Permanence and Asymptotically Periodic Solution for A Cyclic Predator-Prey Model With Sigmoidal Type Functional Response

CHANGJIN XU
Guizhou University of Finance
and Economics
Guizhou Key Laboratory of Economics
System Simulation
Longchongguan Street 276, 550004 Guiyang
CHINA
xcj403@126.com

QIMING ZHANG
Hunan University of Technology
College of science
Beijing Street 42, 412007 Zhuzhou
CHINA
zhqm20082008@sina.com

Abstract: This paper is concerned with a cyclic predator-prey system with Sigmoidal type functional response. By using the differential inequality theory, some sufficient conditions are derived for the permanence of the system. By constructing a suitable Liapunov function, we obtain that the system has a unique asymptotically periodic solution which is globally asymptotically stable. Some numerical simulations that illustrate our analytical predictions are carried out. The paper ends with a brief conclusion.

Key–Words: Predator-prey system, Permanence, Sigmoidal type functional response, Asymptotically periodic solution, Liapunov function; Global stability

1 Introduction

In recent years, the interest in study of the dynamical properties occurring in the predator-prey system with delay has been growing rapidly. For example, Li and Ye [1] had made discussion about the multiple positive almost periodic solutions to an impulsive non-autonomous Lotka-Volterra predator-prey system with harvesting terms. Zhang and Luo [2] analyzed the multiple periodic solutions of a delayed predator-prey system with stage structure for the predator. Dai et al. [3] focused on the multiple periodic solutions for impulsive Gause-type ratio-dependent predator-prey systems with non-monotonic numerical responses. Wang and Fan [4] studied the multiple periodic solutions for a non-autonomous delayed predator-prey model with harvesting terms. Zhang et al. [5] studied the multiplicity of positive periodic solutions to a generalized delayed predator-prey system with stocking. For more investigation about predator-prey models or related topic, one can see [6-67]. It shall be pointed out that all the papers mentioned above are concerned with periodic coefficients. However, the asymptotically periodic system describe our real word more realistic and more accurate than the periodic ones, but the research work about asymptotically periodic predator-prey is scare at present. Recently, Wei and Wang [68] investigated a asymptotically periodic solution multispecies competition predator-prey model with Hilling III functional response. Yang and Chen [69] studied the uniformly strong persistence of a nonlinear asymptotically periodic multispecies competition predator-prey system with general functional response.

In this paper, we will deal with the following cyclic predator-prey system with Sigmoidal type functional response

$$\begin{cases} \dot{x}_1(t) = x_1(t)\Bigg[r_1(t) - a_1(t)x_1(t) \\ \qquad - \dfrac{d_1(t)x_1(t)x_2(t)}{c_1(t)+b_1(t)x_1(t)+x_1^2(t)} \\ \qquad + \dfrac{k_3(t)d_3(t)x_3^2(t)}{c_3(t)+b_3(t)x_3(t)+x_3^2(t)}\Bigg], \\ \dot{x}_2(t) = x_2(t)\Bigg[r_2(t) - a_2(t)x_2(t) \\ \qquad - \dfrac{d_2(t)x_2(t)x_3(t)}{c_2(t)+b_2(t)x_2(t)+x_2^2} \\ \qquad + \dfrac{k_1(t)d_1(t)x_1^2(t)}{c_1(t)+b_1(t)x_1(t)+x_1^2(t)}\Bigg], \\ \dot{x}_3(t) = x_3(t)\Bigg[r_3(t) - a_3(t)x_3(t) \\ \qquad - \dfrac{d_3(t)x_1(t)x_3(t)}{c_3(t)+b_3(t)x_3(t)+x_3^2(t)} \\ \qquad + \dfrac{k_2(t)d_2(t)x_2^2(t)}{c_2(t)+b_2(t)x_2(t)+x_2^2(t)}\Bigg], \end{cases} \quad (1)$$

where x_2 is the predator of x_1, x_3 is the predator of x_2 and x_1 is the predator of x_3, they have dependent density and Sigmoidal functional response. $a_i(t), b_i(t), c_i(t), d_i(t), k_i(t), r_i(t) (i = 1, 2, 3)$ are continuous nonnegative and bounded function within $[0, +\infty)$. Moreover, $a_i(t), c_i(t)(i = 1, 2, 3) > 0$.

Now we define $R_+ = [0, +\infty)$ and introduce the concept of the asymptotically function.

Definition 1 *If $f \in C(R_+, R)$, where $f(t) = g(t) + \alpha(t)$, $g(t)$ is continuous T-periodic function and $\lim_{t\to+\infty} \alpha(t) = 0$, then $f(t)$ is called asymptotically T-periodic function.*

Throughout this paper, we always assume that
(H1) $a_i(t), b_i(t), c_i(t), d_i(t), k_i(t), r_i(t) (i = 1, 2, 3)$ are all continuous positive, bounded asymptotically periodic functions.

This paper is organized as follows. In Section 2, the permanence of system (1) are studied by using the differential inequality theory. In Section 3, the existence and uniqueness of asymptotically periodic solution are investigated by constructing a suitable Liapunov function. Some numerical simulations that illustrate our analytical predictions are carried out in Section 4. A brief conclusion is drawn in Section 5.

2 Permanence

For convenience in the following discussing, we always use the notations:

$$f^l = \inf_{t\in R} f(t), \quad f^u = \sup_{t\in R} f(t),$$

where $f(t)$ is a continuous function. The initial value condition of system (1) is $x_i(0) = \phi_i(0) > 0(i = 1, 2, 3)$. In order to obtain the main result of this paper, we shall first state some definitions and several lemmas which will be useful in the proving the main result.

Definition 2 *We say that system (1) is permanence if there are positive constants m and M such that for each positive solution $(x_1(t), x_2(t), x_3(t))$ of system (1) satisfies*

$$m \le \lim_{t\to+\infty} \inf x_i(t) \le \lim_{t\to+\infty} \sup x_i(t) \le M,$$

where $i = 1, 2, 3$.

Definition 3 *The solution $X(t, t_0, \phi)$ is called ultimately bounded. If there exists $B > 0$ such that for any $t_0 \ge 0, \phi \in C$, there exists $T = T(t_0, \phi) > 0$ when $t \ge t_0 + T, |X(t, t_0, \phi)| \le B$.*

Lemma 4 [70] *If $a > 0, b > 0$ and $\dot{x} \ge x(b - ax)$, when $t \ge 0$ and $x(0) > 0$, we have*

$$\lim_{t\to+\infty} \inf x(t) \ge \frac{b}{a}.$$

If $a > 0, b > 0$ and $\dot{x} \le x(b - ax)$, when $t \ge 0$ and $x(0) > 0$, we have

$$\lim_{t\to+\infty} \sup x(t) \le \frac{b}{a}.$$

Now we state our permanence result for system (1).

