LMI based bounded output feedback control for uncertain systems

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Abstract: The paper provides conditions for constrained dynamic output feedback controller to be cost guaranteeing and assuring asymptotic stability for both continuous and discrete-time systems with quadratically constrained nonlinear/uncertain elements. The conditions are formulated in the form of matrix inequalities, which can be rendered to be linear fixing one of the scalar parameters. An abstract multiplier method is applied. Numerical examples illustrate the application of the proposed method.

Key–Words: robust control; dynamic output feedback; guaranteed cost; uncertain systems; LMI

1 Introduction

Treatment of nonlinearities in dynamical and control systems is one of the research focuses of control theory (see e.g. [7], [10], [29], [31], [32]). The areas of applications of nonlinear control theory cover physics, engineering (see [3], [6], [8], [9], [13], [26], [30], [32], [39] from the recent literature) and also economics (e.g. [19], [21], [27], [28]). The performance of control systems may not be satisfactory because of the presence of exogenous disturbances and of system uncertainties stemming from the mismatch of the model and the real dynamics. A performance index assigned to the system cannot be minimized at the presence of unknown uncertainties, however it is possible to design a controller guaranteeing that the performance index will not exceed a certain bound, and it stabilizes the system for any admissible uncertainties and disturbances (see e.g., [2], [14], [15], [17], [20], [23], [35], [38], [41] and the references therein). It is favorable, if such robust controls can be given in feedback form. However, the state of the system is often not available for feedback. An extended static output feedback is applied e.g. by [33] for continuous-time systems using both the output and its derivatives in the construction of the controller. The same approach is applied to discrete-time systems with polytopic uncertainties in [34]. Paper [40] applies a dynamic output feedback for T-S fuzzy systems with norm bounded uncertainties. A dynamic output feedback can still guarantee an adequate level of system performance and stability (see also [18]). The present paper applies the latter approach for both discrete and continuous-time systems with a broad class of admissible system nonlinearities/uncertainties. The control is also supposed to be quadratically constrained (cf. [4] on stabilization of uncertain linear systems by bounded inputs).

A recently published paper [22] gave a sufficient condition for the existence of robust stabilizing observer-based dynamic output feedback control by solving linear matrix inequalities (LMIs). Unfortunately, this paper contains a technical error. The present paper proposes a method eliminating the mistake, and extends the range of solvable problems in several aspects. In our paper both continuous and discrete-time systems are discussed. We consider quadratically constrained uncertainties. This representation includes, among many others, the norm bounded uncertainty considered in [24] and [22], as a special case. In fact, this approach proposed originally by ([1]) and further developed by ([16]) as an abstract multiplier method allows to treat both uncertainties and system nonlinearities in a common framework, therefore the proposed method of design can be applied to a broad class of dynamic systems. Furthermore, exogenous disturbances are also taken into consideration. The control is also supposed to be quadratically constrained. It is assumed furthermore, that the exact initial state is not known, but it lies in a given ball.

The paper is organized as follows. The problem will be stated, and some preliminary results will be recalled in Section 2. The main results for continuous and discrete time systems will be presented in Section 3. Two numerical examples illustrate the results in

Section 4. Finally, the conclusion will be drawn.

Standard notations are used. The transpose of matrix A is denoted by A^T, I_n is the identity matrix of size $n \times n$, and $P > 0$ (≥ 0) denotes the positive (semi-) definiteness of P. The maximum eigenvalue of the symmetric matrix P is $\lambda_M(P)$. Symbol ∇V stands for the gradient of the multivariable function V, symbol $\otimes$ is used for Kronecker-product, while $\oplus$ is the direct sum. The notation of time-dependence is omitted, if it does not cause any confusion. For the sake of brevity, asterisks replace the blocks in hypermatrices, and matrices in expressions that are inferred readily by symmetry (e.g. $\begin{bmatrix} A & B \\ * & C \end{bmatrix}$ stands for $\begin{bmatrix} A & B \\ B^T & C \end{bmatrix}$, and $(*)PX$ stands for X^TPX). In general, we shall write n_v for the number of coordinates of a vector v, i.e. $v \in \mathbf{R}^{n_v}$.

2 Problem statement and preliminaries

Consider system

$$\delta x = Ax + Bu + E_x w + H_x p_x, \tag{1}$$

$$y = Cx + H_y p_y + E_y w, \tag{2}$$

$$\zeta^T = \left(x^T C_\zeta^T \; u^T D_\zeta^T \right), \tag{3}$$

$$q_x = A_q x + B_q u + G_x p_x, \tag{4}$$

$$q_y = C_q x + D_q u + G_y p_y, \tag{5}$$

where $x \in \mathbf{R}^{n_x}$ is the state, $u \in \mathbf{R}^{n_u}$ is the input, $w \in \mathbf{R}^{n_w}$ is the exogenous disturbance, δx stands for $\dot{x}$ in the continuous-time and x^+ in the discrete-time case. The measured output is $y \in \mathbf{R}^{n_y}$, and $\zeta \in \mathbf{R}^{n_\zeta}$ represents the penalty output, where D_ζ is assumed to be nonsingular.

Uncertainty constraints. All system nonlinearities/uncertainties are represented by functions p_x and p_y possibly depending on t, x and u. Functions q_x and q_y are the uncertain outputs. The only available information about $p^T = \left(p_x^T, p_y^T\right) \in \mathbf{R}^{l_p}$ and $q^T = \left(q_x^T, q_y^T\right) \in \mathbf{R}^{l_q}$ is that their values are constrained by the set $\Omega = \Omega_1 \times ... \times \Omega_s$,

$$\Omega_i = \left\{ \begin{bmatrix} p_i \\ q_i \end{bmatrix} \in \mathbf{R}^{l_{p_i}+l_{q_i}} : \begin{bmatrix} p_i \\ q_i \end{bmatrix}^T \begin{bmatrix} Q_{0i} & S_{0i} \\ S_{0i}^T & R_{0i} \end{bmatrix} \begin{bmatrix} p_i \\ q_i \end{bmatrix} \geq 0 \right\}, \tag{6}$$

