Analysis of Attribute Acceptance Sampling Properties

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Abstract: - Attribute sampling is a powerful quality inspection tool. The Lot Acceptance Sampling Plans (LASPs) for sampling by attributes represent a valuable method of quality inspection. Significant LASPs such as the c=0, Dodge-Romig, Philips as well as ANSI/ASQC Z1.4 which replaced MIL-STD 105E have comprehensive procedures and statistical base. A single LASP can be implemented efficiently if inspection of incoming lots is unpractical and expensive due to the destructive and/or time-consuming procedure. The implementation of LASPs is useful when a large number of purchased items is inspected daily in any complex industrial branch, such as electric-power or gas, but even in large medical laboratories. The significant issues of a complex attribute acceptance sampling statistical frame are considered in the paper. It is demonstrated how the lot size differs over different double sampling plans.

Key-Words: - Attribute acceptance sampling, Attribute case, Quality inspection

1 Introduction

Developed techniques are used for sampling as well as quality control and functionality diagnostics of the primary and secondary equipment in complex industries, such as electric-power or gas. In common inspection, it is necessary to control a large number of items before installation. It is necessary to use the high reliable acceptance sampling method because the assembled items have to be of top-quality level to assure the safety of the electric-power or gas system. At first sight, it seems that 100% inspection would be an appropriate approach to ensure high reliable items. Obviously, 100% inspection is unpractical and expensive if the inspection method of a large number of items is destructive and/or time-consuming. Therefore, an alternative approach should be implemented. In such a case the acceptance sampling is employed and a chosen Lot Acceptance Sampling Plan (LASP) represents the best option because it enables undoubted conclusion on the lot's disposition based on counting the number of defectives in a sample picked from a lot randomly.

The implementation of acceptance sampling can be also useful in other sectors. The authors advocate for using LASPs in medicine, especially in the medical laboratory procurement. Generally, an efficient procurement of laboratory items impacts the quality of laboratory services. Hence, the procurement of laboratory items (such as consumables, glass wares, instruments) is an important issue. The use of LASPs could be especially useful if the laboratory supply chain (as a complex part of the national health system) or large laboratories procured a large number of items for which the top characteristics of quality are often demanded. Furthermore, the tenderers that receive the contract award decisions in the public procurement procedures become new suppliers of goods. If a supplier is new, the quality of goods is always questionable from the purchaser's point of view. When a new supplier provides a shipment of a large number of items, the following question will appear: is the shipment good enough to put into stock? A purchaser must decide whether to accept or reject the received shipment.

Achieving quality in a medical laboratory requires the use of different tools, such as calibrations, procedure manuals, maintenance schedules and quality inspection which encompass the operational techniques and activities used to fulfill prescribed requirements for quality. Testing reliability can be achieved using quality control materials as well as the product quality control. Although the set for evaluating analytical run quality for medical laboratories, i.e. six basic rules created by J.O. Westgard [1]-[2], is often used in the laboratory quality control, it is important to implement a valid sampling acceptance procedure when a purchaser evaluates large lots of materials or products with the aim of fulfilling all requirements for quality.

The acceptance sampling implementation by using the most important LASPs for attribute case, such as Dodge-Romig [3]-[5], Philips [6], MIL-STD 105E [7] (replaced by: ANSI/ASQC Z1.4; ISO 2859) and the c=0, is strongly statistical based, therefore this use is reliable.

Sampling plans developed for sampling by variables, such as the BENDIX and LOT-PLOT, are not considered in this paper which deals with sampling by attributes. Inspection by attributes is a kind of quality inspection in which any a unit is classified either as conforming or nonconforming, with respect to a defined set of requirements.

A process in which a sample (n_i) is picked at random from the lot is carried out by a LASP. Random sampling ensures that all samples of size (n) are equal. Each LASP defines a number of units that should be sampled from a lot and how many defects are allowed in the chosen sample. LASP represents a sampling scheme and a set of rules for making decisions whether a lot should be accepted or rejected [8]. A decision can be made regarding the disposition of the lot (N) on the basis of the resulting information. A point to emphasize is that the aim of acceptance sampling is to decide whether or not the lot is likely to be acceptable, not to estimate the lot quality.

An explanation of the difference between sampling by attributes and sampling by variables is necessary. The characteristic of sampling by attributes is that the item inspection leads to a binary result: either the item is conforming or nonconforming. On the other hand, sampling by variables is characterized by the item inspection leading to a continuous measurement. Sampling by attributes is more common than sampling by variables regardless of the industry sector in which it is used. For instance, many components come in large lots in the case of the electric–power industry, which is often a subject of regulation [9]-[11]. Hence, it is not possible to provide 100% inspection. In such an industry branch, all components have to be of top-quality to ensure the reliability of the power system. In this sense, it is necessary to select a suitable number of components that have to be inspected before installation regardless of using the components for everyday operation or overhaul.

