Assembly Variation Analysis of Three Dimensional Compliant Sheet Metal Parts Under an Over-located Condition

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Abstract: - An over-located condition is very common for most automotive compliant sheet metal assembly process. To accurately analyze the assembly variation can help make an optimization design of assembly process. The existed assembly variation modeling methods based rigid and compliant parts can not fully exactly and rapidly analyze the assembly variation. This paper proposes a new assembly variation modeling method of compliant sheet metal parts to solve these limitations. This proposed new assembly variation modeling method of compliant sheet metal parts combines the rigid and compliant characteristics of sheet metal parts. An assembly variation modeling method of a rigid part under a 3-2-1 locating layout is given using its spatial motion relationship when some fixture locating variation happens. Based on this method, a multi-divided rigid area assembly variation modeling method can be obtained according to part’s bending deforming principle during the locating process. A simplified stiffness solving method for the welding spring-back is given, and an assembly variation modeling method for the welding area of parts is presented using the stiffness comparison of welding parts. Then the assembly variation of measurement points in welding area and out of welding area can be obtained according to the proposed method. A case of two compliant sheet metal parts is used to demonstrate and validate the proposed method. The results show that the proposed method can achieve a rapid and accurate assembly variation analysis through the comparison with the existed rigid and compliant sheet metal assembly variation modeling methods. The proposed assembly method can be used to predict the assembly quality of compliant sheet metal parts, and then an optimization design of the assembly process can be obtained. This provides good help for engineers to design and analyze the assembly problems of automotive body.

Key-Words: - Automotive Body, Compliant Sheet Metal, Assembly Variation, Assembly modeling, over location

1 Introduction

Compliant sheet metal products have been widely used in automotive bodies whose assembly quality greatly influences the outer appearance, passenger comfort and safety. Because compliant sheet metal parts in a manufacturing process are prone to some flexible deformation, there are great difficulties in the assembly quality control.

The variation propagating principle analysis of a compliant sheet metal assembly process has great significance in the assembly quality control and process optimization. A lot of achievements have been done in assembly variation modeling methods of sheet metal parts and some software also have been applied to help automotive body designs. These achievements can be divided into two aspects.

On the one hand, in some compliant sheet metal assembly variation modeling methods, the sheet metal part is assumed to be a rigid body. So the assembly variation is only induced by the rigid motion relationship without any flexible deformation consideration. Chase et al analyzed the 2D and 3D assembly, and obtained some dimensional chain models using the geometries and assembly relationship of parts and could be computed through direct linear method and the Monte Carlo Simulation method [1]-[2]. Apley and Shi set a relationship model between the part variation and fixture locating variation using the geometries and kinematic relationship [3]. Cai et al analyzed the relationship among the manufacturing variation of parts, fixture locating variation and spatial position variation of parts, and obtained the assembly variation model of parts under the determinate location through setting the constraint equations of parts [4]-[6].

On the other hand, the sheet metal part is considered as a compliant body. In these researches, the finite element method is usually used to analyze the flexible deformation. Liu and Hu proposed an offset beam element model for predicting the assembly variation of deformable sheet metal parts joined by a resistance spot welding. The model could be applied to predict the sheet metal assembly variation for one-dimensional models [7]. Liu and Hu proposed the use of Finite Element Methods (FEM) in developing mechanistic variation simulation models for deformable sheet metal parts with complex two or three dimensional free form surfaces. Based on the Method of Influence Coefficients (MIC), they constructed a sensitivity matrix to establish a linear relationship between the incoming part variation and the output assembly variation [8]. Hu proposed a stream-of-variation theory for automotive body assembly. Two assembly configurations, i.e., serial and parallel, were discussed [9]. Long compared a compliant assembly with a traditional rigid-body assembly in assembly mechanisms, locating principles, and variation propagation. A unified variation simulation method was proposed in which the assembly variation consisted of fixture-induced rigid-body motion and fixture-induced deflection [10]. Shiu et al proposed a flexible beam-based modeling method for dimensional control of the sheet metal assembly process. The method included principles of decoupling automobile parts into beam members, beam connectivity selection, beam-to-beam joint geometry modeling, and process locating point identification [11]. Chang and Gossard developed analytical models of compliant parts showing the effects of misalignment within the place, clamp, and fasten stage. These models were limited to a butt-joint configuration spot weld on flanged parts, and only included the expected nominal misalignment using mathematic methods [12]. Camelio et al deduced a variation propagation model for multi-station assembly systems with compliant parts, in which the relationship between assembly variation and part geometric variation, fixture variation and welding gun variation was modeled [13]. Camelio et al analyzed the impact of fixture design on sheet metal assembly variation was analyzed and an optimization algorithm combined finite element analysis and nonlinear programming methods to determine the optimal fixture position such that assembly variation was minimized [14]. Dahlström and Söderberg analyzed the influence of welding sequence and welding type to the final geometry of the sheet metal assembly [15]. Dahlström and Lindkvist proposed a contact modeling technique that could be implemented in the MIC. The contact modeling procedure consisted of a contact detecting algorithm and a solving algorithm to find the position of equilibrium [16]. Liao and Wang proposed a new method based on wavelets analysis and FEM for the variation analysis of non-rigid assemblies [17]. Yu et al set up assembly variation model of compliant sheet metal considering the part material thickness variation [18]-[20]. Besides these achievements, there existed some other achievements which can be used for the automotive body assembly variation management and control, for example [21] and [22].