$i = 1, ..., s$, where $Q_{0i} = Q_{0i}^T$, $R_{0i} = R_{0i}^T \geq 0$ and S_{0i} are constant matrices, $p \in \mathbf{R}^{l_p}$, and $q \in \mathbf{R}^{l_q}$ are partitioned appropriately. We shall use the notations $Q_0 = \text{diag}\{Q_{01}, ..., Q_{0s}\}$, $R_0 = \text{diag}\{R_{01}, ..., R_{0s}\}$, $S_0 = \text{diag}\{S_{01}, ..., S_{0s}\}$. We note that the positive semi-definiteness of R_0 assures that the system (1)-(5) is well posed, i.e. for any (x, u) there is a p so that $\left[p^T, \; q^T\right]^T \in \Omega$. It is worth noting that the considered model of uncertainties involves several types of uncertainties frequently investigated in the literature. For example, if $Q_0 = 0$, $S_0 = I$ and $R_0 = 0$, then one speaks about positive real uncertainty, if $Q_0 = -I$, $S_0 = 0$ and $R_0 = I$, then one has norm-bounded uncertainties, (thus, the uncertainty of [24] and [22] can be obtained as a special case), and if $Q_0 = \frac{1}{2}(K_1^T K_2 + K_2^T K_1)$, $S_0 = \frac{1}{2}(K_1 + K_2)^T$ and $R_0 = I$, then one faces the case of sector-bounded uncertainties.

Control constraints. The control is supposed to be quadratically constrained, i.e.

$$u^T Q_u u \leq 1 \tag{7}$$

must be satisfied for a given matrix $Q_u = Q_u^T > 0$.

State constraints. Since the state is not measured, its initial value is not supposed to be known, but it is assumed that

$$\|x_0\|^2 \leq \rho,$$

where ρ is a given positive constant. We remark however that the initial state x_0 may supposed to be known.

Constraints on disturbances. The disturbances are produced by an exosystem, the input of which is the penalty output ζ of the original system, the output is w, and (ζ, w) satisfy the inequality

$$\|w\|_{S_L}^2 = w^T S_L w \leq \gamma_\Delta \|\zeta\|^2$$

with a given positive definite and symmetric matrix S_L and with $\gamma_\Delta < 1$.

Assign the *cost function*

$$J(x_0, u, w) = \begin{cases} \int\limits_0^\infty L(x(t), u(t), w(t))dt, & \text{if } t \in \mathbf{R}, \\ \sum\limits_{t=1}^\infty L(x(t), u(t), w(t)), & \text{if } t \in \mathbf{Z} \end{cases} \tag{8}$$

to system (1)-(2), where

$$L(x, u, w) = x^T Q_L x + u^T R_L u - w^T S_L w$$

with $Q_L = C_\zeta^T C_\zeta$, $R_L = D_\zeta^T D_\zeta$ and S_L given above. Thus, it follows from their definitions that Q_L, R_L

and S_L are symmetric, Q_L is positive semidefinite, R_L and S_L are positive definite matrices.

The aim is to keep the value of the cost function by the appropriate choice of the control as low as possible for all realizations of the uncertainties and the external perturbations. Because of the presence of uncertainties a minimum (or minimax) value of the cost cannot be achieved; one can only expect a guaranteed upper bound of it. The corresponding guaranteed cost control has to be determined in feedback form. Since the state is not available for feedback, a dynamic output feedback is sought. We look for the controller in the following form:

$$\delta\hat{x} = A_c\hat{x} + L_c y, \qquad \hat{x}(0) = 0, \tag{9}$$

$$u = K_c\hat{x} \tag{10}$$

where $\hat{x} \in \mathbf{R}^{n_x}$.

Introduce the new variable $z = \left(x^T, \hat{x}^T\right)^T$. With this notation, $u = \kappa z$, where $\kappa = (0, K_c)$, and the augmented closed-loop system is

$$\delta z = \mathcal{A}z + \mathcal{E}w + \mathcal{H}p, \tag{11}$$

$$q = \mathcal{A}_q z + \mathcal{G}p, \tag{12}$$

where $\mathcal{G} = \mathrm{diag}\{G_x, G_y\}$,

$$\mathcal{A} = \begin{bmatrix} A & BK_c \\ L_cC & A_c \end{bmatrix}, \ \mathcal{A}_q = \begin{bmatrix} A_q & B_qK_c \\ C_q & D_qK_c \end{bmatrix}, \tag{13}$$

$$\mathcal{E} = \begin{bmatrix} E_x \\ L_cE_y \end{bmatrix}, \qquad \mathcal{H} = \begin{bmatrix} H_x & 0 \\ 0 & L_cH_y \end{bmatrix}. \tag{14}$$

Set $\mathcal{K} = \mathrm{diag}\{I_{n_x}, K_c\}$. The running cost of the augmented closed-loop system is

$$\mathcal{L}(z, w) = z^T\overline{Q}_L z - w^T S_L w,$$

where $\overline{Q}_L = \mathcal{K}^T \mathrm{diag}\{Q_L, R_L\}\mathcal{K}$.

To formulate the notion of guaranteeing cost controller, consider an arbitrary nonlinear/uncertain system

$$\delta z = f(z, u, w, p), \tag{15}$$

$$q = g(z, u, p), \qquad \left[p^T, q^T\right]^T \in \Omega,$$

and a function $\mathcal{V} : \mathbf{R}^{n_z} \to \mathbf{R}^+$.

For system (15) introduce the following notation:

$$\mathcal{V}^*_{(15)}(z, u, w, p) = = \begin{cases} \nabla\mathcal{V}^T(z)\, f(z, u, w, p), & \text{if } t \in \mathbf{R}, \\ \mathcal{V}(f(z, u, w, p)) - \mathcal{V}(z), & \text{if } t \in \mathbf{Z}. \end{cases}$$

Definition 1 *Consider the nonlinear/uncertain system (15) with cost function of the type (8) and with a given set of nonlinearities/uncertainties Ω. The state-feedback $u = k(z)$ is a* guaranteeing cost robust minimax strategy *if there exists a function $\mathcal{V} : \mathbf{R}^{n_z} \to \mathbf{R}^+$ such that*

$$\sup_{\left[p^T,\ q^T\right]^T \in\Omega} \{\mathcal{V}^*_{(15)}(z, k(z), w, p) + L(z, k(z), w)\} < 0 \tag{16}$$

holds for all z and w, $\left[z^T,\ w^T\right] \neq \left[0^T,\ 0^T\right]$. In this case $\mathcal{V}(z_0)$ is called a guaranteed cost.

