Optimization of a Vendor Managed Inventory Supply Chain Based on Complex Fuzzy Control Theory

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Abstract: This paper recommends a scaling factors fine-tuning fuzzy logic control approach to optimize the dynamic performance of one typical vendor managed inventory supply chain with automatic pipeline, inventory and order based production control system (VMI-APIOBPCS), based on complex fuzzy control theory. The first thing is to embed a dual-input single-output fuzzy logic controller into the system based on the classic control engineering model. Then, the fuzzy inputs are given different weights by the way of scaling factors in order to optimize the system further. This methodology can make good use of managers’ experience accumulated in perennial practice and the managers’ rational estimation of different circumstances. Lastly, the simulation results show that, this method can improve the dynamic performance of VMI-APIOBPCS, especially the inventory dynamic behaviors.

Key–Words: VMI-APIOBPCS, fuzzy logic controller, scaling factors, dynamic performance

1 Introduction

With the progress of the information technology the urge of mutual benefit organizations (suppliers, manufacturers, distributors, wholesalers, retailers) in supply chain accommodate their strategies to this new collaborative work tendency[1]. The operational pattern of VMI come into being at the opportune historic moment. As one of the most prevalent integrated styles, the implementation of its effective control is pressing to build modern manufacturing system. However, owing to the intrinsic complexity and turbulent market changes, the effective controlling became unrealistic[2, 4]. Many factors, such as forecast error, the block of information delivery, demand change etc., always result into unexpected overstock and incremental of overall running cost[3, 5, 6, 7, 8, 12].

Those problems, existed in VMI system, are severe in other classic production and inventory control system[11], likewise, which have been discussed heatedly both in practical management and academic field for decades. Early in 1982, Towill adopted control engineering method to optimize the IOBPCS (inventory and order based production control system) by setting the proportion of inventory adjust time ($T_i$) and production delay ($T_p$) and the proportion of demand forecast smoothing time ($T_a$) and production delay ($T_p$), respectively[3]. Later, GA was employed to optimize three control parameters ($T_a$, $T_w$, $T_i$) of APIOBPCS-S, based on stability and robustness of system, and especially, considered the work in progress (WIP) adjust time ($T_w$) in the optimization, a beginning of taking the production and inventory control system as an whole picture[5]. S.M.Disney, based on the research achievement in 2000, synthesized six parameters ($T_a$, $T_w$, $T_i$, $T_q$, $G$, $W$) of VMI-APIOBPCS and made an simulation optimization[6]. The centralized management method, VMI was adopted in this paper to make the manufacturer of APIOBPCS pay more attention to the integrated benefit, thus the distributor’s forecast smoothing time ($T_q$), the proportion of Distributors Safety Stock and Average consumption ($G$) and Ratio of production adaptation to inventory cost ($W$) were considered. In conclusion, the optimization methods mentioned above simply adopted mathematical arithmetic, only reached an ideal combination of control parameters in the mathematical sense.

Although, we researched production-inventory system all-around by the classic control engineering method and came up with numerous of optimization outcomes[19, 20, 21, 23, 28], many supply chains, reported worldwide, still suffered from bad supply chain performance[7]. This situation reminds us to give deep thought about this research angle.

Actually, many researchers investigated this issue in different points. White utilized proportion-integration-differentiation (PID) controller to optimize the IOPBCS, and greatly reduced the inventory
level[16]. B. Samanta combined PID controller with fuzzy logic controller to optimize an inventory control system[18]. At last, the system is capable of pursuing the final system inventory level at the desired level in spite of variations in demand. However, the PID controller is not welcomed in the production-inventory research field for its congenital drawback that its corresponding hardwares is not existed in virtual production-inventory systems[7].

As regards to the control of inventory and production system, a kind of complex social economic system, the element of social sciences is requisite. As the Figure 1 informs us, one critical parameter can be connected with another three or four ones, and mostly are determined by managers based on the relevantly internal and external factors, such as consumer loyalty, long term profits.