The above achievements in the assembly variation modeling methods of sheet metal parts can effectively dispose the variation propagation rules of compliant sheet metal parts during assembly processes. Based on these, some software such as VSA and 3DCS also have been applied in automotive body designs. However, there are still some limitations in solving the real engineering problems. An over-located condition is very common for most automotive compliant sheet metal assembly process. The modeling methods based on rigid parts have simple, quick modeling and analysis process, but they can only solve the assembly variation computation for a 3-2-1 locating layout, and not for a N-2-1 locating layout. Though the modeling method based on compliant parts can more precisely predict the assembly quality, the modeling efficiency is too low because of complication, and it also gives much higher demands for users.

To solve these limitations, this paper proposes an assembly variation modeling method of compliant sheet metal parts to solve a N-2-1 locating assembly problems. In this method, an assembly variation modeling method of a rigid part under a 3-2-1 locating layout is given using its spatial motion relationship when some fixture locating variation happens. Based on this method, a multi-divided rigid area assembly variation modeling method can be obtained according to part's bending deforming principle during the locating process. A simplified stiffness solving method for the welding spring-back is given, and an assembly variation modeling method for the welding area of parts is presented using the stiffness comparison of welding parts. Then the assembly variation of measurement points in welding area and out of welding area can be obtained according to the proposed method.

The remainder of this paper is organized as follows. Firstly, an assembly variation modeling method based on rigid parts under a 3-2-1 locating layout is discussed. Secondly, an assembly variation modeling method of multi welding parts under a N-2-1 locating layout is given. Thirdly, a case of two compliant sheet metal parts is presented to demonstrate and validate the proposed method. Finally, the conclusions are drawn.

2 Assembly Variation Modeling Method of Rigid Parts Under a 3-2-1 Locating Layout

The following Fig. 1 shows a 3-2-1 locating layout of a compliant sheet metal part. In this locating layout, locating blocks A_1, A_2, and A_3 constrain the movements in the z axle, and the rotation motions by y and x axles separately. The locating pin B constrains the movements in x and y axles, and the locating pin C constrains the movement in y axle direction and the rotation motion by z axle.

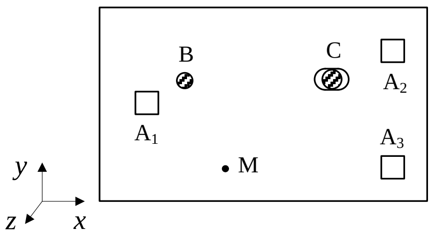

Fig. 1 A 3-2-1 locating layout of a sheet metal part

For automotive body parts, their thickness is usually between 0.7mm and 3mm. So the flexible deformation easily happens in the vertical direction of the sheet metal main surface. In this paper, only the vertical variation of compliant sheet metal parts is considered.