Some important references that deal with acceptance sampling are [12]-[14].

Quality inspection used in industrial branches can also include other approaches. However, the combination of both acceptance sampling and process control can often ensure an efficient quality inspection.

A contribution of this research is in expressing how the right chosen LASP ensures reliability of acceptance of a large number of quality items.

The rest of the paper is structured as follows. Section 2 provides the categories of LASPs. Section 3 describes the average sample number. Section 4 summarizes the properties of the operating characteristic curve. The average amount of inspection per lot is described in section 5. The parameters' comparison for different LASPs is presented in section 6. Finally, section 7 draws some relevant conclusions.

2 The categories of LASPs

LASPs consist of single, double and multiple sampling plans. Making a choice between single and multiple sampling plans is a matter of deciding whether the average sampling savings gained by the multiple plans justifies an additional complexity of these plans and the uncertainty of not knowing how much inspection will be done on a daily basis.

Single sampling plans

A single sampling plan is denoted as (n,c). It is the most common plan used in quality control in which one sample of items (n) is selected at random from a lot (N). The disposition of the lot depends on the resulting information: the lot is rejected if there are more than c defectives (note: c is the acceptance number). This plan has a significant disadvantage, i.e. it requires a large number of samples. Hence, some alternative double or multiple plans should be considered in the case of destructive testing of expensive items when the number of samples is the most important criterion.

Double sampling plans

A double sampling plan has good efficiency [15]. Another sample is taken in the double sampling procedure if the first sample taken from the lot is not informative enough, i.e. the sample (n_2) is taken if the result of the sample (n_1) is not conclusive with regard to accepting or rejecting. The lot is not acceptable if the number of defectives is under the acceptance number (c_2). If the number of defectives is between the acceptance numbers (c_1) and (c_2), the sample (n_2) is taken in order to establish whether the total number of defectives in both samples is bigger or smaller than the acceptance number (c_2). The final decision is based on this information.

One Philips plan can be a single (used if lot size is up to 1,000) or multiple (for example, a double sampling plan is used if the lot size is over 1,000). Nevertheless, Philips is typically a highly characteristic double sampling plan for which:

- there are sample sizes n_1 and n_2 (with the precondition: $n_2 = 2n_1$) and a point of neutral quality p_n,

- the acceptance number (c) is the maximum number of defectives in sample (n) in the case of acceptance of lot (N),
- the relation n_1, c_1, $c_2 = f(N)$ is valid.
- the values of the Acceptable Quality Level (p_a), p_t and the Average Outgoing Quality Level (AOQL) are necessary for drawing the operating characteristic curve (their values are defined by the Philips tables: for example, if a lot size is 1,100 and the acceptable quality level (p_a) is 1.0%, the corresponding p_t is 5.8% with the AOQL of 1.6%).

The usefulness of Philips plan can be demonstrated by using an example. Let some lots be delivered to control. The quality control department will inspect the shipments by using an appropriate sampling plan. According to the contract, the supplier is obligated to send lots of 1,100 units with the Acceptable quality level of 1.0% and Philips double sampling plan is used. To carry out the inspection procedure more easily, it is useful to draw a sampling scheme for this case (Fig. 1). The sampling scheme of double Philips sampling plan is drawn using the values taken from the table of this plan, for N = 1,001–2,000 and p_a = 1%, as follows:

- AOQL = 1,6%,
- p_n = 3% and p_t = 5,8%.

Furthermore, by using p_n = 3% the following values are taken from another table of Philips sampling plan: $n_1 = 45$, $c_1 = 0$ and $c_2 = 3$.