The VMI-APIOBPCS model was rebuilt by fuzzy difference equations, then genetic algorithms (GA) was adopted to search optimal parameters of fuzzy VMI-APIOBPCS model[9]. In final, bullwhip effect was reduced and the overall performance was bettered. Yohanes Kristianto cleverly inserted the fuzzy logic controller with dual-input and one-output into VMI supply chain system, and lastly an ANOVA test, set to assess the assumptions, verified that the inventory response is effectively improved. This method can not only imitate the human thinking, but also absorbed managers’ experience[14]. However, with the turbulent change of modern market and the management environment, the original experience may not be completely adaptable to the new surroundings. Fuzzy logic controller is kind of artificial intelligence and its implementation relies on complex computer techniques. As Filippo Neri said in [10], this kind of model can carry information about the volatility and the correlation among multi-factors, which enables the modern supply chain to be more flexible and accurate.

In view of above drawbacks and requirements, this paper inserts the fuzzy logic controller into the classic VMI-APIOBPCS model built in control engineering[6]. But here the continuous-time version is considered. The potential fuzzy logic controller is connected with a more complex system than VMI with the expectation of extensive revenue. Then different weights are exerted on the dual fuzzy inputs further to enable the experience to suit the present surroundings.

The remaining parts of this paper proceeds as follows: section 2 includes the VMI-APIOBPCS model and introduces the related parameters should be fuzzy; introduces the complex fuzzy control theory and the fuzzy inference system applied in this paper and its optimization; the introduction of objection function. Section 3 shows us the simulation results and corresponding analysis. In section 4, the conclusions are made.

2 Fuzzy VMI-APIOBPCS Control Model

2.1 Construction of VMI-APIOBPCS Model

In 1961, Forrester firstly adopted industrial dynamics (equals to system dynamics) in the research of production and inventory control system. After that, Towill expanded the model into the form of IOBPCS, moreover, carried out a string of optimizations of the system dynamic performance. Simon continuously expanded the model into the more complex form of APIOBPCS with taking WIP into consideration[32]. And the VMI-APIOBPCS is the combination of VMI supply chain and APIOBPCS, which is displayed in Figure 1, in which the variables are classified by different color: words colored green are control parameters in the system: words colored red are parameters been controlled; words colored blue are parameters based on observation or recording; words colored orange are the control parameters limited by consumer loyalty. Overall, the model of VMI-APIOBPCS synthesizes the multi-aspect interactions.

In VMI-APIOBPCS, distributors provide inventory information and data of sales to the supplier. Meanwhile, both of them reach a consensus in terms
of Reorder-point, in order to avoid excess inventory. While the inventory level of distributors below the Reorder-point, the supplier will supply the proper production automatically. Then, the manufacturer of the VMI supply chain will execute the function of APIOBPCS such that makes new production plan or distribution plan according to its inventory level.

Six parts are included in this system: 1) distributors’ demand forecasting policy; 2) factory’s demand forecasting policy; 3) the set of system inventory target; 5) the feedback loop of WIP; 6) production delay. We concluded them into two classes: demand forecasting policy and inventory policy. We can distinctly read the above knowledge in Figure 2(All the ins and outs in the above block diagram mean the connections with other subsystem that will be expressed in the following parts).

All in all, VMI-APIOBPCS, as an integrated management model, effectively slims down the supply chain system, smoothes the information and motivates the agile production.

2.1.1 Demand Forecasting Policy

We use exponential smoothing to predict the demand quantities of distributors and factory. For the sake of convenience, the sample time $\Delta t$ is set for 1 in this continuous-time model.

According to [20], we can obtain the relationship between the factory’s demand forecasting constant $\alpha_a$ and factory’s time to average sales: $\alpha_a = 1/(1 + T_a)$. For same argument, as to distributors, the relationship is: $\alpha_q = 1/(1 + T_q)$. Forecast error $\varepsilon$ is a stochastic variable, with mean zero. At last, the initial input of the whole system is consumer consumption such that market demand

$$C O N S_t = \begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{if } t \geq 0 \end{cases}$$

2.1.2 Inventory Policy

In this paper, $T I N V_t = 0[3, 6]$. $T_p$ is a parameter beyond of control, restricted to manufacturing facility, product type, efficiency of production and so on[3], and in this paper we set $T_p = 4$. From the Figure2, we can obtain that the ORATE is decided by factory’s demand forecasting, product of inventory deviation and $(1/T_i)$, product of WIP deviation and $(1/T_w)$. As to $T_i$, $T_q$ is decided by $\alpha_i, \alpha_a$, such that $\alpha_i = 1/(1 + T_i), \alpha_a = 1/(1 + T_w)[14]$. $T\bar{p}$ is the estimate of the average production delay, and $T\bar{p} = 4$. $G$ is the proportion of distributors’ safety stock between average consumption, reflecting the consumer service level.