Assume that the z-direction variation of measurement point M is V_{Mz}, and the coordinates of locating blocks A_1, A_2, A_3 and measurement point M are $A_1(x_1, y_1, z_1)$, $A_2(x_2, y_2, z_2)$, $A_3(x_3, y_3, z_3)$ and $M(x_M, y_M, z_M)$, respectively.

If there is some fixture locating variation of $V_{A_{1z}}$ in the z-direction for the locating block A_1, and no locating variation for other locating units, the whole compliant sheet metal part will rotate by the A_2-A_3 axle as described in Fig. 2.

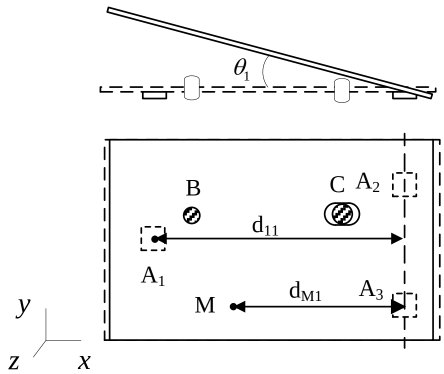

Fig. 2 Part motion caused by fixture locating variation

The rotating angle can be computed as follows:

$$\tan\theta_1 = \frac{V_{A_{1z}}}{d_{11}} = \frac{\delta V_{M1}}{d_{M1}} \tag{1}$$

where δV_{M1} is the z-direction locating variation of the locating block A_1, d_{11} and d_{M1} are the distances from the center of the locating block A_1 and measurement point M to the connecting line between the locating blocks A_2 and A_3, respectively. So the z-direction variation of measurement point M produced by the locating block A_1 is

$$\delta V_{M1} = \frac{d_{M1}}{d_{11}} V_{A_{1z}} \tag{2}$$

In the same way, the z-direction variation of measurement point M produced by the locating blocks A_2 and A_3 are

$$\delta V_{M2} = -\frac{d_{M2}}{d_{22}} V_{A_{2z}} \tag{3}$$

$$\delta V_{M3} = \frac{d_{M3}}{d_{33}} V_{A_{3z}} \tag{4}$$

where $V_{A_{2z}}$ and $V_{A_{3z}}$ are the z-direction locating variation of the locating blocks A_2 and A_3, respectively, d_{22} and d_{33} are the distances from the center of locating block A_2 to the connecting line between the locating blocks A_1 and A_3 and the center of locating block A_3 to the connecting line between the locating blocks A_1 and A_2, respectively, d_{M2} and d_{M3} are the distances from the measurement point M to the connecting line between the locating blocks A_1 and A_3, and A_1 and A_2, respectively.

Notice that when the measurement point and some locating blocks with fixture variation lie in different sides of the bending axle, the variation of measurement point produced by these locating blocks is negative as shown in Equation (3).

According to the former coordinate values, the distance from A_1 to the axle of A_2 and A_3 is

$$d_{11} = \frac{\left|(x_1 - x_2)(y_3 - y_2) - (y_1 - y_2)(x_3 - x_2)\right|}{\sqrt{(y_3 - y_2)^2 + (x_3 - x_2)^2}} \quad (5)$$

The other distance parameters can also be computed as follows.

$$d_{M1} = \frac{\left|(x_M - x_2)(y_3 - y_2) - (y_M - y_2)(x_3 - x_2)\right|}{\sqrt{(y_3 - y_2)^2 + (x_3 - x_2)^2}} \quad (6)$$

$$d_{22} = \frac{\left|(x_2 - x_1)(y_3 - y_1) - (y_2 - y_1)(x_3 - x_1)\right|}{\sqrt{(y_3 - y_1)^2 + (x_3 - x_1)^2}} \quad (7)$$

$$d_{M2} = \frac{\left|(x_M - x_1)(y_3 - y_1) - (y_M - y_1)(x_3 - x_1)\right|}{\sqrt{(y_3 - y_1)^2 + (x_3 - x_1)^2}} \quad (8)$$

$$d_{33} = \frac{\left|(x_3 - x_1)(y_2 - y_1) - (y_3 - y_1)(x_2 - x_1)\right|}{\sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}} \quad (9)$$

$$d_{M3} = \frac{\left|(x_M - x_1)(y_2 - y_1) - (y_M - y_1)(x_2 - x_1)\right|}{\sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}} \quad (10)$$