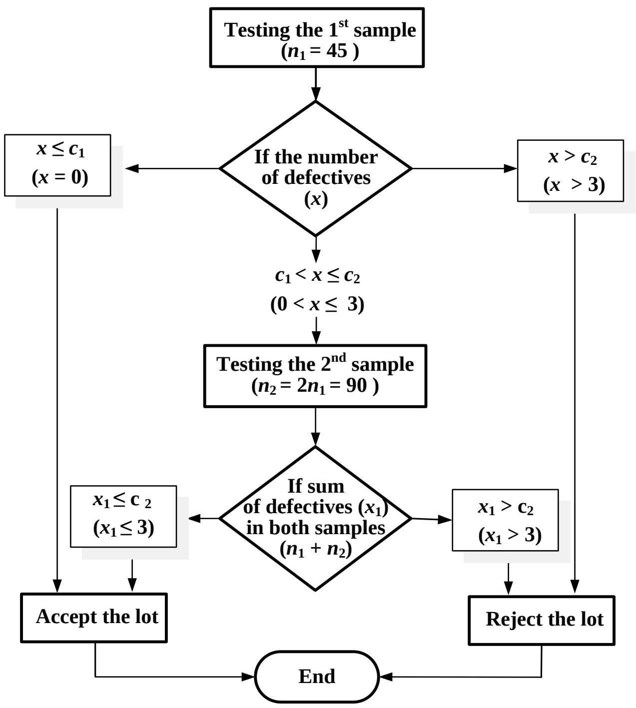

Fig. 1 Sampling scheme for double Philips sampling plan

Multiple sampling plans

A multiple sampling plan introduces an extension of the double sampling plan because additional samples can be drawn after the second sample in the multiple sampling procedure. Consequently, more than two samples are needed to reach a conclusion. In fact, multiple sampling plans were introduced to give a questionable lot another chance to be accepted.

The samples are taken at random and the sampling procedure is reliable. Although multiple sampling plans are complex and the possibility of error is greater, they can reduce the number of samples. These plans have smaller sample sizes and a shorter examination period, which is very useful in the case of complex products [16].

Multiple sampling plans are usually presented in a tabular form. The multiple sampling procedures commence with taking a random sample of size (n_1) from a large lot of size (N) and counting the number of defectives (d_1). If $d_1 \le a_1$, the lot is accepted. Furthermore, if $d_1 \ge r_1$, the lot is rejected. Another sample should be taken if $a_1 < d_1 < r_1$. The first-sample-procedure is repeated sample by sample if subsequent samples (n_2) are required. The total number of defectives (N_r) found at any stage of multiple sampling can be calculated as the sum of all numbers of defectives at each stage (d_j), where j ranges from 1 to the total number of stages in multiple sampling (k).

It is a rule for each stage of multiple sampling procedure that the number of defectives is compared to the acceptance number (a_i) and the rejection number (r_i), until a decision is made. Sometimes acceptance is not allowed at the early stages of multiple sampling. However, the lot's rejection can occur at any stage of the multiple sampling.

Finally, there are some specific sampling plans in use in quality inspection, such as the Sequential sampling plan and the Skip lot sampling plan. The Sequential sampling plan represents the ultimate extension of multiple sampling. Only a fraction of the submitted lots is inspected by using the Skip lot sampling plan.

3 An efficient measure for a multiple sampling scheme

Considering economically, it is important to deal with a rational sample size when inspecting a lot, because a huge number of inspected items would result in high costs.

If single sampling is carried out, only one sample is selected from a lot. The number of items needed in the case of multiple sampling scheme may vary from trial to trial. The Average Sample Number (ASN) represents an average of what might happen over many trials with a fixed incoming defect level (p). ASN measures the efficiency of a multiple sampling scheme. For a single LASP (n,c) each lot (N_i) is known and has a known sample of size (n_i) taken and inspected. However, for double, multiple and sequential LASPs the amount of sampling varies depending on the number of defectives observed. A long term ASN can be calculated for any given double, multiple or sequential plan assuming that all lots have a defect level of (p). Using the ASN curve, a plot of the ASN versus the incoming defect level (p) describes the sampling efficiency of a given LASP scheme. Since the number of samples in the case of multiple sampling may vary from trial to trial, the ASN represents the average of what might happen over many trials with a fixed level of defectives in the incoming lots. A formula for the ASN of a double sampling plan is:

$$\mathrm{ASN} = n_1 P_1 + (n_1 + n_2)(1 - P_1) \qquad (1)$$

where P_1 is the probability of a decision on the first sample.

4 The properties of the Operating Characteristic (OC) Curve

The operating characteristic (OC) curve is an important tool for explaining and researching the properties of LASPs. The OC curve plots the percent of defectives on the x-axis versus the probability of accepting the lot on the y-axis. In fact, the OC curve shows the probability of acceptance depending on the percent of defectives, with the precondition that each lot contains defectives. In an ideal case, after using a chosen LASP, all good lots will be accepted and all bad lots will be rejected. On the other hand, since the decision on acceptation or rejection of the lot depends on the sample taken from the lot, there is a real possibility of making the wrong choice in inspection praxis.