Remark 2 (A) *Similar definitions of guaranteed cost are frequently used in the literature (see e.g. [23], [42], [14], and the references therein). The rationality of this definition is explained by Theorem 7 given below.*
(B) *Observe that a cost guaranteeing control with special choice of matrices Q_L, R_L and S_L is an H_∞ control with the penalty output (3).*

The main problem is to find an appropriate $\mathcal{V}$ and a feedback $k(z)$ because of the need of maximization over Ω. The main idea of the multiplier method is that an equivalent inequality will be solved over a linear space at the expense of introducing a new matrix variable. The method assures that the feasibility set of the new inequality is the same as that of the original problem. In this way, the investigation of the inequality and of the uncertainty bounding set is separated and the problem becomes tractable. Paper [16] presented an abstract multiplier method. We recall here the basic definitions and the lemma to be used. Let $\mathcal{Q} \subset\mathbf{R}^l$ be given.

Definition 3 *([1], [16]) A symmetric matrix M is called a* multiplier matrix *for $\mathcal{Q}$ if $\xi^T M\xi \geq 0$ for all $\xi \in \mathcal{Q}$. If this inequality is strict, then M is called a* positive multiplier matrix *for $\mathcal{Q}$. The set $\mathcal{M}^+$ of positive multiplier matrices for $\mathcal{Q}$ is called a* sufficiently rich set of positive multipliers *for $\mathcal{Q}$, if for any positive multiplier $\overline{M}$ for $\mathcal{Q}$ there exists an element $M \in \mathcal{M}^+$ such that $M \leq \overline{M}$.*

Consider positive constants τ_i and ε_i, $i = 1, ..., s$ and set

$$\underline{\tau} = \mathrm{diag}\left\{\tau_1 I_{l_{p_1}}, ..., \tau_s I_{l_{p_s}}\right\},$$

$$\underline{\underline{\tau}} = \mathrm{diag}\left\{\tau_1 I_{l_{q_1}}, ..., \tau_s I_{l_{q_s}}\right\},$$

$$\underline{\varepsilon} = \mathrm{diag}\left\{\varepsilon_1 I_{l_{p_1}}, ..., \varepsilon_s I_{l_{p_s}}\right\},$$

$$\underline{\underline{\varepsilon}} = \mathrm{diag}\left\{\varepsilon_1 I_{l_{q_1}}, ..., \varepsilon_s I_{l_{q_s}}\right\}.$$

We note that, if $s = 1$, matrices $\underline{\tau}$, $\underline{\underline{\tau}}$, $\underline{\varepsilon}$ and $\underline{\underline{\varepsilon}}$ consist of a single block, thus two scalar parameters can be used instead. In order to avoid the repetition of big formulas, we shall use the matrix notations in the special case of $s = 1$, as well.

Lemma 4 *([17]) The set*

$$\mathcal{M}^+ = \left\{ M : \; M = \begin{bmatrix} \underline{\tau} Q_0 + \underline{\varepsilon} & \underline{\tau} S_0 \\ S_0^T \underline{\tau} & \underline{\underline{\tau}} R_0 + \underline{\underline{\varepsilon}} \end{bmatrix}, \right.$$
$$\left. \tau_i, \varepsilon_i > 0, \; i = 1, ..., s \right\} \quad (17)$$

consists of positive multiplier matrices for Ω. If $s = 1$, then $\mathcal{M}^+$ is sufficiently rich.

The recently published paper [22] gave a sufficient condition for the existence of robust stabilizing feedback based on a Luenberger type observer for continuous-time systems with norm-bounded uncertainties. The condition was formulated as an LMI. It was stated that the given LMI contains three adjustable parameters. In fact, there is only one free parameter. The source of the error was that authors failed to multiply the 6th and the 8th term from the left hand side and the 7th and 9th term from the right hand side by P^{-1} in equation (10). If the Schur complement is applied two more times after the correct congruence transformation, it turns out that only parameter ε_1 is adjustable. This certainly results in a lower $\alpha_{\max}$ in the second numerical example of that paper. We made several experiments for fixed $\varepsilon_4 = 0.01$ and for different values of ε_2 and ε_3 with changing magnitudes. It was found that $\alpha_{\max} < 0.98$ for the considered parameter combination.

The present paper solves a more general problem. Both continuous and discrete-time systems are examined with a far broader class of uncertainties/nonlinearities, and exogenous disturbances are considered, too. Also state and control constraints can a priori be given.

3 Main results

Assumption 1 *Inequalities* (1) $R_0 \geq 0$ *and*

(2) $Q_0 + \mathcal{G}^T S_0^T + S_0 \mathcal{G} + \mathcal{G}^T R_0 \mathcal{G} < 0$
hold true.

The second inequality of the Assumption 1 implies that $\left[p^T, \; p^T \mathcal{G}^T\right]^T \in \Omega$ if and only if $p = 0$, thus the origin is an equilibrium point of the unperturbed uncertain/nonlinear system. Moreover, the set of uncertain input vectors satisfying $\left[p^T, q^T\right]^T \in \Omega$ is bounded if q is defined by (4)-(5) and (x, u) comes from a bounded set, which is also a reasonable requirement. Similar conditions are applied e.g. in [41].

Set $N = 5n_x + n_u + n_w + l_p + l_q$, $\Xi = \operatorname{diag}\{Q_L, R_L, -S_L\}$. Introduce the $2n_x \times 2n_x$ matrix

$$\phi = \begin{cases} \phi_c = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}, & \text{if } t \in \mathbf{R}, \\ \phi_d = \begin{bmatrix} -I & 0 \\ 0 & I \end{bmatrix}, & \text{if } t \in \mathbf{Z}, \end{cases}$$

and the matrices

$$\mathcal{L}_1^T = \begin{bmatrix} I & \mathcal{A}^T & \mathcal{K}^T & 0 & 0 & \mathcal{A}_q^T \\ 0 & \mathcal{E}^T & 0 & I & 0 & 0 \end{bmatrix},$$
$$\mathcal{L}_0^T = \begin{bmatrix} 0 & \mathcal{H}^T & 0 & 0 & I & \mathcal{G}^T \end{bmatrix}. \quad (18)$$

Lemma 5 *Suppose that Assumption 1 holds true for the set Ω given by (6). The dynamic output feedback controller (9)-(10) defined by the matrices A_c, L_c, K_c yields a guaranteeing cost robust minimax strategy $k(z) = \kappa z$ and $\mathcal{V}(z_0)$ is the guaranteed cost with $\mathcal{V}(z_0) = z_0^T P z_0$, $P = P^T > 0$ if there exists an $M \in \mathcal{M}^+$ such that P, M satisfy the matrix inequality*

$$[*] \; diag\left\{\phi \otimes P, \; \Xi, \; M\right\} \left[\mathcal{L}_1, \mathcal{L}_0\right] < 0, \quad (19)$$

where $\mathcal{L}_1$, $\mathcal{L}_0$ correspond to matrices A_c, L_c, K_c as defined by (13), (14) and (18). The existence of $M \in \mathcal{M}^+$ is also necessary, if the uncertainty is unstructured, i.e. if $s = 1$.