So the final z-direction assembly variation of measurement point M produced by the fixture locating variations of locating blocks A_1, A_2 and A_3 are

$$V_{ML} = D_{MA_1A_2A_3} \cdot V_{A_1A_2A_3} + V_{Mz} \quad (11)$$

where $D_{MA_1A_2A_3} = \begin{bmatrix} \frac{d_{M1}}{d_{11}} & -\frac{d_{M2}}{d_{22}} & \frac{d_{M3}}{d_{33}} \end{bmatrix}$ is the position parameter, and $V_{A_1A_2A_3} = \begin{bmatrix} V_{A_{1z}} \\ V_{A_{2z}} \\ V_{A_{3z}} \end{bmatrix}$ is the fixture locating variation of locating blocks A_1, A_2 and A_3.

3 Assembly Variation Modeling Method of Multi Welding Parts Under a N-2-1 Locating Layout

A N-2-1 locating layout is widely used in sheet metal manufacturing industries, such as automobiles, airplanes, high speed trains, and so on. All these industries have high demands in the control of outer body dimensions. This section can provide a more effective assembly variation modeling method, which can help these industries better predict and increase the assembly quality of products.

3.1 Multi-divided Rigid Areas Assembly Variation Modeling Method

The N-2-1 locating layout is usually used for automotive body compliant sheet metal parts to decrease the bending deformation in locating and welding processes. It adopts more than three locating blocks in the biggest surface compared to the 3-2-1 locating layout.

Under the N-2-1 locating layout, the locating process of parts is usually from a 3-2-1 location gradually to a N-2-1 location, i.e., the compliant sheet metal part is firstly located in a 3-2-1 locating layout, and then located by the additional locating blocks one by one. If the additional locating blocks exist some locating variation, some bending deformation of compliant sheet metal parts inevitably happens during the locating process as shown in Fig. 3.

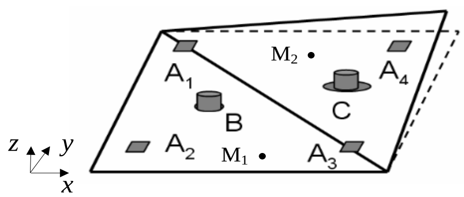

Fig. 3 The bending deformation of a sheet metal part under an over-located condition

In this figure, the part is firstly located in blocks A_1, A_2, and A_3 in the z-direction, and then the block A_4 is located finally. If there is some locating variation in block A_4, the part will bend by the axle of A_1 and A_3. Then the whole part is divided into two rigid areas, i.e., $A_1A_2A_3$ and $A_1A_3A_4$. The two divided parts are located by blocks A_1, A_2 and A_3, and blocks A_1, A_3 and A_4. So the assembly variation of measurement points can be computed by the locating blocks which these measurement points lie in. For example, the assembly variation models of measurement points M_1 and M_2 in Fig. 3 are shown as follows.

$$\begin{cases} V_{M_1L} = D_{M_1A_1A_2A_3} \cdot V_{A_1A_2A_3} + V_{M_1z} \\ V_{M_2L} = D_{M_2A_1A_3A_4} \cdot V_{A_1A_3A_4} + V_{M_2z} \end{cases} \tag{12}$$

When more additional locating blocks are adopted, the divided rigid areas must be determined for the assembly variation computation of measurement points according to the locating sequence of these additional locating blocks. In Fig. 4, with a 6-2-1 locating layout and the locating sequence of $A_1A_2A_3$-A_4-A_5-A_6, the compliant sheet metal part is divided into four rigid areas by dashed lines. Determine where the measurement point lie and its assembly variation can be computed by these relevant locating blocks.