An example of the OC curve is shown in Fig. 2. Regarding the OC curve the following is valid:

- the lot size (N) is very large compared to the sample size (n), therefore, removing the sample doesn't significantly change the remainder of the lot (no matter how many defectives are in the sample which is taken at random),
- the distribution of the number of defectives (d) in a random sample of (n) items is approximately binomial with parameters (n) and (p).

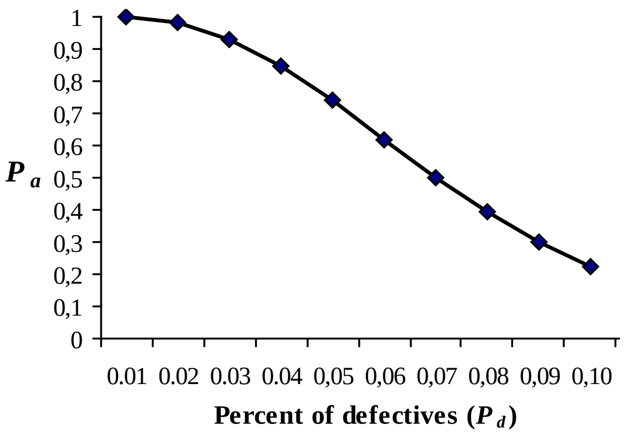

Fig. 2 The OC curve (for the LASP: n = 52, c = 3)

The probability of observing exactly (d) defectives is given by the following formula:

$$f(d) = P(d) = p^d (1-p)^{n-d} \frac{n!}{d!(n-d)!} \qquad (2)$$

Obviously, this formula corresponds to the binomial distribution. Furthermore, the probability of a lot acceptance is the probability that the number of defectives (x) is less than or equal to the acceptance number (c). This means that:

$$P_a = \sum_{x=0}^{c} p^x (1-p)^{n-x} \frac{n!}{x!(n-x)!} \qquad (3)$$

where $P_a = P\{x \le c\}$.

The following combinations of values for (p_a,p_d) are calculated using P_d = 0.01,…, 0.12, n = 52 and c = 3 in (2) and (3):

P_a	P_d	P_a	P_d	P_a	P_d
0.998	0.01	0.739	0.05	0.300	0.09
0.980	0.02	0.620	0.06	0.223	0.10
0.930	0.03	0.502	0.07	0.162	0.11
0.845	0.04	0.394	0.08	0.115	0.12

These data represent the coordinates of the points at the OC curve for a sampling plan n = 52, c = 3.

The calculation of a sampling plan with a given OC curve is based on complex formulas. Let us design such a sampling plan that the probability of acceptance is 1-α for the lots with fraction defective (p_1) and the probability of acceptance is (β) for the lots with fraction defective (p_2). Typical notions for such a case are:

- α is the supplier's risk that a good lot is rejected (type I error),
- β is the consumer's risk that a bad lot is accepted (type II error),

- p_1 is the Acceptable Quality Level (AQL),
- p_2 is the Lot Tolerance Percent Defective (LTPD).

If binomial sampling is valid, the possibility of lot acceptance with defectives (p_1), for the sample size (n) and the acceptance number (c), is:

$$P_a = \sum_{d=0}^{c} p_1^{\ d} \left(1 - p_1\right)^{n-d} \frac{n!}{d!\left(n-d\right)!} \tag{4}$$

where $P_a = 1 - \alpha$.

Furthermore, the possibility of lot acceptance with defectives (p_2) is:

$$P_a = \sum_{d=0}^{c} p_2^{\ d} \left(1 - p_2\right)^{n-d} \frac{n!}{d!\left(n-d\right)!} \tag{5}$$

where $P_a = \beta$.

Obviously, there is no direct (simple) calculation solution because (4) and (5) are nonlinear simultaneous equations. However, iterative techniques are available that give good approximate solutions.

The lot's inspection in praxis is often done by the consumer's inspection department after the lots were received from the supplier. Assuredly, the lots will not always contain the same percent of defectives, and it is necessary to use the OC curve regardless of the fact that the method does not completely ensure that the accepted lot will be good, and there is also a certain possibility that a good lot can be rejected.

The OC curve is just a picture of a certain sampling plan, so that each sampling plan has a unique OC curve. The OC curve is defined by the sample size and the acceptance number. The shapes of ideal and real (round) OC curves are shown in Fig. 3.

In praxis, it is important to know how the sample responds to a range of percentages of defectives in the lot. The OC curve includes this basic characteristic of each sampling plan. Nevertheless, it should be accentuated that this method does not guarantee with 100% that each accepted lot is a good lot. Moreover, there is a certain probability that a good lot is denied. In fact, the probability of occurrence of certain defectives in the lot can be obtained using the OC curve.