Lemma 5 *The set $R^n_+ = \{(x_1, x_2, x_3)|x_i > 0, i = 1, 2, 3.\}$ is the positively invariant set of system (1).*

Proof: It follows from the initial value condition $x_i(0) = \phi_i(0 > 0(i = 1, 2, 3)$ that

$$\begin{cases} x_1(t) = x_1(0) \exp\left\{ \int_0^t \left[r_1(s) - a_1(s)x_1(s) \right.\right. \\ \qquad - \dfrac{d_1(s)x_1(s)x_2(s)}{c_1(s) + b_1(s)x_1(s) + x_1^2(s)} \\ \qquad \left.\left. + \dfrac{k_3(s)d_3(s)x_3^2(s)}{c_3(s) + b_3(s)x_3(s) + x_3^2(s)} \right] ds \right\}, \\ x_2(t) = x_2(0) \exp\left\{ \int_0^t \left[r_2(s) - a_2(s)x_2(s) \right.\right. \\ \qquad - \dfrac{d_2(s)x_2(s)x_3(s)}{c_2(s) + b_2(s)x_2(s) + x_2^2(s)} \\ \qquad \left.\left. + \dfrac{k_1(s)d_1(s)x_1^2(s)}{c_1(s) + b_1(s)x_1(s) + x_1^2(s)} \right] ds \right\}, \\ \dot{x}_3(t) = x_3(0) \exp\left\{ \int_0^t \left[r_3(s) - a_3(s)x_3(s) \right.\right. \\ \qquad - \dfrac{d_3(s)x_1(s)x_3(s)}{c_3(s) + b_3(s)x_3(s) + x_3^2(s)} \\ \qquad \left.\left. + \dfrac{k_2(s)d_2(s)x_2^2(s)}{c_2(s) + b_2(s)x_2(s) + x_2^2(s)} \right] ds \right\}. \end{cases} \tag{2}$$

The proof of Lemma 5 is complete.

Theorem 6 *Let M_1, M_2, M_3 are defined by (3),(5) and (7), respectively. In addition to the condition (H1), suppose that the following condition*

$$(H2)\ b_1^l r_1^l > d_1^u M_2, b_2^l r_2^l > d_2^u M_3, b_3^l r_3^l > d_3^u M_1$$

hold, then system (1) is permanent, that is, there exist positive constants $m_i, M_i (i = 1, 2, 3)$ which are independent of the solution of system (1), such that for any positive solution $(x_1(t), x_2(t), x_3(t))$ of system (1) with the initial condition

$$x_i(0) \ge 0(i = 1, 2, 3),$$

one has

$$m_i \le \lim_{t\to+\infty} \inf x_i(t) \le \lim_{t\to+\infty} \sup u(t) \le M_i.$$

Proof: It is easy to see that system (1) with the initial value condition $(x_1(0), x_2(0), x_3(0))$ has positive solution $(x_1(t), x_2(t), x_3(t))$ passing through $(x_1(0), x_2(0), x_3(0))$. Let $(x_1(t), x_2(t), x_3(t))$ be any positive solution of system (1) with the initial condition $(x_1(0), x_2(0), x_3(0))$. It follows from the first equation of system (1) that

$$\begin{aligned}\frac{dx_1(t)}{dt} &= x_1(t)\Big[r_1(t) - a_1(t)x_1(t)\\ &\quad -\frac{d_1(t)x_1(t)x_2(t)}{c_1(t)+b_1(t)x_1(t)+x_1^2(t)}\\ &\quad +\frac{k_3(t)d_3(t)x_3^2(t)}{c_3(t)+b_3(t)x_3(t)+x_3^2(t)}\Big]\\ &\le x_1(t)\Big[r_1(t) - a_1(t)x_1(t)\\ &\quad +k_3(t)d_3(t)\Big]\\ &\le x_1(t)\left[r_1^u + k_3^u d_3^u - a_1^l x_1(t)\right].\end{aligned}$$

It follows from Lemma 4 that

$$\lim_{t\to+\infty}\sup x_1(t) \le \frac{r_1^u + k_3^u d_3^u}{a_1^l} := M_1. \tag{3}$$

For any positive constant $\varepsilon_1 > 0$, it follows (3) that there exists a $T_1 > 0$ such that for all $t > T_1$,

$$x_1(t) \le M_1 + \varepsilon. \tag{4}$$

By the second equation of system (1) that

$$\begin{aligned}\frac{dx_2(t)}{dt} &= x_2(t)\Big[r_2(t) - a_2(t)x_2(t)\\ &\quad -\frac{d_2(t)x_2(t)x_3(t)}{c_2(t)+b_2(t)x_2(t)+x_2^2}\\ &\quad +\frac{k_1(t)d_1(t)x_1^2(t)}{c_1(t)+b_1(t)x_1(t)+x_1^2(t)}\Big]\\ &\le x_2(t)\Big[r_2(t) - a_2(t)x_2(t)\\ &\quad +k_1(t)d_1(t)\Big]\\ &\le x_2(t)\left[r_2^u + k_1^u d_1^u - a_2^l x_2(t)\right].\end{aligned}$$

It follows from Lemma 4 that

$$\lim_{t\to+\infty}\sup x_2(t) \le \frac{r_2^u + k_1^u d_1^u}{a_2^l} := M_2. \tag{5}$$

For any positive constant $\varepsilon > 0$, it follows (5) that there exists a $T_2 > 0$ such that for all $t > T_2$,

$$x_2(t) \le M_2 + \varepsilon. \tag{6}$$

By the third equation of system (1) that

$$\begin{aligned}\frac{dx_3(t)}{dt} &= x_3(t)\Big[r_3(t) - a_3(t)x_3(t)\\ &\quad -\frac{d_3(t)x_1(t)x_3(t)}{c_3(t)+b_3(t)x_3(t)+x_3^2(t)}\\ &\quad +\frac{k_2(t)d_2(t)x_2^2(t)}{c_2(t)+b_2(t)x_2(t)+x_2^2(t)}\Big]\\ &\le x_2(t)\Big[r_3(t) - a_3(t)x_3(t)\\ &\quad +k_2(t)d_2(t)\Big]\\ &\le x_2(t)\left[r_3^u + k_2^u d_2^u - a_3^l x_3(t)\right].\end{aligned}$$

It follows from Lemma 4 that

$$\lim_{t\to+\infty}\sup x_3(t) \le \frac{r_3^u + k_2^u d_2^u}{a_3^l} := M_3. \tag{7}$$