Proof. Introduce function $F : \mathbf{R}^{2n_x + n_w + n_p} \to \mathbf{R}$ with the definition

$$F(z, w, p) = [*] \; [*] \; [\phi \otimes P] \begin{bmatrix} I \; 0 \; 0 \\ \mathcal{A} \; \mathcal{E} \; \mathcal{H} \end{bmatrix} \begin{bmatrix} z \\ w \\ p \end{bmatrix}$$
$$+ [*] \; [*] \; \Xi \begin{bmatrix} \mathcal{K} \; 0 \; 0 \\ 0 \; I \; 0 \end{bmatrix} \begin{bmatrix} z \\ w \\ p \end{bmatrix}$$

Then inequality (16) with respect to (11) is equivalent to

$$\sup_{\left[p^T, \; q^T\right]^T \in \Omega} F(z, w, p) < 0 \quad (20)$$

for all z and w, $\left[z^T, \quad w^T\right] \neq \left[0^T, \quad 0^T\right]$.

Set $\Psi = \operatorname{diag}\{\phi \otimes P, \; \Xi, \; 0\}$ and $\mathcal{B}_0 = \operatorname{im}\mathcal{L}_0$, $\mathcal{B}_1 = \operatorname{im}\mathcal{L}_1$, $\mathcal{B} = \operatorname{im}(\mathcal{L}_1, \mathcal{L}_0)$. Then $\mathcal{B} = \mathcal{B}_1 \oplus \mathcal{B}_0$ and $\mathcal{B}_1 \cap \mathcal{B}_0 = \{0\}$. A straightforward calculation shows that $F(z, w, p) = y^T \Psi y$, if $y \in \mathcal{B}$, i.e. if $y = \mathcal{L}_1 \left[z^T \quad w^T\right]^T + \mathcal{L}_0 \, p$. Set

$$V = \begin{bmatrix} 0 \; 0 \; 0 \; 0 \; I \; 0 \\ 0 \; 0 \; 0 \; 0 \; 0 \; I \end{bmatrix} \in \mathbf{R}^{(l_p + l_q) \times N},$$

and

$$\mathcal{B}_\Omega = \left\{ y \in \mathcal{B} \subset \mathbf{R}^N : \quad Vy \in \Omega \right\}.$$

Thus inequality (20) is equivalent to

$$y^T \Psi y < 0 \quad \text{ for all } y \in \mathcal{B}_\Omega,\ y \neq 0. \tag{21}$$

It was proven in ([16]) that inequality (21) holds true if there exists an $M \in \mathcal{M}^+$ such that

$$\Psi + V^T M V < 0 \quad \text{ for all } y \in \mathcal{B},$$

which is identical to (19). The necessity of the existence of $M \in \mathcal{M}^+$ with this property has been proven in ([16]), as well. □

Remark 6 *Since $\hat{x}(0)$ is fixed, the guaranteed cost depends on $x(0)$ only. Moreover, since any matrix in $\mathcal{M}^+$ is determined by two scalar parameters τ and ε, the existence of an appropriate $M \in \mathcal{M}^+$ is equivalent to the existence of these two scalar parameters.*

In what follows, we shall show that a guaranteeing cost controller in the sense of Definition 1 yields an upper bound of the cost function and a closed-loop system for which the origin is asymptotically stable. This gives the rationality of Definition 1.

Denote the ellipsoid in $\mathbf{R}^{2n_x}$ as

$$\Gamma(P, \alpha) = \left\{ \xi \in \mathbf{R}^{2n_x} : \xi^T P \xi \leq \alpha \right\}.$$

Theorem 7 *Consider the augmented closed-loop system (11)-(12) with Ω satisfying Assumption 1, and suppose that for a given $P = P^T > 0$, inequality (19) holds true. Then $\alpha = \lambda_M(P)\rho$ is an upper bound of the cost function for any admissible initial state, disturbance and uncertainty. Moreover, the ellipsoid $\Gamma(P, \alpha)$ is positively invariant and the origin is asymptotically stable for the closed-loop uncertain system.*

Proof. If inequality (19) holds true then there exists a $\delta > 0$ such that

$$\begin{aligned} &[*]\ \mathrm{diag}\left\{\phi \otimes P,\ \Xi,\ M\right\}\left[\mathcal{L}_1, \mathcal{L}_0\right] \\ &\qquad + \ \delta\ \mathrm{diag}\left\{\ I_{2n_x},\ \ 0,\ \ 0\ \right\} < 0, \end{aligned}$$

This means in compliance with Lemma 5 that for $k(z) = \kappa z$

$$\begin{aligned} &\mathcal{V}^*_{(11)}(z, k(z), w, p) \\ &\qquad + \ L(z, k(z), w) + \delta \|z\|^2 < 0 \end{aligned} \tag{22}$$

holds true for any $(z, w) \neq (0, 0)$ and for any uncertainty/nonlinearity satisfying $\left[p^T, q^T\right]^T \in \Omega$.