It is necessary to explain the values shown in Fig. 3. Acceptable Quality Level – AQL (p_a) is the maximal percent of nonconforming items, or the maximal number of nonconformities per hundred items, which is considered a satisfying process average for inspection purposes. Therefore, it is not lot specific. The AQL is the maximal percent of defectives that is acceptable for both producer and consumer, i.e. the poorest level of quality (percent of nonconformities) that the process can tolerate.

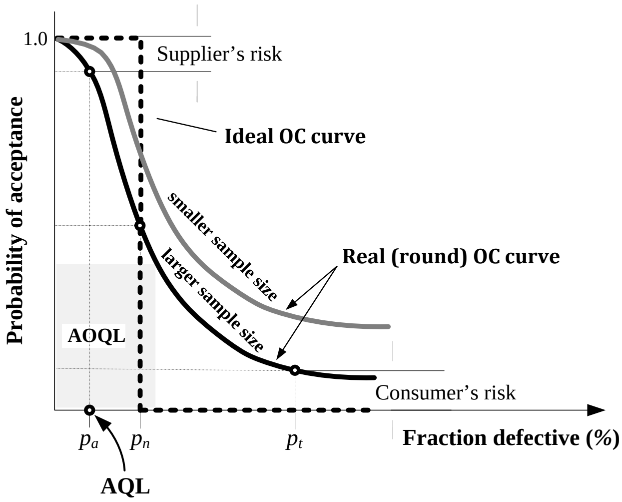

Fig. 3 The shape of OC curve

The producer prefers to design a sampling plan so that there is a high probability of accepting a lot that has a defect level less than or equal to the AQL. The AQL is stated in the standard as a percent (so, an AQL of 0.2 is a rate of 0.2 nonconforming items per 100 items or 0.2%). For example, if the AQL is 2.0 then the resulting product is acceptable to both parties, i.e. the producer is ready to produce approximately 2% of defectives. Furthermore, the Neutral Quality (p_n) is an indifference quality level: i.e. there is a 50% chance that the bad lot can be accepted and a 50% chance that the good lot can be rejected. Finally, the Average Outgoing Quality Level (AOQL) represents the maximal possible rectangle under the real OC curve. The AOQL is the worst possible quality that results from the rectifying inspection programme. It is the maximum AOQ for all possible levels of incoming quality.

Furthermore, the next rule is valid: the round OC curve is for smaller values (n) and (c) while the ideal OC curve is for bigger values (n) and (c). When the round OC curve is used, there is a certain probability that the entire lot is rejected if it contains less defectives than LTPD, however it is also possible that the lot is accepted if it contains more defectives than LTPD.

With the OC curve there is a probability that the lot is rejected if the defect level is less than the AQL, but also that the lot is accepted if the defect level is higher. There is a possibility that the lot contains less defectives than the AQL, but α-risk (the supplier's risk) still exists. Such a case occurs at the acceptable quality level (p_a). Usually, α-risk is 5%. This is a type I error (α), which is the probability for a given (n,c) sampling plan of rejecting a lot that has a defect level equal to the AQL.

On the other hand, there is a possibility that a lot with an unacceptable quality is accepted. Obviously, the consumer suffers when such a risk occurs. This risk occurs at the tolerance quality level (p_t). Usually, β-risk is 10%. This is a type II error (β), which is the probability for a given (n,c) sampling plan of accepting a lot with a defect level equal to the LTPD.

Regarding decision on a lot, four possible outcomes are shown in Table 1.

Generally, the OC curve is characterized by its AQL and LTPD. In fact, the AQL describes what the sampling plan generally accepts. Formally, it is the percent of defectives with a 95% chance of acceptance. The LTPD describes what the sampling plan generally rejects. Formally, it is the percent of defectives with a 10% chance of acceptance. The LTPD is a designated high defect level that would be unacceptable to the consumer. By using the LASP and AQL, the producer is ready to produce the number of defectives that will be accepted by the consumer.

Table 1 Outcomes in inspection praxis

Supplier’s risk of rejecting good lot (α –risk; type I error). Consumer's risk of accepting bad lot (β –risk; type II error).		Supplier's (producer's) activity	
		The lot conforms	The lot does not conform
Consumer's (purchaser's) decision	Accept	+	β-risk
	Reject	α-risk	+

Normally, each consumer prefers a sampling plan with a low probability of accepting a lot with a defect level the same as the LTPD. The LTPD is an important criterion for the LASP [17], which can be used in the case of limited resources in the quality control department when a minimal size of samples is used.