For any positive constant $\varepsilon > 0$, it follows (7) that there exists a $T_3 > T_2 > 0$ such that for all $t > T_3$,

$$x_3(t) \le M_3 + \varepsilon. \tag{8}$$

For $t \ge T_3$, from (6) and the first equation of system (1), we have

$$\begin{aligned}\frac{dx_1(t)}{dt} &= x_1(t)\Big[r_1(t) - a_1(t)x_1(t)\\ &\quad -\frac{d_1(t)x_1(t)x_2(t)}{c_1(t)+b_1(t)x_1(t)+x_1^2(t)}\\ &\quad +\frac{k_3(t)d_3(t)x_3^2(t)}{c_3(t)+b_3(t)x_3(t)+x_3^2(t)}\Big]\\ &\ge x_1(t)\Big[r_1(t) - a_1(t)x_1(t)\\ &\quad -\frac{d_1(t)x_2(t)}{b_1(t)}\Big]\\ &\ge x_1(t)\Big[r_1^l - a_1^u x_1(t)\\ &\quad -\frac{d_1^u(M_2+\varepsilon)}{b_1^l}\Big].\end{aligned}$$

Thus, as a direct corollary of Lemma 4, one has

$$\lim_{t\to+\infty}\inf x_1(t) \ge \frac{b_1^l r_1^l - d_1^u(M_2+\varepsilon)}{a_1^u b_1^l}. \tag{9}$$

Setting $\varepsilon \to 0$, it follows that

$$\lim_{t\to+\infty}\inf x_1(t) \ge \frac{b_1^l r_1^l - d_1^u M_2}{a_1^u b_1^l} := m_1. \tag{10}$$

For $t \geq T_3$, from (8) and the second equation of system (1), we have

$$\begin{aligned}\frac{dx_2(t)}{dt} &= x_2(t)\bigg[r_2(t) - a_2(t)x_2(t) \\ &\quad -\frac{d_2(t)x_2(t)x_3(t)}{c_2(t)+b_2(t)x_2(t)+x_2^2} \\ &\quad +\frac{k_1(t)d_1(t)x_1^2(t)}{c_1(t)+b_1(t)x_1(t)+x_1^2(t)}\bigg] \\ &\geq x_2(t)\bigg[r_2(t)-a_2(t)x_2(t) \\ &\quad -\frac{d_2(t)x_3(t)}{b_2(t)}\bigg] \\ &\geq x_2(t)\Big[r_2^l - a_2^u x_2(t) \\ &\quad -\frac{d_2^u(M_3+\varepsilon)}{b_2^l}\Big]. \end{aligned} \quad (11)$$

It follows Lemma 2.1 and (11) that

$$\lim_{t\to+\infty} \inf x_2(t) \geq \frac{b_2^l r_2^l - d_2^u(M_3+\varepsilon)}{a_2^u b_2^l}. \quad (12)$$

Setting $\varepsilon \to 0$ in (12) leads to

$$\lim_{t\to+\infty} \inf x_2(t) \geq \frac{b_2^l r_2^l - d_2^u M_3}{a_2^u b_2^l} := m_2. \quad (13)$$

For $t \geq T_3 > T_1 > 0$, from (4) and the second equation of system (1), we have

$$\begin{aligned}\frac{dx_3(t)}{dt} &= x_3(t)\bigg[r_3(t) - a_3(t)x_3(t) \\ &\quad -\frac{d_3(t)x_1(t)x_3(t)}{c_3(t)+b_3(t)x_3(t)+x_3^2(t)} \\ &\quad +\frac{k_2(t)d_2(t)x_2^2(t)}{c_2(t)+b_2(t)x_2(t)+x_2^2(t)}\bigg] \\ &\geq x_3(t)\bigg[r_3(t)-a_3(t)x_3(t) \\ &\quad -\frac{d_3(t)x_1(t)x_3(t)}{b_3(t)}\bigg] \\ &\geq x_3(t)\bigg[r_3^l - a_3^u x_3(t) \\ &\quad -\frac{d_3^u(M_1+\varepsilon)}{b_3^l}\bigg]. \end{aligned}$$

It follows Lemma 4 that

$$\lim_{t\to+\infty} \inf x_3(t) \geq \frac{b_3^l r_3^l - d_3^u(M_1+\varepsilon)}{a_3^u b_3^l}. \quad (14)$$

Setting $\varepsilon \to 0$ in (2.13) leads to

$$\lim_{t\to+\infty} \inf x_3(t) \geq \frac{b_3^l r_3^l - d_3^u M_1}{a_3^u b_3^l} := m_3. \quad (15)$$

In view of (3),(5),(7),(10), (13) and (15), we can conclude that system (1) is permanent. The proof of Theorem 6 is complete.

Denote

$$\Omega = \{(x_1,x_2,x_3)^T \in R_+ | m_i \leq x_i \leq M_i, i = 1,2,3\}.$$

Corollary 7 *The set Ω is the ultimately bounded set of system (1).*

3 Existence and uniqueness of asymptotically periodic solution

Let us consider the asymptotically periodic system as follows

$$\frac{dx}{dt} = f(t, x_t), \quad (16)$$

where $f \in C([-r,0], R^n)$ and for any $x_t \in C$. Define $x_t(\theta) = x(t+\theta), \theta \in [-r,0]$. For any $x = (x_1, x_2, \cdots, x_n) \in R^n$, we define $|x| = \sum_{i=1}^n |x_i|$, from the above proof, we can see that there exists $H > 0$ such that $|x| \leq nM_i < H$. For any $\phi \in C$, define $||\phi|| = \sup_{-r\leq\theta\leq0} |\phi(\theta)|$. Let $C_H = \{\phi \in C, ||\phi|| < H\}$ and $\bar{S}_H = \{x \in R^n, |x| < H\}$. In order to focus on the existence and uniqueness of asymptotically periodic solution of system (16), we consider the adjoint system

$$\begin{cases} \frac{dx}{dt} = f(t, x_t), \\ \frac{dy}{dt} = f(t, y_t). \end{cases} \quad (17)$$

Then we begin with our analysis with Lemma 3.1.

Lemma 8 (Yuan [71]) *Let $V \in C(R_+ \times S_H \times S_H, R_+)$ satisfy*
(i) $a(|x-y| \leq V(t,x,y) \leq b(|x-y|)$, where $a(r)$ and $b(r)$ are continuously positively increasing functions;
(ii) $|V(t,x_1,y_1) - V(t,x_2,y_2)| \leq l(|x_1-x_2| + |y_1-y_2|)$, where l is a constant and satisfies $l > 0$;
(iii) there exists continuous non-increasing function $P(s)$, such that for $s > 0, P(s) > s$. And as $P(V(t,\phi(0),\phi(0))) > V(t+\theta,\phi(\theta),\phi(\theta)), \theta \in [-r,0]$, it follows that $V'_{(17)}(t,\phi(0),\phi(0)) \leq -\delta V(t,\phi(0),\phi(0))$, where δ is a constant and satisfies $\delta > 0$. Furthermore, system (16) has a solution

$\xi(t)$ for $t \geq t_0$ and satisfies $||\xi_t|| \leq H$. Then system (16) has a unique asymptotically periodic solution, which is uniformly asymptotically stable.