For the sake of definiteness, suppose that we are facing the continuous-time case. (The discrete-time case is completely analogous.) Integrating inequality (22) from 0 to $T > 0$, we obtain that

$$\begin{aligned} &\mathcal{V}(z(T)) - \mathcal{V}(z(0)) + \int_0^T z(t)^T \overline{Q}_L z(t) dt \\ &- \int_0^T w(t)^T S_L w(t) dt + \delta \int_0^T \|z(t)\|^2 dt < 0. \end{aligned} \tag{23}$$

Omitting the first and the last (nonnegative) terms on the left hand side, we obtain that for all $T > 0$

$$\int_0^T \mathcal{L}(z(t), w(t)) dt < \mathcal{V}(z(0)). \tag{24}$$

For the considered $w(.)$, $\mathcal{L}(z(t), w(t)) \geq 0$ for all t, therefore the integral on the left hand side of (24) is convergent as $T \to \infty$, and it tends to the value of the cost function. Since $\hat{x}(0) = 0$, for any x_0 with $\|x_0\|^2 \leq \rho$, we have that

$$\mathcal{V}(z(0)) \leq \lambda_M(P)\rho = \alpha.$$

From (23) it follows that $\mathcal{V}(z(T)) < \alpha$ for any $T > 0$, thus the ellipsoid $\Gamma(P, \alpha)$ is invariant. Furthermore P is assumed to be positive definite, thus it follows from (22) that function $\mathcal{V}$ is an appropriate Lyapunov-function having a derivative along the solutions of the closed-loop system (11) strictly smaller than $-\delta\|z\|^2$. Therefore the origin is asymptotically stable with a basin of attraction containing $\Gamma(P, \alpha)$.

Corollary 8 *If $z_0^T P z_0 \leq 1$ for any $z_0 = (x_0^T,\ 0^T)^T$ with $\|x_0\|^2 \leq \rho$, then $\Gamma(P, 1)$ is invariant for the closed-loop uncertain system. Moreover, if*

$$\begin{bmatrix} P & * \\ \kappa & Q_u^{-1} \end{bmatrix} \geq 0, \tag{25}$$

then for any $z \in \Gamma(P, 1)$, the control $u = \kappa z$ satisfies the control constraint (7).

Proof. The first part of the statement immediately follows from Theorem 7. The second part follows from (25) using Schur complements. □

We remark that other types of exogenous disturbances can be treated too. For example, disturbances of finite 'energy' are formulated as

$$\int_0^\infty w(t)^2 dt \leq \eta, \qquad \text{if } t \in \mathbf{R},$$

$$\sum_{t=1}^\infty w(t)^2 \leq \eta, \qquad \text{if } t \in \mathbf{Z},$$

where η is a given positive constant. A similar statement can be proven in this case, but the invariant ellipsoid is slightly different.

In what follows we propose methods to determine matrices P, A_c, L_c, K_c, and scalars ε_i, τ_i, $(i = 1, ..., s)$ in the discrete and in the continuous-time case. In order to obtain the matrix inequalities on the basis of which these parameters can be determined, we apply an approach similar to that of [12]. Represent matrix P and its inverse as

$$P = \begin{bmatrix} X & N_1 \\ N_1^T & Z \end{bmatrix}, \quad P^{-1} = \begin{bmatrix} Y & N_2 \\ N_2^T & W \end{bmatrix} \tag{26}$$

with $X = X^T > 0$, $Y = Y^T > 0$, and consider matrices

$$F_1 = \begin{bmatrix} X & I \\ N_1^T & 0 \end{bmatrix}, \quad F_2 = \begin{bmatrix} I & Y \\ 0 & N_2^T \end{bmatrix}, \tag{27}$$

where each block is of dimension $n_x \times n_x$. Clearly,

$$F_1^T P^{-1} F_1 = \begin{bmatrix} X & I \\ I & Y \end{bmatrix}, \quad P^{-1} F_1 = F_2. \tag{28}$$

Introduce furthermore the notations

$$\widetilde{K} = K_c N_2^T, \qquad \widetilde{L} = N_1 L_c, \tag{29}$$

$$\widetilde{A} = XAY + XB\widetilde{K} + \widetilde{L}CY + N_1 A_c N_2^T. \tag{30}$$

Now we derive a matrix inequality equivalent to (19), which is linear in all of the unknown matrices except for parameters τ_i.

3.1 The continuous-time case

In this subsection ϕ is fixed as $\phi = \phi_c$.

Theorem 9 *Inequality (19) holds true for the symmetric and positive definite matrix P partitioned as in (26) and for the coefficient matrices A_c, L_c, K_c of the controller and for the positive scalars τ_i and ε_i if and only if X, Y, $\widetilde{A}$, $\widetilde{L}$, $\widetilde{K}$, ε_i and τ_i $(i = 1, \ldots, s)$ satisfy the following matrix inequalities:*

$$\begin{bmatrix} \Phi_{11} & * \\ \Phi_{21} & \Phi_{22} \end{bmatrix} < 0, \qquad \begin{bmatrix} X & I \\ I & Y \end{bmatrix} > 0, \tag{31}$$

$$\Phi_{11} = \begin{bmatrix} \varphi_{11}^{11} & * & * & * \\ \varphi_{21}^{11} & \varphi_{22}^{11} & * & * \\ \varphi_{31}^{11} & \varphi_{32}^{11} & -S_L & * \\ \varphi_{41}^{11} & \varphi_{42}^{11} & 0 & \varphi_{44}^{11} \end{bmatrix},$$

$$\varphi_{11}^{11} = A^T X + XA + C^T \widetilde{L}^T + \widetilde{L} C,$$
$$\varphi_{22}^{11} = AY + B\widetilde{K} + Y^T A^T + \widetilde{K}^T B^T,$$
$$\varphi_{44}^{11} = Q_0 \underline{\tau}^{-1} + S_0 \mathcal{G} \underline{\tau}^{-1} + \underline{\tau}^{-1} \mathcal{G}^T S_0^T,$$
$$\varphi_{21}^{11} = A + \widetilde{A}^T,$$
$$\varphi_{31}^{11} = E_x^T X + E_y^T \widetilde{L}^T, \quad \varphi_{32}^{11} = E_x^T,$$
$$\varphi_{41}^{11} = \underline{\tau}^{-1} \begin{bmatrix} H_x^T X \\ H_y^T \widetilde{L}^T \end{bmatrix} + S_0 \begin{bmatrix} A_q \\ C_q \end{bmatrix},$$
$$\varphi_{42}^{11} = \underline{\tau}^{-1} \begin{bmatrix} H_x^T \\ 0 \end{bmatrix} + S_0 \begin{bmatrix} A_q Y + B_q \widetilde{K} \\ C_q Y + D_q \widetilde{K} \end{bmatrix},$$