The LASP is based on a clearly defined correlation with the lot size. Generally, a lot is rejected if any defectives are found in a sample. The procedure gives the proportion of the lot which must be sampled. The Schilling Table [18] can be used for this procedure. The manufacturing process should be run at the average quality level of less than 5% of LTPD in order to achieve a reasonably small probability of rejecting the good lot if c=0 for the chosen LASP. In the case that this average cannot be ensured one should choose another type of LASP with a bigger sample size, because the LASP which has c = 0 is obviously not a good choice.

As mentioned beforehand, the LASP is designed in such a way that the OC curve passes through two designated points corresponding to AQL and LTPD. This presents a problem if an error occurs. Nevertheless, some corrections of the acceptance procedure can be done in order to ensure that the OC curve passes through the aforementioned points. This can be shown, but only if the OC curve passes through (AQL, 1-α) and (LTPD, β). The LASP's errors can have a huge influence on the sample size needed to achieve the desired position of the points on the OC curve [19]. The sampling plans can be used for a variety of purposes depending on past history and other circumstances. One must provide justification for the AQLs and LTPDs used in the quality control department. This requires that the aim of each inspection is defined.

For example, an AQL of 1.0% is specified as the aim of inspection of the main defects. This AQL is not necessarily equal to the sampling plan AQL (so it is noted as 'AQL-Alternative' to make this distinction clear). The AQL-Alternative is interpreted as the maximum percent defective for which acceptance is desired. However, the AQL-Alternative does not mean that there is a permission to deliver bad lots.

All lots above the AQL-Alternative are best rejected. On the other hand, all lots below the AQL-Alternative are best accepted. The break-even quality between acceptance and rejection is, therefore, represented by the AQL-Alternative. The cost of carrying out a 100% inspection will exceed the benefits of doing so in terms of fewer defects released if lots come with percent defectives below the AQL-Alternative. All lots should be 100% inspected if a manufacturing process always produces lots with percent defectives above the AQL-Alternative. The quality control department could use a sampling plan to screen out lots not requiring 100% inspection if some lots are below the AQL-Alternative.

A sampling plan with a LTPD the same as the AQL-Alternative can be used to ensure that lots worse than the AQL-Alternative are rejected, but with the risk of rejecting some acceptable lots. The single sampling plan with n = 230 and c = 0 (which has a LTPD of 1.0%) is appropriate for an AQL-Alternative of 1.0%.

For example, the required sample size for zero acceptance number and desired confidence level of 95% is: n = 300 / AQL-Alternative. Finally, the following rule is valid: the AQL should be used when producing many lots of a product, and the LTPD should be used when producing smaller number of lots.

If sampling and testing is non-destructive, a common procedure is to use the 100% inspection of the rejected lots and to replace all defectives with good units. In such a case, all rejected lots no longer contain any defectives and the only defectives left are those in the lots that were accepted during the sampling procedure as good ones.

Assume that all lots come in with exactly a proportion of defectives (p). After having inspected a rejected lot, the final fraction defective is zero for that lot. Nevertheless, the accepted lots have a fraction defective (p). Therefore, the outgoing lots from the quality control department are a mixture of the lots with a fraction defective (p) and zero.

Each LASP ensures a certain average quality as a result of all received lots. The Average Outgoing Quality (AOQ) is the expected average quality of outgoing items for a given value of incoming product quality. The AOQ can be expressed as (process average)×(probability of acceptance)×10^6, where 10^6 is a conversion to parts per million (ppm).

The AOQs refer to the long term defect level for the combined LASP and 100% inspection of rejected lots process.

If all delivered lots have a defect level of exactly (P), and the chosen LASP (n,c) indicates a probability (P_a) of accepting such a lot, over the long run the AOQ can be calculated as:

$$\mathrm{AOQ} = \frac{1}{N}\left[P_a P\left(N-n\right)\right] \tag{6}$$

If the sample size is assumed to be very small compared to the lot size, the AOQ is practically the same as the product of P_a and P.

For example, let N = 10,000, sampling plan with n = 52 and c = 3. Let the quality of incoming lots be 0.03. It follows that P_a = 0.930 for P = 0.03, based on the table of the OC curve. Furthermore, by using (6) the calculated AOQ is 0.0278.

5 The average amount of inspection per lot

An important question of acceptance sampling is: what is the total amount of inspection when rejected lots are screened?