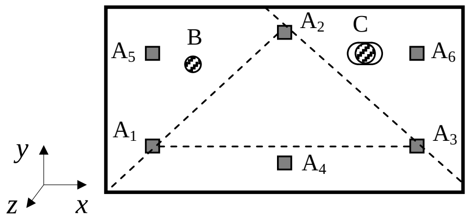

Fig. 4 Bending deformation of a 6-2-1 locating layout

3.2 Welding Spring-back Computation of Parts

When two or more sheet metal parts are welded together by spot welding, some welding deformation may happen because of the existence of variation in the welding areas of parts. To compute the welding deformation, the welding spring-back must be determined. Though the welding spring-back computation of compliant sheet metal parts can be obtained by the finite element method, it needs lots of finite element computation, and has low efficiency. Here a simplified method is presented to compute the welding spring-back.

The part stiffness in welding points must be given before the computation of welding spring-back. The stiffness computation is shown as follows. Firstly, the finite element models of parts are built with zero variation in all corresponding boundary conditions. Secondly, apply a unit force in the vertical direction of a welding point, so the variation of the welding point can be obtained as C_i. When the force F is applied, the variation of the welding point is

$$V_i = C_i F \tag{13}$$

Then $$F = \frac{1}{C_i} V_i = K_i V_i \tag{14}$$

So the stiffness of the part in this welding point is $K_i = \frac{1}{C_i}$.

Assume that V_1 and V_2 are the variation of welding points in part 1 and part 2 after they are located, K_1 and K_2 are the stiffness of parts in corresponding welding points, and V_W is the assembly variation of welding point after welded, so the welding force is

$$F = F_1 + F_2 = K_1V_1 + K_2V_2 = KV_W \tag{15}$$

where F_1 and F_2 are the forces applied separately to the welding points of part 1 and part 2, K is the stiffness of the welded assembly.

Then the assembly variation is

$$V_W = \frac{K_1}{K} V_1 + \frac{K_2}{K} V_2 \tag{16}$$

Because of one-dimensional structure, there exists $K = K_1 + K_2$. The Equation (16) is changed as follows.

$$V_W = \frac{K_1}{K_1 + K_2} V_1 + \frac{K_2}{K_1 + K_2} V_2 \tag{17}$$

Then the assembly spring-back of part 1 in this welding point is

$$\Delta V_1 = V_W - V_1 = \frac{K_2}{K_1 + K_2}(V_2 - V_1) \tag{18}$$

3.3 The Assembly Variation Computation Under an Over-located Condition

In Fig. 5 two compliant sheet metal parts are assembled. A 3-2-1 locating layout is adopted for part 1 and a 4-2-1 locating layout is for part 2. M_1, M_2 and M_3 are measurement points, and the corresponding welding points between part 1 and part 2 are W_{11}/W_{21} , W_{12}/W_{22} and W_{13}/W_{23}.

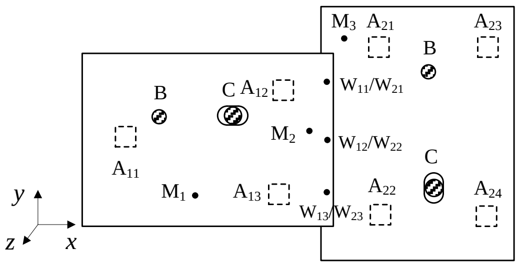

Fig. 5 An assembly of two sheet metal parts

The measurement points are divided into two types according to whether they are influenced by the welding process. One is that some measurement points locates away from the welding area, and are not influenced by the welding process, e.g. the measurement point M_1 whose variation is only affected by the locating process. The variation computation of these measurement points can be obtained according to Section 3.1, then the assembly variation of measurement point M_1 can be obtained as

$$V_{M_1} = V_{M_1L} = D_{M_1A_{11}A_{12}A_{13}} \cdot V_{A_{11}A_{12}A_{13}} + V_{M_1z} \tag{19}$$

The others are the measurement points lying in the welding area whose assembly variation is affected by the welding process. Their assembly variation computation is divided into three steps.