$$\Phi_{21} = \begin{bmatrix} 0 & 0 & 0 & \underline{\tau}^{-1} \\ \varphi_{21}^{21} & \varphi_{22}^{21} & 0 & R_0^{1/2} \mathcal{G} \underline{\tau}^{-1} \\ \varphi_{31}^{21} & \varphi_{32}^{21} & 0 & \mathcal{G} \underline{\tau}^{-1} \\ C_\zeta & C_\zeta Y & 0 & 0 \\ 0 & \widetilde{K} & 0 & 0 \end{bmatrix},$$

$$\varphi_{21}^{21} = R_0^{1/2} \begin{bmatrix} A_q \\ C_q \end{bmatrix},$$
$$\varphi_{22}^{21} = R_0^{1/2} \begin{bmatrix} A_q Y + B_q \widetilde{K} \\ C_q Y + D_q \widetilde{K} \end{bmatrix},$$
$$\varphi_{31}^{21} = \begin{bmatrix} A_q \\ C_q \end{bmatrix}, \quad \varphi_{32}^{21} \begin{bmatrix} A_q Y + B_q \widetilde{K} \\ C_q Y + D_q \widetilde{K} \end{bmatrix},$$

$$\Phi_{22} = \mathrm{diag}\left\{ -\underline{\varepsilon}^{-1}, -\underline{\underline{\tau}}^{-1}, -\underline{\underline{\varepsilon}}^{-1}, -I, R_L^{-1} \right\}.$$

Proof. Consider inequality (21) with an arbitrary $M \in \mathcal{M}^+$ given in (18), and multiply the middle block-diagonal matrix from left and right by $L^T L = I$, where L is an appropriate permutation matrix to obtain that

$$[*] \begin{bmatrix} 0 & * & * & * & * & * \\ P & 0 & * & * & * & * \\ 0 & 0 & -S_L & * & * & * \\ 0 & 0 & 0 & \underline{\tau} Q_0 + \underline{\varepsilon} & * & * \\ 0 & 0 & 0 & 0 & \Upsilon & * \\ 0 & 0 & 0 & S_0^T \underline{\tau} & 0 & \underline{\underline{\tau}} R_0 + \underline{\underline{\varepsilon}} \end{bmatrix} \times$$
$$\times \begin{bmatrix} I & 0 & 0 \\ \mathcal{A} & \mathcal{E} & \mathcal{H} \\ 0 & I & 0 \\ 0 & 0 & I \\ \mathcal{K} & 0 & 0 \\ \mathcal{A}_{\mathrm{II}} & 0 & \mathcal{G} \end{bmatrix} < 0 \tag{32}$$

with $\Upsilon = \mathrm{diag}\{Q_L, R_L\}$. Applying the definition $Q_L = C_\zeta^T C_\zeta$ and the linearization lemma (see [37]) one obtains that (32) is equivalent to

$$\begin{bmatrix} \mathcal{A}^T P + P\mathcal{A} & * & * & * & * \\ \mathcal{E}^T P & -S_L & * & * & * \\ \mathcal{H}^T P + \underline{\tau} S_0 \mathcal{A}_q & 0 & \vartheta + \underline{\varepsilon} & * & * \\ \mathcal{C}_\zeta \mathcal{K} & 0 & 0 & -\mathcal{R}_L^{-1} & * \\ \mathcal{A}_q & 0 & \mathcal{G} & 0 & -(\underline{\tau} R_0 + \underline{\underline{\varepsilon}}) \end{bmatrix} < 0,$$

where the notations $\vartheta = \underline{\tau} Q_0 + \underline{\tau} S_0 \mathcal{G} + \mathcal{G}^T S_0^T \underline{\tau}$ and $\mathcal{C}_\zeta = \operatorname{diag}\{C_\zeta, I\}$ and $\mathcal{R}_L = \operatorname{diag}\{I, R_L\}$ have been used. Now we can apply the Schur complement lemma to get rid of the inverse of $(\underline{\tau} R_0 + \underline{\underline{\varepsilon}})$. Then apply the Schur complement again, the congruence transformation with $\operatorname{diag}\{P^{-1}, I, \underline{\tau}^{-1}, I, I, I, I\}$ and notations

$$\theta_1 = \mathcal{A}^T P^{-1} + P^{-1} \mathcal{A}^T, \quad \theta_2 = \underline{\tau}^{-1} \mathcal{H}^T + S_0 \mathcal{A}_q P^{-1}$$
$$\theta_3 = R_0^{1/2} \mathcal{A}_q P^{-1}, \quad \theta_4 = R_0^{1/2} \mathcal{G} \underline{\tau}^{-1}, \quad \theta_5 = \mathcal{C}_\zeta \mathcal{K} P^{-1}$$

to derive the equivalent inequality

$$\begin{bmatrix} \theta_1 & * & * & * & * & * & * \\ \mathcal{E}^T & -S_L & * & * & * & * & * \\ \theta_2 & 0 & \varphi_{44}^{11} & * & * & * & * \\ 0 & 0 & \underline{\tau}^{-1} & -\underline{\varepsilon}^{-1} & * & * & * \\ \theta_3 & \theta_4 & 0 & 0 & \underline{\tau}^{-1} & * & * \\ \mathcal{A}_q P^{-1} & 0 & \mathcal{G}\underline{\tau}^{-1} & 0 & 0 & \underline{\underline{\varepsilon}}^{-1} & * \\ \theta_5 & 0 & \mathcal{G}\underline{\tau}^{-1} & 0 & 0 & 0 & -\mathcal{R}_L^{-1} \end{bmatrix} < 0. \quad (33)$$

Multiply (33) by $\operatorname{diag}\{F_1^T, I, I, I, I, I, I\}$ from the left and by its transpose from the right, and take into consideration (27)-(28) to obtain that

$$F_1^T \mathcal{A} F_2 = \begin{bmatrix} \varphi_{11}^{11} & * \\ \varphi_{21}^{11} & \varphi_{22}^{11} \end{bmatrix}, \quad \mathcal{C}_\zeta \mathcal{K} F_2 = \begin{bmatrix} C_\zeta & C_\zeta Y \\ 0 & \widetilde{K} \end{bmatrix},$$
$$\mathcal{E}^T F_1 = \left[\varphi_{31}^{11}, \varphi_{32}^{11}\right],$$
$$\underline{\tau}^{-1} \mathcal{H} F_1 + S_0 \mathcal{A}_q F_2 = \left[\varphi_{41}^{11}, \varphi_{42}^{11}\right],$$
$$R_0^{1/2} \mathcal{A}_q F_2 = \left[\varphi_{21}^{21}, \varphi_{22}^{21}\right], \quad \mathcal{A}_q F_2 = \left[\varphi_{31}^{21}, \varphi_{32}^{21}\right]. \quad \square$$