Three cases are possible:

- all lots are inspected and the amount to be inspected is (N) if all inspected items are defective,
- no lot is rejected if all inspected samples (n_i) from the lots (N_i) contain zero defectives,
- if the lot quality is $0 < p < 1$, the average number of the inspected items per lot vary between the sample size (n) and the lot size (N).

The Average Total Inspection (ATI) denotes the average number of units that will be inspected for a particular incoming quality level and probability of acceptance. ATI presents the expected number of units inspected after rectifying inspection for a given quality level.

If all rejected lots are 100% inspected, it is easy to calculate ATI when lots come consistently with a defect level of (p). For a LASP (n,c) the formula for ATI is:

$$\mathrm{ATI} = n + \left(1-P_a\right)\left(N-n\right) \tag{7}$$

where P_a is the probability of accepting a lot, N is the lot size and n is the sample size.

After replacing the defectives, and if during the replacement procedure some type of error occurs, the formula for ATI is:

$$\mathrm{ATI} = \frac{1}{1-p_e}\left[n + \left(1-P_{a(e)}\right)\left(N-n\right)\right] \tag{8}$$

If defectives are not replaced, the formula for ATI is:

$$\mathrm{ATI} = n + \left(1-P_{a(e)}\right)\left(N-n\right) \tag{9}$$

Formula for the ATI curve for double sampling plans is:

$$\mathrm{ATI} = P_a^1 n_1 + P_a^2\left(n_1+n_2\right) + N\left(1-P_a\right) \tag{10}$$

where P_a^i is the probability of acceptance on the i^{th} sample.

The calculation of some values of ATI is presented in Table 2.

Table 2 The calculation of the ATI using some different values of P

P	n	N	P_a	1 - P_a	N - n	ATI
0.05			0.739	0.261		2,648
0.04			0.845	0.155		1,594
0.03	52	10,000	0.930	0.070	9,948	748
0.02			0.980	0.020		251
0.01			0.998	0.002		72

Example:
What is the value of ATI for the chosen sampling plan n = 52 and c = 3 if N = 10,000 and P = 0.04? By using the coordinates for the points at the specified sampling plan OC curve, it can be noticed that P = 0.04 matches P_a = 0.845. Hence, it follows that ATI = 1,594 by using (7). It means that a total of 1,594 items was inspected for the considered sampling plan, i.e. as high as 15.94% of the whole lot (N). Obviously, this would result in unacceptably high costs. Therefore, it is much better to use a smaller P: for example, ATI = 251 if P = 0.02 or even better ATI = 72 if P = 0.01.

6 Parameters' comparison for different LASPs

Sampling by attributes is a method of quality inspection in which every 100 items is classified by percentage of conformance or non-conformance. For example, 3 non-conforming would be a 3% rate. Acceptance sampling is used to determine whether to accept or reject a specific quantity of goods or materials [20]. Philips, Dodge-Romig and MIL-STD 105E form a group of classic sampling plans used in the sampling inspection praxis over many years. As a matter of fact, MIL-STD 105E was cancelled three times: first in 1995, then in 2001 and finally in 2008. MIL-STD 105E was simply replaced by ANSI/ASQC Z1.4 in such a way that Z1.4 was actually a reprint of 105E. MIL-STD 105E was cancelled to reduce costs through the elimination of duplication. Thereafter, ANSI/ASQ Z1.4-2008 with its sampling procedures and tables for inspection by attributes replace ANSI/ASQ Z1.4-2003 in 2008. The ANSI/ASQ Z1.4-2008 follows its previous (2003) version, so the procedures and tables remain practically unchanged from MIL-STD 105E.

Moreover, other standards such as ISO2859-1, DIN40080, NF06-022 and BS6001 exist as equivalents of the ANSI/ASQ Z1.4.

In fact, MIL-STD-105E is nearly the same as ANSI/ASQ Z1.4 (1993) in relation to the tables. ANSI/ASQ Z1.4 has changes in the explanatory text that do not affect procedures, however, it has different switching rules. ANSI/ASQ Z1.4 uses the limit numbers for switching and it contains some additional OC curves that describe the protection provided by the switching procedure for the periods of constant quality.