Theorem 9 *Let $\theta_1, \theta_2, \theta_3$ and δ are defined by (24), (25), (26) and (27), respectively. In addition to the conditions (H1) and (H2), assume further that $\delta > 0$ is satisfied, then there exists a unique asymptotically periodic solution of system (1) wich is uniformly asymptotically stable.*

Proof: By Theorem 6 (or Corollary 7), we know that the solution of system (1) is ultimately bounded. Ω is the region of ultimately bounded. We consider the adjoint system of system (1) as follows

$$\begin{cases} \dot{x}_1(t) = x_1(t)\Big[r_1(t) - a_1(t)x_1(t) \\ \qquad - \frac{d_1(t)x_1(t)x_2(t)}{c_1(t)+b_1(t)x_1(t)+x_1^2(t)} \\ \qquad + \frac{k_3(t)d_3(t)x_3^2(t)}{c_3(t)+b_3(t)x_3(t)+x_3^2(t)}\Big], \\ \dot{x}_2(t) = x_2(t)\Big[r_2(t) - a_2(t)x_2(t) \\ \qquad - \frac{d_2(t)x_2(t)x_3(t)}{c_2(t)+b_2(t)x_2(t)+x_2^2} \\ \qquad + \frac{k_1(t)d_1(t)x_1^2(t)}{c_1(t)+b_1(t)x_1(t)+x_1^2(t)}\Big], \\ \dot{x}_3(t) = x_3(t)\Big[r_3(t) - a_3(t)x_3(t) \\ \qquad - \frac{d_3(t)x_1(t)x_3(t)}{c_3(t)+b_3(t)x_3(t)+x_3^2(t)} \\ \qquad + \frac{k_2(t)d_2(t)x_2^2(t)}{c_2(t)+b_2(t)x_2(t)+x_2^2(t)}\Big], \\ \dot{u}_1(t) = u_1(t)\Big[r_1(t) - a_1(t)u_1(t) \\ \qquad - \frac{d_1(t)u_1(t)u_2(t)}{c_1(t)+b_1(t)u_1(t)+u_1^2(t)} \\ \qquad + \frac{k_3(t)d_3(t)u_3^2(t)}{c_3(t)+b_3(t)u_3(t)+u_3^2(t)}\Big], \\ \dot{u}_2(t) = u_2(t)\Big[r_2(t) - a_2(t)u_2(t) \\ \qquad - \frac{d_2(t)u_2(t)u_3(t)}{c_2(t)+b_2(t)u_2(t)+u_2^2} \\ \qquad + \frac{k_1(t)d_1(t)u_1^2(t)}{c_1(t)+b_1(t)u_1(t)+u_1^2(t)}\Big], \\ \dot{u}_3(t) = u_3(t)\Big[r_3(t) - a_3(t)u_3(t) \\ \qquad - \frac{d_3(t)u_1(t)u_3(t)}{c_3(t)+b_3(t)u_3(t)+u_3^2(t)} \\ \qquad + \frac{k_2(t)d_2(t)u_2^2(t)}{c_2(t)+b_2(t)u_2(t)+u_2^2(t)}\Big]. \end{cases} \tag{18}$$

For $X(t) = (x_1(t), x_2(t), x_3(t))$ and $U(t) = (u_1(t), u_2(t), u_3(t))$ are the solutions of system (18) in $\Omega \times \Omega$. Let $x_i^*(t) = \ln x_i(t), u_i^*(t) = \ln u_i(t), i = 1, 2, 3$. Now we construct a Lyapunov functional as follows

$$V(t) = \sum_{i=1}^{3} |x_i^*(t) - u_i^*(t)|. \tag{19}$$

Taking $a(r) = b(r) = \sum_{i=1}^{3} |x_i^*(t) - u_i^*(t)|$ and using the inequality $||a| - |b|| \leq |a - b|$, we can easily prove that (i) and (ii) in Lemma 8 hold true. In the sequel, we will investigate (iii) of Lemma 8. It follows from (16) that

$$\begin{aligned} & D^+V(t) = \sum_{i=1}^{3}\left(\frac{\dot{x}_i(t)}{x_i(t)} - \frac{\dot{u}_i(t)}{u_i(t)}\right) \\ & \times \mathrm{sign}(x_i(t) - u_i(t)) \\ & \leq -a_1^l|x_1(t) - u_1(t)| \\ & + \Big[\frac{d_1(t)x_1(t)x_2(t)}{c_1(t) + b_1(t)x_1(t) + x_1^2(t)} \\ & - \frac{d_1(t)u_1(t)u_2(t)}{c_1(t) + b_1(t)u_1(t) + u_1^2(t)}\Big] \\ & + \Big[\frac{k_3(t)d_3(t)x_3^2(t)}{c_3(t) + b_3(t)x_3(t) + x_3^2(t)} \\ & - \frac{k_3(t)d_3(t)u_3^2(t)}{c_3(t) + b_3(t)u_3(t) + u_3^2(t)}\Big] \\ & - a_2^l|x_2(t) - u_2(t)| \\ & + \Big[\frac{d_2(t)x_2(t)x_3(t)}{c_2(t) + b_2(t)x_2(t) + x_2^2} \\ & - \frac{d_2(t)u_2(t)u_3(t)}{c_2(t) + b_2(t)u_2(t) + u_2^2}\Big] \\ & + \Big[\frac{k_1(t)d_1(t)x_1^2(t)}{c_1(t) + b_1(t)x_1(t) + x_1^2(t)} \\ & - \frac{k_1(t)d_1(t)u_1^2(t)}{c_1(t) + b_1(t)u_1(t) + u_1^2(t)}\Big] \\ & - a_3^l|x_3(t) - u_3(t)| \\ & + \Big[\frac{d_3(t)x_1(t)x_3(t)}{c_3(t) + b_3(t)x_3(t) + x_3^2(t)} \\ & - \frac{d_3(t)u_1(t)u_3(t)}{c_3(t) + b_3(t)u_3(t) + u_3^2(t)}\Big] \\ & + \Big[\frac{k_2(t)d_2(t)x_2^2(t)}{c_2(t) + b_2(t)x_2(t) + x_2^2(t)} \\ & - \frac{k_2(t)d_2(t)u_2^2(t)}{c_2(t) + b_2(t)u_2(t) + u_2^2(t)}\Big] \\ & \leq \Big[-a_1^l + \frac{c_1^M d_1^M M_2 + d_1^M M_1^2 M_2}{(c_1^l + b_1^l m_1 + m_1^2)^2}\Big] \end{aligned}$$