We observe that the matrix inequality (31) is nonlinear in the unknown parameters τ_i. However, if τ_i's are fixed, this inequality becomes linear in variables X, Y, $\widetilde{K}$, $\widetilde{L}$, $\widetilde{A}$ and ε_i^{-1}, $i = 1, ..., s$. If (31) is feasible, inequality

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} > 0,$$

is equivalent to $I - XY < 0$, hence the left hand side is factorizable as $N_1 N_2^T = I - XY$, where N_1 and N_2 are invertible, i.e. matrices K_c, L_c and A_c can be expressed uniquely from the solution of (31) employing (29)-(30).

3.2 The discrete-time case

In this subsection ϕ is fixed as $\phi = \phi_d$.

Theorem 10 *Inequality (19) holds true for the symmetric and positive definite matrix P partitioned as in (26) and for the coefficient matrices A_c, L_c, K_c, of the controller and for the positive scalars τ_i and ε_i if and only if X, Y, $\widetilde{A}$, $\widetilde{L}$, $\widetilde{K}$, ε_i and τ_i, $(i = 1, ..., s)$, satisfy the following matrix inequality:*

$$\begin{bmatrix} \psi_{11} & * & * & * & * & * & * & * \\ 0 & -S_L & * & * & * & * & * & * \\ \psi_{31} & 0 & \psi_{33} & * & * & * & * & * \\ \psi_{41} & \psi_{42} & \psi_{43} & \psi_{44} & * & * & * & * \\ 0 & 0 & I & 0 & -\underline{\varepsilon}^{-1} & * & * & * \\ \psi_{61} & 0 & \mathcal{G} & 0 & 0 & -\underline{\underline{\varepsilon}}^{-1} & * & * \\ \psi_{71} & 0 & R_0^{1/2}\mathcal{G} & 0 & 0 & 0 & -\underline{\underline{\tau}}^{-1} & * \\ \psi_{81} & 0 & 0 & 0 & 0 & 0 & 0 & -\mathcal{R}_L^{-1} \end{bmatrix} < 0, \quad (34)$$

where

$$\psi_{11} = -\begin{bmatrix} X & I \\ I & Y \end{bmatrix}, \ \psi_{31} = \tau S_0 \psi_{71},$$
$$\psi_{33} = \underline{\tau} Q_0 + \underline{\tau} S_0 \mathcal{G} + \mathcal{G}^T S_0^T \underline{\tau},$$
$$\psi_{41} = \begin{bmatrix} XA + \widetilde{L}C & \widetilde{A} \\ A & AY + B\widetilde{K} \end{bmatrix},$$
$$\psi_{42} = \begin{bmatrix} XE_x + \widetilde{L}E_y \\ E_x \end{bmatrix},$$
$$\psi_{43} = \begin{bmatrix} XH_x & \widetilde{L}H_y \\ H_x & 0 \end{bmatrix}, \ \psi_{44} = \psi_{11},$$
$$\psi_{61} = \begin{bmatrix} A_q & A_q Y + B_q \widetilde{K} \\ C_q & C_q Y + D_q \widetilde{K} \end{bmatrix},$$
$$\psi_{71} = R_0^{1/2} \psi_{61}, \quad \psi_{81} = \begin{bmatrix} C_\zeta & C_\zeta Y \\ 0 & \widetilde{K} \end{bmatrix},$$

Proof. The theorem can be proved completely analogously to the previous one, the details are omitted for the lack of space. $\square$

Inequality (34) is nonlinear in he unknown parameters τ_i, but for any fixed values, it is an LMI in the remaining unknown matrices. Observations similar to the continuous-time case can be made concerning the computation of the coefficient matrices of the controller, as well.

3.3 Control constraint LMIs

For a given value of parameters τ_i, Theorems 9 and 10 provide LMIs on the basis of which one can obtain the solution of the formulated problem, if no control constraint is imposed. Next we shall derive additional LMIs to assure the satisfaction of the control constraint presuming that the initial value x_0 is admissible.

Theorem 11 *Assume that the condition of Corollary 8 is valid. Suppose that in addition to (31) in the continuous-time case and to (34) in the discrete-time case inequalities*

$$X - \frac{1}{\rho} I \leq 0, \tag{35}$$

$$\begin{bmatrix} X & I & 0 \\ I & Y & \widetilde{K}^T \\ 0 & \widetilde{K} & Q_u^{-1} \end{bmatrix} \geq 0 \tag{36}$$

hold true. Then $z(t)^T P z(t) \leq 1$ for any $t \geq 0$, and $u(t) = \kappa z(t)$ satisfies the control constraint (7) for all $t \geq 0$.

Proof. Suppose that P is partitioned according to (26) and $\|x_0\|^2 \leq \rho$, $\hat{x}(0) = 0$. Then we have $z_0^T P z_0 = x_0^T X x_0$, thus it follows from (35) that $z_0^T P z_0 \leq 1$. Therefore, Corollary 8 involves that $z(t)^T P z(t) \leq 1$ for any $t \geq 0$. On the other hand, if

$$z(t)^T \kappa^T Q_u \kappa z(t) \leq z(t)^T P z(t), \tag{37}$$

then (7) holds true for any $t \geq 0$. Inequality (37) holds true, if

$$P - \kappa^T Q_u \kappa \geq 0,$$

which is equivalent to

$$\begin{bmatrix} P & * \\ \kappa & Q_u^{-1} \end{bmatrix} \geq 0. \tag{38}$$

If we apply the congruence transformation for (38) with diag $\left\{ P^{-1} F_1,\ I \right\}$, we get that the latter inequality is equivalent to the required one (36). □