The so-called 'c=0' described in [21] by Squeglia presents broadly used sampling plans in the attribute inspection today. The c=0 plans, in which the acceptance number is zero in all cases (so all OC curves present a special case when c = 0), are defined by the OC curves and they have practically the same (LTPD, β) point as the corresponding ANSI/ASQ Z1.4 single normal plan. The c=0 plans set β = 0.1. It tries to match the ANSI/ASQ Z1.4 plans at the RQL (or LTPD) point. However, the main difference is that the c=0 plans are oriented to the LTPD whilst ANSI/ASQ Z1.4 are oriented to the AQL which is set by a quality engineer in praxis (the supplier's process average should be less than the chosen AQL). Furthermore, the calculations in the ANSI/ASQ Z1.4 use the binomial or Poisson distribution, while they use the hyper-geometric distribution in the c=0 plan. Considering the OC curves for both plans, a clear advantage of using much smaller sample sizes in the case of c=0, compared to the corresponding ANSI/ASQ Z1.4 plans, is evident. For example, if N = 1,300 and AQL = 4.0%, the sample size (n) is 125 in the case of ANSI/ASQ Z1.4 but only 18 in the case of c=0. Consequently, c = 10 and c = 0. Hence, the inspection costs drop from 125 to 18 items if the c=0 is used instead of the corresponding plan ANSI/ASQ Z1.4.

It should be mentioned that ANSI/ASQ Z1.4 is probably the most commonly used standard for the attribute acceptance case in the last ten years. This standard offers seven levels of inspection: the reduced, normal and tightened inspections as well as four levels of special inspection (that should be used only when small sample sizes are necessary and large risks can be tolerated).

Selecting a plan from ANSI/ASQ Z1.4-2003 implicates the step-by-step procedure: select an AQL; select the inspection level; determine the lot size (n); find the appropriate sample size code letter in the table of the standard; determine the type of sampling plan – single, double or multiple; enter the appropriate table to find the plan to be used by using the selected AQL and sample size code letter; determine the normal, reduced and tightened plans as required from the corresponding tables. A more detailed explanation of the sampling plan procedure is outside the scope of this paper.

A comparison of some double sampling plans is provided in Table 3.

By observing data shown in Table 3, it can be concluded that MIL-STD 105E protects the supplier, Dodge-Romig protects the consumer, while Philips is somewhere in between.

Considering the ($n_1 + n_2$) criterion, Philips is the most demanding plan with $n_1 + n_2$ = 405. Dodge-Romig follows with $n_1 + n_2$ = 370 for p_t and 385 for AOQL. MIL-STD 105E with n_1+n_2=250 is the least demanding plan.

Table 3 Parameters of the double Philips, Dodge-Romig and MIL-STD 105E sampling plans

LASP type	DOUBLE SAMPLING PLAN			
	PHILIPS	DODGE-ROMIG		MIL-STD 105E
Lot size	N = 3,500			
Criterion	p_n (%)	p_t (%)	AOQL (%)	p_a (%)
1st sample	n_1=135 c_1=0	n_1=140 c_1=0	n_1=145 c_1=0	n_1=125 c_1=0
2nd sample	n_2=270 c_2=3	n_2=230 c_2=3	n_2=240 c_2=3	n_2=125 c_2=3
AOQL (%)	0.5	0.52	0.5	0.62
p_a (%)	0.35	–	–	0.4
p_n (%)	1	–	–	–
p_t (%)	2	2	1.9	–

Furthermore, AOQL is 0.62% for MIL-STD 105E, while for the other two plans AOQL is almost the same (0.5 or 0.52%). It should be emphasized that ANSI/ASQ Z1.4 is the same as MIL-STD 105E by the consumer's protection criterion in this consideration due to the fact that Z1.4 is just a mirror version of 105E as mentioned before. Furthermore, the c=0 sampling plans provide equal or even greater consumer protection with less overall inspection compared to the corresponding MIL-STD-105-E.

At the end of this section, it should be stressed that there is a need to consider the total cost of using the sampling plan which includes the costs of inspection, reworks and defective items returned by the consumers [22].

A broader context related to the topic considered in this paper is presented in the following noteworthy papers [23]-[31].

7 Conclusion

The acceptance sampling is very useful when inspecting large lots. As shown in this paper, different Lot Acceptance Sampling Plans (LASPs) developed for the attribute acceptance sampling represent a powerful inspection tool used by the quality control experts.

In praxis, it is often too expensive and even practically impossible to use 100% inspection when huge purchased lots should be inspected by the quality control staff. Hence, the authors advocate for the LASPs' use when large lots are inspected in industrial branches, such as the electric-power or gas branch as well as in medical laboratory procurement. In fact, the ordinary usage of LASPs saves money and time without influencing the AOQ of the lots.

Showing how the right chosen LASP can ensure a high reliability of acceptance of a large number of quality items is a contribution of this research.

Finally, it should be pointed out that it is of significant importance to know the LASPs' characteristics as well as their comprehensive mathematical base, especially due to the existing differences between the considered sampling plans.

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