$$
\begin{aligned}
&\times|x_1(t)-u_1(t)|\\
&+\left[\frac{c_1^M d_1^M M_1+d_1^M b_1^M M_1^2+d_1^M M_1^3}{(c_1^l+b_1^l m_1+m_1^2)^2}\right]\\
&\times|x_2(t)-u_2(t)|\\
&+\left[\frac{2k_3^M c_3^M d_3^M M_3+2k_3^M b_3^M d_3^M M_3^2}{(c_3^l+b_3^l m_3+m_3^2)^2}\right]\\
&\times|x_3(t)-u_3(t)|\\
&+\left[-a_2^l+\frac{d_2^M c_2^M M_3+d_2^M M_2^2 M_3}{(c_2^l+b_2^l m_2+m_2^2)^2}\right]\\
&\times|x_2(t)-u_2(t)|\\
&+\left[\frac{d_2^M c_2^M M_2+d_2^M b_2^M M_2^2+d_2^M M_2^3}{(c_2^l+b_2^l m_2+m_2^2)^2}\right]\\
&\times|x_3(t)-u_3(t)|\\
&+\left[-a_3^l+\frac{d_3^M c_3^M M_1+2d_3^M M_3 M_1}{(c_3^l+b_3^l m_3+m_3^2)^2}\right]\\
&\times|x_3(t)-u_3(t)|\\
&+\left[\frac{d_3^M c_3^M M_3+2d_3^M b_3^M M_3+2d_3^M M_3^2}{(c_3^l+b_3^l m_3+m_3^2)^2}\right]\\
&\times|x_1(t)-u_1(t)|. \qquad (20)
\end{aligned}
$$

Nothing that

$$
\begin{aligned}
&|x_i(t)-u_i(t)|\\
&=|\exp(x_i^*(t))-\exp(u_i^*(t))|\\
&=\exp(\xi_i(t))||x_i^*(t)-u_i^*(t)|, \qquad (21)
\end{aligned}
$$

where $\xi_i(t)$ lies between $x_i(t)$ and $u_i(t)$. Then we have

$$
\begin{aligned}
&m_i|x_i^*(t)-u_i^*(t)|\\
&\le|x_i(t)-u_i(t)|\\
&\le M_i|x_i^*(t)-u_i^*(t)|, i=1,2,3. \qquad (22)
\end{aligned}
$$

It follows from (20)and (22) that

$$
\begin{aligned}
&D^+V(t)\le\left[-a_1^l+\frac{c_1^M d_1^M M_2+d_1^M M_1^2 M_2}{(c_1^l+b_1^l m_1+m_1^2)^2}\right]\\
&\times m_1|x_1^*(t)-u_1^*(t)|\\
&+\left[\frac{c_1^M d_1^M M_1+d_1^M b_1^M M_1^2+d_1^M M_1^3}{(c_1^l+b_1^l m_1+m_1^2)^2}\right]\\
&\times M_2|x_2^*(t)-u_2^*(t)|\\
&+\left[\frac{2k_3^M c_3^M d_3^M M_3+2k_3^M b_3^M d_3^M M_3^2}{(c_3^l+b_3^l m_3+m_3^2)^2}\right]\\
&\times M_3|x_3^*(t)-u_3^*(t)|\\
&+\left[-a_2^l+\frac{d_2^M c_2^M M_3+d_2^M M_2^2 M_3}{(c_2^l+b_2^l m_2+m_2^2)^2}\right]\\
&\times m_2|x_2^*(t)-u_2^*(t)|\\
&+\left[\frac{d_2^M c_2^M M_2+d_2^M b_2^M M_2^2+d_2^M M_2^3}{(c_2^l+b_2^l m_2+m_2^2)^2}\right]\\
&\times M_3|x_3^*(t)-u_3^*(t)|\\
&+\left[-a_3^l+\frac{d_3^M c_3^M M_1+2d_3^M M_3 M_1}{(c_3^l+b_3^l m_3+m_3^2)^2}\right]\\
&\times m_3|x_3^*(t)-u_3^*(t)|\\
&+\left[\frac{d_3^M c_3^M M_3+2d_3^M b_3^M M_3+2d_3^M M_3^2}{(c_3^l+b_3^l m_3+m_3^2)^2}\right]\\
&\times M_1|x_1^*(t)-u_1^*(t)|\\
&=-\theta_1|x_1^*(t)-u_1^*(t)|\\
&-\theta_2|x_2^*(t)-u_2^*(t)|\\
&-\theta_3|x_3^*(t)-u_3^*(t)|, \qquad (23)
\end{aligned}
$$

where

$$
\begin{aligned}
\theta_1=&\left[a_1^l-\frac{c_1^M d_1^M M_2+d_1^M M_1^2 M_2}{(c_1^l+b_1^l m_1+m_1^2)^2}\right]m_1\\
&-\left[\frac{d_3^M c_3^M M_3+2d_3^M b_3^M M_3+2d_3^M M_3^2}{(c_3^l+b_3^l m_3+m_3^2)^2}\right]\\
&\times M_1, \qquad (24)\\
\theta_2=&\left[a_2^l-\frac{d_2^M c_2^M M_3+d_2^M M_2^2 M_3}{(c_2^l+b_2^l m_2+m_2^2)^2}\right]m_2\\
&-\left[\frac{c_1^M d_1^M M_1+d_1^M b_1^M M_1^2+d_1^M M_1^3}{(c_1^l+b_1^l m_1+m_1^2)^2}\right]\\
&\times M_2, \qquad (25)\\
\theta_3=&\left[a_3^l-\frac{d_3^M c_3^M M_1+2d_3^M M_3 M_1}{(c_3^l+b_3^l m_3+m_3^2)^2}\right]m_3\\
&-\left[\frac{2k_3^M c_3^M d_3^M M_3+2k_3^M b_3^M d_3^M M_3^2}{(c_3^l+b_3^l m_3+m_3^2)^2}\right]\\
&\times M_3\\
&-\left[\frac{d_2^M c_2^M M_2+d_2^M b_2^M M_2^2+d_2^M M_2^3}{(c_2^l+b_2^l m_2+m_2^2)^2}\right]\\
&\times M_3. \qquad (26)
\end{aligned}
$$

Let

$$\delta=\min\{\theta_1,\theta_2,\theta_3\}. \qquad (27)$$

It follows from (23) and (27) that

$$D^+V(t)\le-\delta V(t). \qquad (28)$$

Then (iii) of Lemma 3.1 is fulfilled. Therefore system (1) has a unique positive asymptotically periodic solution in domain Ω, which is uniformly asymptotically stable. The proof is complete.