2.1.3 Optimization of Parameters

Towill built the IOBPCS model, and acquired the optimal parameters by analyzing the sensitivity of the control parameters. Disney optimized the control variables ($T_a, T_i, T_w, T_q, G, W$) by simulation, and the results were assessed by $ITAE$ (Product of Time and Absolute Error)[6]. Darya Kastsian adopted normal vector method to optimized the control parameter ($T_a, T_i, T_w, T_q$), based on the stability and robustness of system[33]. Kuo Ping Lin combined fuzzy mathematics and GA to obtain optimal dynamic performance of VMI-APIOBPCS [9]. Yohanes Kristianto pointed out there is a drawback for the forecast changed can unilaterally decided the smoothing constant. The decision support system should take more errors or inevitable deviations[14].

Disney obtained strict optimal parameters ($T_a, T_i, T_w$), based on the stability and robustness of system, then, analyzed the impact of change of single control parameter on the dynamic inventory response and received that the change of $T_i$ incurred maximum variation of the dynamic inventory level[5]. To discern the indication of the fuzzy logic controller optimization more distinctly, we just choose $T_i$ as our optimization parameter.

Furthermore, in order to make a more pragmatic optimization, we additionally select two deviations as the fuzzy inputs. According to [14], we select demand change and the difference between inventory level and demand as inputs.
2.2 Complex Fuzzy Control Theory

The variables of complex system always have no definite relationships in mathematical sense, even are impossible to quantitative analysis with diverse assumptions[13, 15]. Traditional control theory is confined. In contrast, fuzzy control can make good use of experts’ knowledge and experience, relying on fuzzy inference and decision-making to realize the control of complex system, especially the complex social economic system with nonlinear lumped or distributed parameter [29, 30, 17].

The human factors in decision-making mainly include attitude to risk, intuition, experiences, or the combination of some of them. Those factors can directly act on the result of decision-making[1, 31]. In practical, managers can accumulate lot of experience for long time, expressed as a language such as straight lines, triangular, trapezoids, haversine, exponential[22]. And we adopt the simple and effective triangular one as our membership function, which is expressed in Table1. We can understand it by Figure4 more intuitively.

In this paper, the two input variables are demand change ($\delta_t$) and deviation between system level and demand change ($\varepsilon_t$). $\delta_t = D_t - D_{t-1}$, $\varepsilon_t = AINV_t - CONS_t$. $\varepsilon_t$ can be produced by system itself in the simulation. Besides we assume $\delta_t = 0.2$ such that the demand change of system is 0.2 (Actually, the demand change is a stochastic variable, which is dependent on season, promotion, product life cycle and so on. Here we assume it as a constant just for simplifying simulation process).

2) Database

Database stores all the membership functions that are used in input and output, providing data to the inference system. In this paper, the membership functions of inputs and output are in form of Figure4, collaboratively.

Finally, the combination of database and rule-base produces the fuzzy inference system, which is illustrated by Figure5. After a series of fuzzy operations, all the rules can form the fuzzy rule curved surface, like Figure6.

In Figure4 and Figure6, ECa means smoothing constant $\alpha_t$; DEF means $\varepsilon$; DC means $\delta$.

