5.34, K_2 = 17.53]$  $K_3 = 8.84, T_1 = 32s, T_2 = 154s, T_3 = 112s, T_4 = 1.4, y_0 = 35$ ]. Each local plant model is approximated to a Ziegler-Nichols (ZN) model  $P_k(s) = K_k \cdot e^{-\tau k \cdot s} \cdot (T_k s + 1)^{-1}$  based on the Taylor's series expansion of the time delay element  $e^{-\tau k.s} \approx (\tau_k s + 1)^{-1}$ , where  $\tau_1 = \tau_2 = \tau_3 = T_4$  since  $\tau_k = \min(T_k, T_4)$ . The ZN local plant models enable the application of engineering methods for fast tuning of standard local linear controllers. Here PI local controllers with transfer functions  $C_k(s) = K_{\text{pk}}(1+1/T_{\text{ik}}s)$  are initially tuned accounting for the corresponding local linear ZN plant models. The tuning criterion is a desired overshoot and settling time of the local linear closed loop systems and the method used is empirical [14-16] - $K_{pk}=A*T_k/(\tau_k.K_k)$ ,  $T_{ik}=B*T_k$ . The ranges for the tuning parameters  $(K_{pk}, T_{ik})$  are determined by the coefficients  $A \in [0.1, 1.4]$  and B=0.5/A which define various acceptable overshoots. The discrete values of the MFs for the three linearization zones make the combinations (9). These combinations result from all possible measured values for the temperature and determine the weighted average in mixing the outputs of the three local linear plants.

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For each couple of coefficients (A, B) eleven equivalent open loop system Nyquist plots  $W_{OL}^{eqi}(j\omega)$  and closed loop system spectrums  $A_{CL}^{eqi}(\omega)$ ,  $i=1\div11$ , are computed and depicted in Fig. 5. The accepted requirement in the frequency domain for the PDC tuning is  $M^{eq}=M^{eq}_{d}=1$ , i.e. overdamped step responses of the nonlinear PDC closed loop system ( $\sigma=0\%$ ). The subset of the equivalent open loop system Nyquist plots and the equivalent closed loop system spectrums that satisfy this requirement, are shown in Fig. 6.

The optimal PDC tuning parameters  $\mathbf{q}_{\text{PDC}}^{\circ}$  are determined among these that ensure M<sup>eq</sup>=1 using an additional criterion for minimal product of  $K_{\text{pk}}/T_{\text{ik}}$ ,  $k=1\div3$ . Thus  $\mathbf{q}_{\text{PDC}}^{\circ}=[K_{\text{p1}}=0.87, K_{\text{p2}}=1.27, K_{\text{p3}}=1.83, T_{\text{i1}}=81, T_{\text{i2}}=386 \text{ s}, T_{\text{i3}}=280 \text{ s}]$  and are computed for (A°=0.2, B°=2.5) and *i*=6 that defines  $\mu=[\mu_1=0, \mu_2=1, \mu_3=0]$ . The equivalent Nyquist plot and system spectrum for the PDC optimal parameters are outlined in green in Fig. 6.



**Fig.5.** Equivalent open loop system Nyquist plots and equivalent closed loop system spectrums for all values of (A, B)



**Fig.6.** Equivalent open loop system Nyquist plots and equivalent closed loop system spectrums with  $M^{eq}=1$ , in green - for the optimal PDC parameters

# **5** Investigation of the Designed PDC system via Simulations and in Real Time Temperature Control

The investigation of the designed PDC system pursuits several goals:

- to prove that the frequency domain based approach for the design of PDC ensures nonlinear system stability and good performance by simple means;

- to prove that the system performance indices do not surpass the used in the PDC design despite the plant nonlinearity expressed in different parameters when operating in different operation points (zones), so the design criteria are observed;

- to compare the PDC system performance with the performance of the empirically designed in [3, 18] model-free Mamdani FLC system, a design of a PDC based on more knowledge about the plant contained in its TSK model gives grounds for better responding to the plant nonlinearity and hence for PDC system performance improvement;

- to assess the energy efficiency of the control;

- to assess the impact of the TSK plant model accuracy on the PDC design.

The step responses for various temperature references, that cover the operation range of the plant, are obtained for three different closed loop control systems:

- Mamdani FLC system in real time temperature control;

- PDC system with the derived TSK plant model in simulations;

- PDC system in real time control of the air temperature in the laboratory scale dryer.

The investigation of the systems via simulations and in real time control is carried out using the facilities of MATLAB<sup>TM</sup> and its Toolboxes [19, 20];

The temperature step responses of the simulated PDC-TSK system, of the PDC system from the real time control and of the Mamdani FLC system from the real time control are depicted in Fig.7. The control actions are shown in Fig.8.

The analysis of the results shows:

• The PDC-TSK system step responses from simulation are fast and overdamped in all operation points and correspond to the PDC frequency domain based design criterion.

• The PDC system step responses from real time control are close to the PDC-TSK system step responses but have a small overshoot which can be explained by the measurement noise that causes oscillations in the control *u* and the real time control restrictions. This proves the good accuracy of the TSK plant model and hence the precise tuning of the local PI controllers of the PDC;

• In real time control the PDC system is 2-5 times faster than the Mamdani FLC system with settling times in the range –[60, 200]s against [200, 450]s;

• The PDC system control u is 2-3 times smaller and with shorter settling time than the Mamdani FLC system control except for the short peaks at the reference step changes. This is an evidence for an improved energy efficiency of the control – good systems performance is achieved with less energy for the control - low u means low duty ratio and less energy consumed by the dryer's heater and fan.

#### 6 Conclusion and Future Research

A novel frequency domain approach is suggested



Fig.7. Temperature step responses from real time control in PDC and Mamdani PI FLC systems and in simulated PDC-TSK system



**Fig.8.** Control action *u* from real time control in PDC and Mamdani PI FLC systems and in simulated PDC-TSK system

for the design of a parallel distributed compensation from requirements for global nonlinear system stability, robustness and good performance. It considers modified transfer functions based TSK plant model and PDC and defined for them equivalent linear systems frequency responses and frequency domain performance indicators.

The approach is demonstrated for the design of a PDC for the real time control of the air temperature in a laboratory scale dryer. The step responses to different reference changes during the real time PDC control prove the closed loop system stability and closeness to the desired specifications used in the design procedure despite the plant nonlinearity. The PDC control reduced the settling time and the control action compared to the Mamdani FLC. The PDC design is based on relatively accurate modified TSK plant model derived and validated from processed experimental input-output data rich in magnitudes and frequencies.

The suggested frequency domain approach can be used with all transfer function based design methods, developed for linear control systems. In this way it facilitates the design and tuning of more sophisticated PDCs with various local linear controllers such as Smith predictor, internal model controllers, multivariable controllers, etc.

The future research will focus on the development of the suggested approach for the purpose of the PDC design for MIMO plants.

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