4 Numerical example

To illustrate the theoretical results, we present some numerical simulations. Let us consider the following cyclic predator-prey system with Sigmoidal type functional response:

$$\begin{cases} \dot{x}_1(t) = x_1(t)\Big[r_1(t) - a_1(t)x_1(t) \\ \qquad - \dfrac{d_1(t)x_1(t)x_2(t)}{c_1(t)+b_1(t)x_1(t)+x_1^2(t)} \\ \qquad + \dfrac{k_3(t)d_3(t)x_3^2(t)}{c_3(t)+b_3(t)x_3(t)+x_3^2(t)}\Big], \\ \dot{x}_2(t) = x_2(t)\Big[r_2(t) - a_2(t)x_2(t) \\ \qquad - \dfrac{d_2(t)x_2(t)x_3(t)}{c_2(t)+b_2(t)x_2(t)+x_2^2} \\ \qquad + \dfrac{k_1(t)d_1(t)x_1^2(t)}{c_1(t)+b_1(t)x_1(t)+x_1^2(t)}\Big], \\ \dot{x}_3(t) = x_3(t)\Big[r_3(t) - a_3(t)x_3(t) \\ \qquad - \dfrac{d_3(t)x_1(t)x_3(t)}{c_3(t)+b_3(t)x_3(t)+x_3^2(t)} \\ \qquad + \dfrac{k_2(t)d_2(t)x_2^2(t)}{c_2(t)+b_2(t)x_2(t)+x_2^2(t)}\Big], \end{cases} \tag{29}$$

where

$$\begin{cases} b_1(t) = 20 + 0.2\sin t, b_2(t) = 10 + 0.4\cos t, \\ b_3(t) = 15 + 0.3\sin t, a_1(t) = 10 + \sin t, \\ a_2(t) = 11 + \cos t, a_3(t) = 12 - \cos t, \\ r_1(t) = 10 + 0.3\sin t, r_2(t) = 12 + 0.2\cos t, \\ r_3(t) = 13 - 0.2\sin t, d_1(t) = 0.2 + 0.2\cos t, \\ d_2(t) = 0.2 + 0.1\sin t, d_3(t) = 0.1 + 0.1\cos t, \\ k_1(t) = 1 + \sin t, k_2(t) = 1 + \sin t, \\ k_3(t) = 1 + \sin t. \end{cases}$$

Then

$$\begin{cases} b_1^l = 19.8, b_1^l = 9.6, b_3^l = 14.7, \\ r_1^l = 9.7, r_2^l = 11.8, r_3^l = 12.8, \\ a_1^l = 9, a_2^l = 10, a_3^l = 11, \\ r_1^u = 10.3, r_2^u = 12.2, r_3^u = 13.2, \\ d_1^u = 0.4, d_2^u = 0.3, d_3^u = 0.2, \\ M_1 = 1.1889, M_2 = 1.3, M_3 = 1.2181. \end{cases}$$

It is easy to check that the coefficients of system (29) satisfy all the conditions in Theorem 9. The phase diagram of system (29) is illustrated in Figures 1-3. Numerical simulations show that system (29) has a unique positive periodic solution which is globally asymptotically stable.

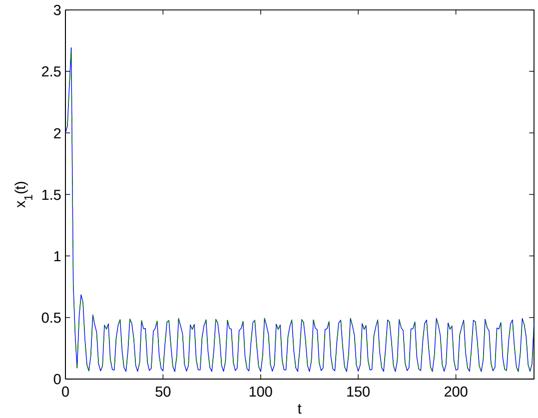

Fig.1. The dynamical behavior of the solution $(x_1(t), x_2(t), x_3(t))$ of system (29).

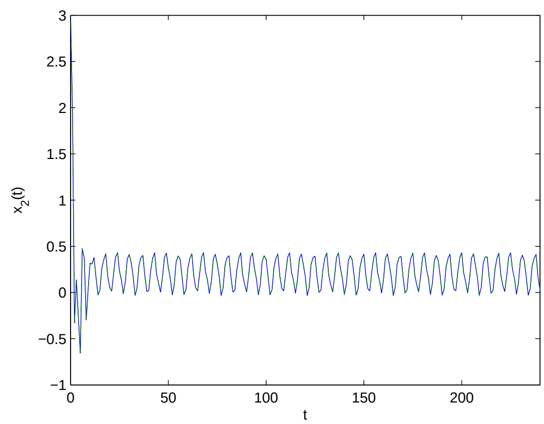

Fig.2. The dynamical behavior of the solution $(x_1(t), x_2(t), x_3(t))$ of system (29).

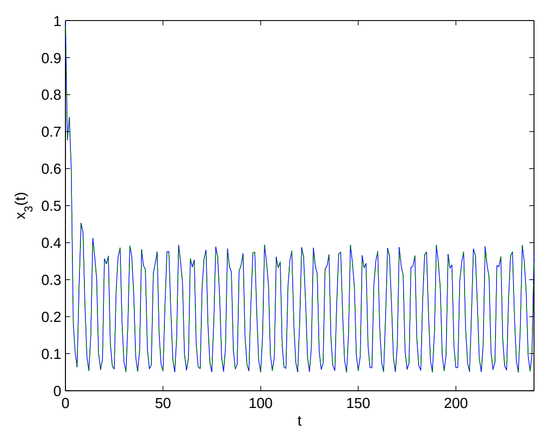

Fig.3. The dynamical behavior of the solution $(x_1(t), x_2(t), x_3(t))$ of system (29).

5 Conclusions

In this paper, we have analyzed a cyclic predator-prey system with Sigmoidal type functional response. Applying the differential inequality theory, we obtain some sufficient conditions for the permanence of the system. By constructing a suitable Liapunov function, we find that under some suitable conditions, the system has a unique asymptotically periodic solution which is globally asymptotically stable. Numerical simulations show the feasibility of our main results.

Acknowledgements: The research was supported by This work is supported by National Natural Science Foundation of China(No.11261010, No.11101126), Soft Science and Technology Program of Guizhou Province(No.2011LKC2030), Natural Science and Technology Foundation of Guizhou Province(J[2012]2100), Governor Foundation of Guizhou Province([2012]53), Natural Science and Technology Foundation of Guizhou Province(2014), Natural Science Innovation Team Project of Guizhou Province ([2013]14) and Doctoral Foundation of Guizhou University of Finance and Economics (2010).

References:

[1] Y.K. Li, and Y. Ye, Multiple positive almost periodic solutions to an impulsive non-autonomous Lotka-Volterra predator-prey system with harvesting terms, *Commun. Nonlinear Sci. Numer. Simul.* 18, 2013, pp. 3190–3201.

[2] Z.Q. Zhang, and J.B. Luo, Multiple periodic solutions of a delayed predator-prey system with stage structure for the predator, *Nonlinear Anal.: Real World Appl.* 11, 2010, pp. 4109–4120.

[3] B.X. Dai, Y. Li, and Z.G. Luo, Multiple periodic solutions for impulsive Gause-type ratio-dependent predator-prey systems with non-monotonic numerical responses, *Appl. Math. Compu.* 217, 2011, pp. 7478–7487.

[4] L.L. Wang, and Y.H. Fan, Multiple periodic solutions for a non-autonomous delayed predator-prey model with harvesting terms, *Appl. Math. Comput.* 217, 2011, pp. 9552–9561.