3) Rule-base

The Rule-base of fuzzy logic controller is based on experts’ knowledge and frontline workers’ experience accumulated for long time, expressed as a lan-

![Figure 3: Fuzzy logic controller](image)

![Figure 4: Membership function](image)

<table>
<thead>
<tr>
<th>Linguistic scale</th>
<th>input $\Delta(\delta, \varepsilon)$</th>
<th>Smoothing constant($\alpha$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very High (VH)</td>
<td>$0.75 \leq \Delta \leq \infty$</td>
<td>0.5:1:1</td>
</tr>
<tr>
<td>High (H)</td>
<td>$0.51 \leq \Delta \leq 0.74$</td>
<td>0.25:0.75:1</td>
</tr>
<tr>
<td>Medium (M)</td>
<td>$0.26 \leq \Delta \leq 0.50$</td>
<td>0.25:0.5:0.75</td>
</tr>
<tr>
<td>Low (L)</td>
<td>$0.05 \leq \Delta \leq 0.25$</td>
<td>0:0.25:0.75</td>
</tr>
<tr>
<td>Very Low (VL)</td>
<td>$\Delta \leq 0.04$</td>
<td>0:0:0.5</td>
</tr>
</tbody>
</table>

Table 1: Membership function
Finally, the combination of database and rule-base produces the fuzzy inference system, which is illustrated by Figure 5. After a series of fuzzy operations, all the rules can form the fuzzy rule curved surface, like Figure 6.

4) Defuzzification output interface

Defuzzification output interface can transform the fuzzy outputs into the normal form the control system can identify and accept. In this paper, we adopt the frequently-used centroid calculation, known as the center of gravity of area defuzzification (the explicit process, which is carried out in the fuzzy logic controller (FLC in Figure 7, is in [14]). After defuzzification, we get the smoothing constant \( \alpha_i \), consequently converted through a series mathematical calculations into the form of \( Ti \) based on \( \alpha_i = 1/(1 + Ti) \). The

subsystem of the calculations in simulink is displayed in Figure 7.

2.2.2 Optimization of Fuzzy Logic Controller

Generally, in the simple fuzzy logic controller, all inputs have the same influence on the fuzzy logic controller such that we rigidly follow the original experts’ experience. However, this natural extraction of experts’ a priori knowledge is not always easy or possible to realize [37], for some of the experience may not be suitable for present situation due to uncertainties. In this paper, after exerting different weights on the inputs by scaling factors, managers can flexibly master the inputs to be better for the control [24, 25, 26].

\( \mu, \delta \) and \( \varepsilon \) are the fuzzy variable of their own discourse domain, so the control table of the simple fuzzy logic controller can be expressed by the following analysis formula.

\[
\mu = \langle (\delta + \varepsilon) / 2 \rangle 
\]

(1)

In order to enable the fuzzy logic controller to suit for different surroundings, we need to expand the control table to have more space to be revised. In this paper, we expand (1) into

\[
\mu = \langle K1 \times \delta + K2 \times \varepsilon \rangle
\]

(2)

\( K1, K2 \in (0,1) \). \( K1, K2 \) are independent from each other, used to regulate the degree of impact of inputs on fuzzy control. In other words, we consider the scaling factors \( K1, K2 \) as the subjective weights of inputs given by managers in different situations. Meantime, we assume \( K1 + K2 = 1, K1, K2 \in (0,1) \). Then we can adjust the scaling factors to find the optimal result.

2.3 Evaluation of Fuzzy VMI-APIOBPCS Control System

The complex of production-inventory system directly makes its evaluation intractable, for it is involved in versatile factors, such as dynamic response time, errors, deviations etc. But in this paper, we have only one goal for evaluation of the system control—minimum cost. According to this criterion, we can draw up the objective function.

Towell evaluated the IOBPCS by

\[
P.I. = \int_0^\infty (COMRATE)^2 + \mu^2 (INV.DEV)^2 dt
\]

, \( COMRATE \) means completion rate, \( INV.DEV \) means deviation between inventory and inventory tar-
the three parts of the objective will be interpreted explicitly.

1) \( VR = \left[ \frac{\int_0^{t_s} (\text{ORATE}(t))^2 dt}{\int_0^{t_s} (\text{CONS}(t))^2 dt} \right]^2 \)

2) \( \text{ITAE}_{\text{EINV}} = \frac{\int_0^{t_s} |\text{E}_{\text{EINV}}|^2 dt}{\alpha} \), \( \text{ITAE}_{\text{ECON}} = \frac{\int_0^{t_s} |\text{E}_{\text{ECON}}|^2 dt}{\beta} \)

As to the meaning of \( |\text{E}_{\text{EINV}}|, |\text{E}_{\text{ECON}}| \), we can refer to [6].