(1) Compute the variation caused by the fixture locating variation. In this stage, the variation of measurement points and welding points must all be computed. For example, in this case, the variation of measurement point M_2 and M_3, and welding point W_{11}, W_{12} and W_{13} are obtained as follows.

$$\begin{cases} V_{M_2L} = D_{M_2A_{11}A_{12}A_{13}} \cdot V_{A_{11}A_{12}A_{13}} + V_{M_2z} \\ V_{M_3L} = D_{M_3A_{21}A_{22}A_{23}} \cdot V_{A_{21}A_{22}A_{23}} + V_{M_3z} \end{cases} \tag{20}$$

$$\begin{cases} V_{W_{11}L} = D_{W_{11}A_{11}A_{12}A_{13}} \cdot V_{A_{11}A_{12}A_{13}} + V_{W_{11}z} \\ V_{W_{12}L} = D_{W_{12}A_{11}A_{12}A_{13}} \cdot V_{A_{11}A_{12}A_{13}} + V_{W_{12}z} \\ V_{W_{13}L} = D_{W_{13}A_{11}A_{12}A_{13}} \cdot V_{A_{11}A_{12}A_{13}} + V_{W_{13}z} \end{cases} \tag{21}$$

$$\begin{cases} V_{W_{21}L} = D_{W_{21}A_{21}A_{22}A_{23}} \cdot V_{A_{21}A_{22}A_{23}} + V_{W_{21}z} \\ V_{W_{22}L} = D_{W_{22}A_{21}A_{22}A_{23}} \cdot V_{A_{21}A_{22}A_{23}} + V_{W_{22}z} \\ V_{W_{23}L} = D_{W_{23}A_{21}A_{22}A_{23}} \cdot V_{A_{21}A_{22}A_{23}} + V_{W_{23}z} \end{cases} \tag{22}$$

(2) Compute the welding spring-back of welding points. According to Section 3.2, the welding spring-back of welding points are obtained as follows.

$$\begin{cases} \Delta V_{W_{11}} = \dfrac{K_{21}}{K_{11}+K_{21}}(V_{W_{21}L} - V_{W_{11}L}) \\ \Delta V_{W_{12}} = \dfrac{K_{22}}{K_{12}+K_{22}}(V_{W_{22}L} - V_{W_{12}L}) \\ \Delta V_{W_{13}} = \dfrac{K_{23}}{K_{13}+K_{23}}(V_{W_{23}L} - V_{W_{13}L}) \end{cases} \tag{23}$$

$$\begin{cases} \Delta V_{W_{21}} = \dfrac{K_{11}}{K_{11}+K_{21}}(V_{W_{11}L} - V_{W_{21}L}) \\ \Delta V_{W_{22}} = \dfrac{K_{12}}{K_{12}+K_{22}}(V_{W_{12}L} - V_{W_{22}L}) \\ \Delta V_{W_{23}} = \dfrac{K_{13}}{K_{13}+K_{23}}(V_{W_{13}L} - V_{W_{23}L}) \end{cases} \tag{24}$$

(3) Compute the final assembly variation. Find the nearest welding point to the computed measurement point, and compute the variation of the measurement points caused by the welding spring-back of the welding point. Because usually there is little difference between the variation of welding points, some variation affected by other welding points are ignored. So the variation of measurement points M_2 and M_3 produced by the welding spring-back is obtained as follows.

$$\begin{cases} V_{M_2S} = D_{M_2W_{12}A_{12}A_{13}} \cdot \Delta V_{W_{12}} \\ V_{M_3S} = D_{M_3W_{21}A_{21}A_{22}} \cdot \Delta V_{W_{21}} \end{cases} \tag{25}$$

Then the final assembly variation of measurement points M_2 and M_3 after welded is

$$\begin{cases} V_{M_2} = V_{M_2L} + V_{M_2S} \\ V_{M_3} = V_{M_3L} + V_{M_3S} \end{cases} \tag{26}$$

4 Case Analysis of a Compliant Sheet Metal Assembly

4.1 Case Description

An assembly of two compliant sheet metal parts is shown in Fig. 6. The sizes of the parts used in this case are $100\times100\times1\text{mm}^3$ (part 1) and $100\times100\times2\text{mm}^3$ (part 2). The material of both parts is mild steel with Young's modulus E=20700N/mm^2 and Poison's ratio v=0.3.

The two parts are all located by a 4-2-1 locating layout whose fixture locating layout and measurement points are shown in Fig. 6. A_{ij} denotes the jth locating block in the ith part, and M_{ij} denotes the jth measurement point in the ith part. The sign of W_{ij} denotes the jth welding point in the ith part. The letters in brackets after locating block symbols in Fig. 6 denote the constrained motion axles. The locating sequences of these two parts are $A_{11}A_{12}A_{13}$-A_{14}, and $A_{21}A_{22}A_{23}$-A_{24}.