[5] Z.Q. Zhang, Z.T. Hou, and L. Wang, Multiplicity of positive periodic solutions to a generalized delayed predator-prey system with stocking, *Nonlinear Anal.* 68, 2008, pp. 2608–2622.

[6] M. Fan, K. Wang, and D.Q. Jiang, Existence and global attractivity of positive periodic solutions of periodic n-species Lotka-Volterra competition systems with several deviating arguments, *Math. Biosci.* 160, 1999, pp. 47–61.

[7] Z.J. Liu, R.H. Tan, Y.P. Chen, and L.S. Chen, On the stable periodic solutions of a delayed two-species model of faculative mutualism, *Appl. Math. Comput.* 196, 2008, pp. 105–117.

[8] D.W. Hu, and Z.Q. Zhang, Four positive periodic solution to a Lotka-Volterra cooperative system with harvesting terms, Nonlinear Anal.: Real World Appl., 11, 2010, pp. 1115-1121.

[9] Y.K. Li, and G.T. Xu, Positive periodic solutions for an integrodifferential model of mutualism, *Appl. Math. Lett.* 14, 2001, pp. 525–530.

[10] Y.H. Xia, J.D. Cao, and S.S. Cheng, Multiple periodic solutions of a delayed stage-structured predator-prey model with non-monotone functional responses, *Appl. Math. Modelling* 31, 2007, pp. 1947–1959.

[11] Z.Q. Zhang, and H.L. Wang, Existence and global attractivity of positive periodic solutions for a generalized predator-prey system with time delay, *Math. Comput. Modelling* 44, 2006, pp. 188–203.

[12] Y.H. Fan, and L.L. Wang, Multiplicity of periodic solutions for a delayed ratio-dependent predator-prey model with Holling type III functional response and harvesting terms, *J. Math. Anal. Appl.* 365, 2010, pp. 525–540.

[13] F.Q. Zhang, and C.W. Zheng, Positive periodic solutions for the neutral ratio-dependent predator-prey model, *Comput. Mat. Appl.* 61, 2011, pp. 2221-2226.

[14] K. Wang, Periodic solutions to a delayed predator-prey model with Hassell-Varley type functional response, *Nonlinear Anal.: Real World Appl.* 12, 2011, pp. 137–145.

[15] Z.W. Cai, L.H. Huang, and H.B. Chen, Positive periodic solution for a multispecies competition-predator system with Holling III functional response and time delays, *Appl. Math. Comput.* 217, 2011, pp. 4866-4878.

[16] B. Sahoo, S. Poria, Effects of supplying alternative food in a predator-prey model with harvesting, *Appl. Math. Comput.* 234, 2014, pp. 150–166.

[17] H.W. Yin, X.Y. Xiao, X.Q. Wen, and K. Liu, Pattern analysis of a modified Leslie-Gower predator-prey model with Crowley-Martin functional response and diffusion, *Comput. Math. Appl.* 67, 2014, pp. 1607–1621.

[18] Y. Zhang, Q.L. Zhang, and X.G. Yan, Complex dynamics in a singular LeslieCGower predator-prey bioeconomic model with time delay and stochastic fluctuations, *Phys. A* 404, 2014, pp. 180–191.

[19] Q. Liu, Y.L. Liu, and X. Pan, Global stability of a stochastic predator-prey system with infinite delays, *Appl. Math. Comput.* 235, 2014, pp. 1–7.

[20] Y.H. Huang, and P.X. Weng, Periodic traveling wave train and point-to-periodic traveling wave for a diffusive predator-prey system with Ivlev-type functional response, *J. Math. Anal. Appl.* 417, 2014, pp. 376–393.

[21] Z. Ling, L. Zhang, Z.G. Lin, Turing pattern formation in a predator-prey system with cross diffusion, *Appl. Math. Modelling*, in Press, 2014.

[22] J.D. Flores, and E. González-Olivares, Dynamics of a predator-prey model with Allee effect on prey and ratio-dependent functional response, *Ecological Complexity*, in press, 2014.

[23] P.J. Pal, and P.K. Mandal, Bifurcation analysis of a modified LeslieCGower predator-prey model with Beddington-DeAngelis functional response and strong Allee effect, *Math. Comput. Simul.* 97, 2014, pp. 123–146.

[24] G.D. Zhang, Y. Shen, and B.S. Chen, Bifurcation analysis in a discrete differential-algebraic predator-prey system, *Appl. Math. Modelling*, in press, 2014.

[25] Y.F. Lv, and R. Yuan, Existence of traveling wave solutions for Gause-type models of predator-prey systems Original Research Article *Appl. Math. Comput.* 229, 2014, pp. 70–84.

[26] Z.J. Du, and Z.S. Feng, Periodic solutions of a neutral impulsive predator-prey model with Beddington-DeAngelis functional response with delays, *J. Comput. Appl. Math.* 258, 2014, pp. 87–98.

[27] O. Makarenkov, Topological degree in the generalized Gause preyCpredator model *J. Math. Anal. Appl.* 410, 2014, pp. 525–540.

[28] X.X. Liu, Impulsive periodic oscillation for a predatorCprey model with Hassell-Varley-Holling functional response Original Research Article *Appl. Math. Modelling* 38, 2014, pp. 1482–1494.

[29] S. Bhattacharya, M. Martcheva, and X.Z. Li, A predator-prey-disease model with immune response in infected prey, *J. Math. Anal. Appl.* 411, 2014, pp. 297-313.

[30] S. Shabani, Diffusive Holling type-II predator-prey system with harvesting of prey, *J. Math. Anal. Appl.* 410, 2014, pp. 469–482.

[31] L.N. Guin, Existence of spatial patterns in a predatorCprey model with self- and cross-diffusion, *Appl. Math. Comput.* 226, 2014, pp. 320–335.

[32] B.S.R.V. Prasad, M. Banerjee, P.D.N. Srinivasu, Dynamics of additional food provided predator-prey system with mutually interfering predators, *Math. Biosci.* 246, 2013, pp. 176–190.

[33] M. Javidi, and N. Nyamoradi, Dynamic analysis of a fractional order preyCpredator interaction with harvesting, *Appl. Math. Modelling* 37, 2013, pp. 8946–8956.

[34] M.M. Molina, M.A. Moreno-Armendáriz, and J.C.S. Tuoh Mora, On the spatial dynamics and oscillatory behavior of a predator-prey model based on cellular automata and local particle swarm optimization, *J. Theor. Biol.* 336, 2013, pp 173–184.

[35] M. Liu, and C.Z. Bai, On a stochastic delayed predator-prey model with Levy jumps, *Appl. Math. Compu.* 228, 2014, pp. 563–570.

[36] M. Fras, and M. Gosak, Spatiotemporal patterns provoked by environmental variability in a predator-prey model, *Biosystems* 114, 2013, pp. 172–177.

[37] A.J. Terry, A predator-prey model with generic birth and death rates for the predator, *Math. Biosci.* 248, 2014, pp. 57–66.