But the meaning of \( \alpha, \beta \) is different. The function of \( \alpha, \beta \) in \( \text{ITAE}_{\text{EINV}}, \text{ITAE}_{\text{ECON}} \) is to simplify the value to the same order of magnitude. However, the coefficients inevitably change the proportion between

In this paper, VR is made to be the measurement of bullwhip effect, which is obviously different from the expression \( \omega_N = \int_0^\infty |\text{ORATE}(\omega)|^2 d\omega \) in related works [3, 5].

In this paper, all the simulations are operated in time domain in which the tradition expression is refractory. Here, in order to gain precise data, we create a new form of metric for bullwhip effect strictly based on the definition of bullwhip effect (a tendency for small changes in end-consumer demand to be amplified as one moves further up the supply chain[8]. In communication engineering, \( W' = \int_{-\infty}^{\infty} (f(t))^2 dt \) means the total power of signal (equals to the spectral density estimate). Naturally, \( W' = \int_{0}^{t_s} (f(t))^2 dt \) (\( t_0 \) means particular moment) means the power of signal in a period of time. \( O = \int_{0}^{t_s} (\text{ORATE}(t))^2 dt \) represents the total variations of order rate from the beginning of response to the last stability. The same argument, \( I = \int_{0}^{t_s} (\text{CONS}(t))^2 dt \) represents for the total variations of consumer consumption from the beginning of response to the last stability. Then \( VR = O/I \). So

\[
VR = \left[ \frac{\int_0^{t_s} (\text{ORATE}(t))^2 dt}{\int_0^{t_s} (\text{CONS}(t))^2 dt} \right]^2
\]

is can be competent for the measurement of bullwhip effect. Its calculation subsystem is in the Figure8.
ITAE_{AINV} and ITAE_{VCON} in the objective function. The subsystem of calculating ITAE in Simulink is displayed in Figure 9.

When $a = 250$, the value of the subsystem is ITAE for inventory response, noted as ITAE_{AINV}; When $a = 10$, the value of the subsystem is ITAE for virtual demand, noted as ITAE_{VCON}.

At last, the above three subsystems are assembled together to be the system of objective function, which is expressed in Figure 10.

3 Simulation Results and Analysis

In this paper, the simulation was implemented in the Simulink of Matlab 7.0.1. The block diagram is shown in Figure 11.

We selected nine groups of data for the control parameters in simulation and every group is simulated in the conditions with and without FLC. Besides, under the condition with FLC, the fuzzy input variables are given nine different weights ($K_1 = 0.9, K_2 = 0.1$; $K_1 = 0.8, K_2 = 0.2$; $K_1 = 0.7, K_2 = 0.3$; $K_1 = 0.6, K_2 = 0.4$; $K_1 = 0.5, K_2 = 0.5$; $K_1 = 0.4, K_2 = 0.6$; $K_1 = 0.3, K_2 = 0.7$; $K_1 = 0.2, K_2 = 0.8$; $K_1 = 0.1, K_2 = 0.9$). Take the simplicity of human thinking into consideration, we just choose the simple and intuitive numbers as the weights given to the fuzzy inputs.

3.1 Overall Dynamic Performance Comparison

In Table 3, we can find that the values of objective function with FLC are obviously smaller than that without FLC. And after the regulation of scaling factors, the performance is further optimized. From Figure 13, the overall dynamic performances in the three different conditions are compared. The value of D without FLC is about three times larger than that with FLC. Figure 14 shows the effect of fine-tuning of the scaling factors on the performance of the whole system.

The distinct comparisons are the strongest evidence of optimizing quality. After the connection with fuzzy logic controller, the dynamic performance of VMI-APIOBPCS is greatly optimized. Although, the scaling factor cannot change the performance obviously, the fine-tuning can enable the managers to flexibly manipulate the business activities, so as to preserve the maximum profit in spite of disturbances.