Remark 12 *Every feasible solution of systems (31), (35)-(36) or (34), (35)-(36), provides a cost guaranteeing controller. Several types of objective functions can be assigned to the systems of inequalities. For example, paper [11] proposes to minimize tr$P$ to obtain the largest set of admissible states of the augmented system. (Matrix P was kept there as the unknown of the LIMs.) Similar purpose can be achieved by minimizing $\mu := 1/\varrho$.*

4 Numerical examples

Example 1. ([24], [22]) To illustrate the effectiveness of our approach we consider the same example as [24] and [22]. The system is described by the following parameters:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 1 \\ 1 & -2 & -5 \end{bmatrix},\ B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix},\ A_q = \begin{bmatrix} 0 & 0 & \alpha \\ 0 & \beta & 0 \\ \gamma & 0 & 0 \end{bmatrix},$$

$$p = \begin{bmatrix} p_x \\ p_y \end{bmatrix} \in \mathbf{R}^4,\ q = \begin{bmatrix} q_x \\ q_y \end{bmatrix} \in \mathbf{R}^4,\ p_i = F_i(t) q_i,$$

with $|F_i(t)| \leq 1,\ i = 1, \ldots, 4$, (i.e. $Q_{0i} = -1$, $S_{0i} = 0$, $R_{0i} = 1$,) and $E_x = 0$, $H_x = I_3$, $C = (1\ 0\ 1)$, $E_y = 0$, $H_y = 1$, $C_q = (0\ \delta\ 0)$ $G_x = 0$, $G_y = 0$. Similarly to [24] and [22], we assumed that $\alpha = \beta = \gamma = \delta$. The maximum value of α achieved by [24] was 1.35, while inequality (31) has a feasible stabilizing solution up to $\alpha_{max} = 3.48$. (The results of the second paper are not comparable, the best result that we could achieve with the corrected inequality and with a wide range of parameter combinations was $\alpha_{max} < 0.98$. Figure 1 illustrates that the dynamic

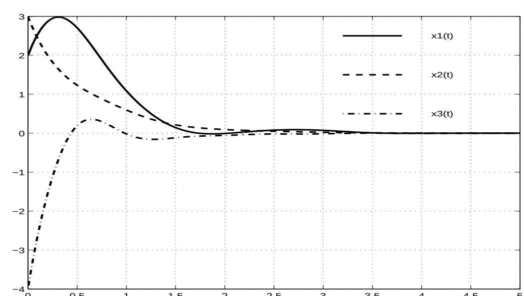

Figure 1: Time evolution of the state variables.

output feedback control obtained with $\alpha_{max} = 3.4$ provides a quick convergence and smaller deviations than in [24], when the initial state and the uncertainties in the simulation are same as there.

Example 2. ([5]) To illustrate the applicability of our approach to nonlinear systems we consider the example of a flexible joint robotic arm investigated e.g. in [5]. The the dynamics of this model contains a sector bounded nonlinearity. [5] constructed a stabilizing predictive control supposing that the state was available for feedback. We applied here the dynamic output feedback control (9)-(10) supposing that only x_1 and x_3 were measured. Moreover, we allowed the effect of exogenous disturbances with $w(t) \in \mathbf{R}$, $E_x^T = \begin{bmatrix} 0 & 1 & 0 & 1 \end{bmatrix}$. In our representation, the problem to be solved was characterized by matrices

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -48.6 & -1.25 & 48.6 & 0 \\ 0 & 0 & 0 & 1 \\ 19.5 & 0 & -16.7 & 0 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 21.6 \\ 0 \\ 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 1\ 0\ 0\ 0 \\ 0\ 0\ 1\ 0 \end{bmatrix},\ H_x^T = \begin{bmatrix} 0\ 0\ 0\ -3.33 \end{bmatrix},$$

$A_q = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$, $Q_0 = -1$, $S_0 = 1$, $R_0 = 0$, $Q_u = 1/2.25$, $C_\zeta = \text{diag}\{1, \sqrt{0.1}, 1, \sqrt{0.1}\}$, $R_L = 0.1$, and $p_x = \sin x_3 + x_3$, $q_x = x_3$. The initial state is $x_0 = \begin{pmatrix} 1.2 & 0 & 0 & 0 \end{pmatrix}^T$ and we set $\rho = 1.21$ and $S_L = Q_L$. Figures 2 illustrates that the dynamic

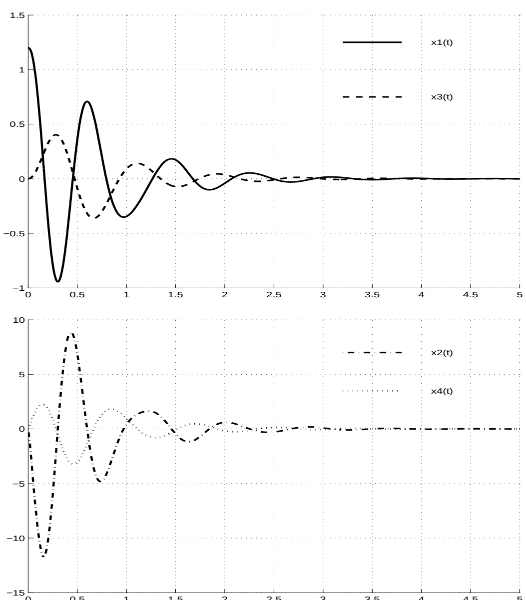

Figure 2: Time evolution of the state variables.

output feedback control still provides a quick convergence at the presence of exogenous disturbances. The disturbances were simulated as $w(t) = 0.1\sin(t)x(t)$, which was admissible with any $0.01 \leq \gamma_\Delta < 1$.

The computations were made in both examples using YALMIP ([25]) and MATLAB.

5 Conclusion

The paper establishes sufficient (and necessary) conditions for dynamic output feedback to be cost guaranteeing and stabilizing in the case of systems with quadratically constrained nonlinearities/uncertainties. It is shown that this condition was sufficient for the boundedness of the cost and the trajectories, if the constructed dynamic feedback is applied. The considered class of nonlinearities/uncertainties permits to treat a great number of nonlinearity/uncertainty types by the appropriate choice of system parameters. Both the discrete and continuous-time cases are examined. The conditions are formulated as matrix inequalities. When one scalar parameter is fixed, the matrix inequality system to be solved is linear. The proposed method extends the results of a recently published paper in several aspects. Numerical examples illustrate the application of the proposed method.

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