[38] Y.K. Li, and Y. Ye, Multiple positive almost periodic solutions to an impulsive non-autonomous Lotka-Volterra predator-prey system with harvesting terms, *Commun. Nonlinear Sci. Numer. Simul.* 18, 2013, pp. 3190–3201.

[39] H.K. Kim, and H. Baek, The dynamical complexity of a predator-prey system with Hassell-Varley functional response and impulsive effect,*Math. Comput. Simul.* 94, 2013, pp. 1–14.

[40] J. Zhou, and J.P. Shi, The existence, bifurcation and stability of positive stationary solutions of a diffusive Leslie-Gower predator-prey model with Holling-type II functional responses, *J. Math. Anal. Appl.* 405, 2013, pp. 618–630.

[41] C. Miao, and Y. Ke, Positive periodic solutions of a generalized Gilpin-Ayala competitive system with time delays, *WSEAS Trans. Math.*, 12 (3), 2013, pp.277–285.

[42] L. Zhang, Y. Li, Q. Ren, and Z. Huo, Global dynamics of an SEIRS epidemic model with constant immigration and immunity, *WSEAS Trans. Math.*, 12 (5), 2013, pp.630–640.

[43] K. Zhuang, and H. Zhu, Stability and bifurcation analysis for an improved HIV model with time delay and cure rate, *WSEAS Trans. Math.*, 12 (8), 2013, pp.860–869.

[44] K. Zhao, and L. Ding, Multiple periodic solutions for a general class of delayed cooperative systems on time scales, *WSEAS Trans. Math.*, 12 (10), 2013, pp.957–966.

[45] Y. Pei, Y. Liu, and C. Li, Dynamic study of mathe-matical models on antibiotics and immunologic adjuvant against Toxoplasmosis, *WSEAS Trans. Math.*, 11 (11), 2012, pp.1018–1027.

[46] C. Miao, and Y. Ke, Positive periodic solutions of a generalized Gilpin-Ayala competitive system with time delays, *WSEAS Trans. Math.*, 12 (3), 2013, pp.277–285.

[47] A. V. Doroshin, and F. Neri, Open research issues on Nonlinear Dynamics, Dynamical Systems and Processes, *WSEAS Trans. Syst.*, 13, 2014, in press.

[48] C. Ciufudean, and F. Neri, Open research issues on Multi-Models for Complex Technological Systems, *WSEAS Trans. Syst.*, 13, 2014, in press.

[49] F. Neri, Open research issues on Computational Techniques for Financial Applications, *WSEAS Trans. Syst.*, 13, 2014, in press.

[50] P. Karthikeyan, and F. Neri, Open research issues on Deregulated Electricity Market: Investigation and Solution Methodologies, *WSEAS Trans. Syst.*, 13, 2014, in press.

[51] M. Panoiu, and F. Neri, Open research issues on Modeling, Simulation and Optimization in Electrical Systems, *WSEAS Trans. Syst*, 13, 2014, in press.

[52] F. Neri, Open research issues on Advanced Control Methods: Theory and Application, *WSEAS Trans. Syst.*, 13, 2014, in press.

[53] P. Hajek, and F. Neri, An introduction to the special issue on computational techniques for trading systems, time series forecasting, stock market modeling, financial assets modeling, *WSEAS Transa. Bus. Econ.*, 10, 2013, pp. 201-292.

[54] M. Azzouzi, and F. Neri, An introduction to the special issue on advanced control of energy systems, *WSEAS Trans. Power Syst.*, 8, 2013, pp. 103.

[55] Z. Bojkovic, and F. Neri, An introduction to the special issue on advances on interactive multimedia systems, *WSEAS Trans. Syst.*, 12, 2013, pp. 337-338.

[56] L. Pekar, and F. Neri, An introduction to the special issue on advanced control methods: Theory and application, *WSEAS Trans. Syst.*, 12, 2013, pp. 301–303.

[57] C. Guarnaccia, and F. Neri, An introduction to the special issue on recent methods on physical polluting agents and environment modeling and simulation, *WSEAS Trans. Syst.*, 12, 2013, pp. 53–54.

[58] F. Neri, An introduction to the special issue on computational techniques for trading systems, time series forecasting, stock market modeling, and financial assets modeling *WSEAS Trans. Syst.*, 11, 2012, pp. 659–660.

[59] M. Muntean, and F. Neri, Foreword to the special issue on collaborative systems, *WSEAS Trans. Syst.*, 11, 2012, pp. 617.

[60] L. Pekar, and F. Neri, An introduction to the special issue on time delay systems: Modelling, identification, stability, control and applications, *WSEAS Trans. Syst.*, 11, 2012, pp. 539–540.

[61] C. Volos, and F. Neri, An introduction to the special issue: Recent advances in defense systems: Applications, methodology, technology, *WSEAS Trans. Syst.*, 11, 2012, pp. 477–478.

[62] A.V. Doroshin, Exact solutions in attitude dynamics of a magnetic dual-spin spacecraft and a generalization of the lagrange top, *WSEAS Trans. Syst.*, 12, 2013, pp. 471–482.

[63] A.V. Doroshin, Synthesis of attitude motion of variable mass coaxial bodies, *WSEAS Trans. Syst. Control* 3, 2008, pp. 50–61.

[64] L. Zhang, Y. Li, Q. Ren, and Z. Huo, Global dynamics of an SEIRS epidemic model with constant immigration and immunity, *WSEAS Trans. Math.*, 12, 2013, pp.630–640.

[65] K. Zhuang, and H. Zhu, Stability and bifurcation analysis for an improved HIV model with time delay and cure rate, *WSEAS Trans. Math.*, 12, 2013, pp.860–869.

[66] K. Zhao, and L. Ding, Multiple periodic solutions for a general class of delayed cooperative systems on time scales, *WSEAS Trans. Math.*, 12, 2013, pp.957–966.

[67] Y. Pei, Y. Liu, and C. Li, Dynamic study of mathe-matical models on antibiotics and immunologic adjuvant against Toxoplasmosis, *WSEAS Trans. Math.*, 11, 2012, pp.1018–1027.

[68] F.Y. Wei, K. Wang, Uniform persistence of asymptotically periodic multispecies competition predator-prey systems with Holling III type of functional response, *Appl. Math. Comput.* 170, 2005, pp. 994–998.

[69] Y. Yang, and W.C. Chen, Uniformly strong persistence of an nonlinear asymptotically periodic multispecies competition predator-prey system with general functional response, *Appl. Math. Comput.* 183, 2006, pp. 423–426.

[70] F. Montes de Oca, and M. Vivas, Extinction in a two dimensional Lotka-Volterra sysstem with infinite delay, *Nonlinear Anal.: Real World Appl.* 7, 2006, pp. 1042–1047.

[71] R. Yuan, Existence of almost periodic solutions of functional differential equations of neutral type, *J. Math. Anal. Appl.* 165, 1992, pp. 524–538.