3.2 Management Insights

In this paper, the fuzzy logic controller is applied to optimize the dynamic performance of VMI-
<table>
<thead>
<tr>
<th>Value of control parameters ($Ta, Ti, Tq, Tw, G, W$)</th>
<th>With FLC</th>
<th>Without FLC</th>
<th>Optimal Sf</th>
</tr>
</thead>
<tbody>
<tr>
<td>($6, 7, 6, 42, 1, 1$)</td>
<td>4.05</td>
<td>1.33</td>
<td>1.28</td>
</tr>
<tr>
<td>($2, 16, 3, 35, 4, 0.2$)</td>
<td>6.09</td>
<td>3.25</td>
<td>3.15</td>
</tr>
<tr>
<td>($3, 3, 2, 4, 1, 0.05$)</td>
<td>1.10</td>
<td>0.65</td>
<td>0.64</td>
</tr>
<tr>
<td>($7, 12, 6, 63, 2, 5$)</td>
<td>9.00</td>
<td>2.48</td>
<td>2.46</td>
</tr>
<tr>
<td>($1, 5, 1, 5, 2, 0.01$)</td>
<td>1.21</td>
<td>1.02</td>
<td>0.96</td>
</tr>
<tr>
<td>($10, 20, 6, 63, 4, 20$)</td>
<td>27.89</td>
<td>4.80</td>
<td>4.79</td>
</tr>
<tr>
<td>($7, 27, 6, 63, 8, 5$)</td>
<td>28.95</td>
<td>9.55</td>
<td>9.48</td>
</tr>
<tr>
<td>($14, 27, 2, 63, 16, 1$)</td>
<td>37.53</td>
<td>10.78</td>
<td>10.36</td>
</tr>
<tr>
<td>($30, 26, 3, 35, 32, 1$)</td>
<td>135.24</td>
<td>26.00</td>
<td>24.65</td>
</tr>
</tbody>
</table>

◊ Sf is the abbreviation of Scaling factor
◊ All the value of control parameters are recommended settings in[3].

Table 3: Performance comparison

![Block diagram of simulation](image_url)
APIOBPCS model, furthermore, the fuzzy input variables are given different weights to adjust the knowledge in the new surroundings. After the investigation, we can reach the several important conclusions about innovations in this paper:

With the application human intelligence in the model, we can get new results that the fuzzy controller can greatly optimize the integrated performance of VMI-APIOBPCS. The scaling factors can tune the system performance finely, which makes the system optimization flexible to different new surroundings.

The dual-input, single input fuzzy logic controller can take demand and deviation between the inventory level and demand into consideration to make a more rational decision.

What’s more, an auxiliary benefit, the inventory dynamic performance is largely improved. It is profitable for the inventory control.

After the conclusions, several points of management insight are obtained:

In the practice of forecasting, we should take more factors, both direct and indirect, into consideration according to the feature of our own business. Production-inventory system is complex social economic system, in which human being play irreplaceable roles. Therefore, in the designing of optimization method, the unique thinking pattern needs to be taken into account.

In the process of management, we not only absorb the lessons but also conclude the precious experience. And we need to ponder how use the experience in the future work. This is the ideal of fuzzy control in this paper, meanwhile, the philosophy of learning organization[41].

When we adopt control engineering to research or optimize the production-inventory system, we should assess the practicability and whether it is proper for the control of production-inventory system, for it instinctively differently from the system like electronic and mechanical system[38, 39].

In a word, fuzzy control, as one kind of artificial intelligence, can imitate the way of human thinking, exploit experts’ knowledge and experience, and renew the knowledge constantly. It is greatly significant to improve operational performance, reduce management cost, and elevate the flexibility[34, 40].

4 Conclusion

In this paper, the novel artificial intelligence–fuzzy control is adopted to optimize the production-inventory system. The simulation results verify that the overall dynamic performance is greatly improved.
and the inventory dynamic response is obviously improved. Fuzzy control can imitate thinking of human beings and make good use of experts' knowledge and experience to avoid the turbulent fluctuations in inventory dynamic changes.

By the way, further research can adopt optimization algorithm, liking GA, to initiatively search the optimal scaling factors, or investigate the effect of other variables on the determination of control parameters. Besides, the excellent inventory response may lay much pressure on the production, and this can be further researched. The relationship between the optimal performance and scaling factors can also be explored.

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