The coordinates of the fixture locating blocks, the measurement points and the welding points are shown in Table 1.

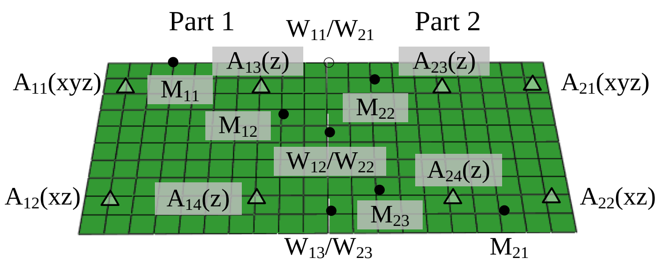

Fig. 6 Finite element model of a compliant sheet metal assembly

Table 1 Coordinates of this case

Name	Coordinate	Name	Coordinate
A_{11}	(-90,30,0)	M_{11}	(-70,40,0)
A_{12}	(-90,-30,0)	M_{12}	(-20,10,0)
A_{13}	(-30,30,0)	W_{11}	(0,40,0)
A_{14}	(-30,-30,0)	W_{12}	(0,0,0)
		W_{13}	(0,-40,0)
A_{21}	(90,30,0)	M_{21}	(70,-40,0)
A_{22}	(90,-30,0)	M_{22}	(20,30,0)
A_{23}	(50,30,0)	M_{23}	(20,-30,0)
A_{24}	(50,-30,0)	W_{21}	(0,40,0)
		W_{22}	(0,0,0)
		W_{23}	(0,-40,0)

4.2 Assembly Variation Computation

According to the given information in this case, the assembly variation expressions of the measurement points and welding points after parts are located are obtained as follows.

$$\begin{cases} V_{M_{11}L} = 0.8333V_{A_{11}} - 0.1667V_{A_{12}} + 0.3333V_{A_{13}} + V_{M_{11}z} \\ V_{M_{12}L} = -0.1667V_{A_{12}} + 0.6667V_{A_{13}} + 0.5V_{A_{14}} + V_{M_{12}z} \\ V_{M_{21}L} = 0.5V_{A_{22}} - 0.1667V_{A_{23}} + 0.6667V_{A_{24}} + V_{M_{21}z} \\ V_{M_{22}L} = -0.75V_{A_{22}} + V_{A_{23}} + 0.75V_{A_{24}} + V_{M_{22}z} \\ V_{M_{23}L} = -0.75V_{A_{22}} + 1.75V_{A_{24}} + V_{M_{23}z} \end{cases} \quad (27)$$

$$\begin{cases} V_{W_{11}L} = -0.5V_{A_{12}} + 1.1667V_{A_{13}} + 0.3333V_{A_{14}} + V_{W_{11}z} \\ V_{W_{12}L} = -0.5V_{A_{12}} + 0.5V_{A_{13}} + V_{A_{14}} + V_{W_{12}z} \\ V_{W_{13}L} = -0.5V_{A_{12}} + 0.1667V_{A_{13}} + 1.6667V_{A_{14}} + V_{W_{13}z} \\ V_{W_{21}L} = -1.25V_{A_{22}} + 1.1667V_{A_{23}} + 1.0833V_{A_{24}} + V_{W_{21}z} \\ V_{W_{22}L} = -1.25V_{A_{22}} + 0.5V_{A_{23}} + 1.75V_{A_{24}} + V_{W_{22}z} \\ V_{W_{23}L} = -1.25V_{A_{22}} + 0.1667V_{A_{23}} + 2.4167V_{A_{24}} + V_{W_{23}z} \end{cases} \quad (28)$$

During the welding process, the variation of the measurement points M_{12}, M_{22} and M_{23} are influenced by the welding spring-back. From Fig. 6, M_{12}, M_{22} and M_{23} are mainly influenced by the welding points W_{12}, W_{21} and W_{23}, respectively. Then the welding spring-back of these welding points can be computed as follows.

$$\begin{cases} \Delta V_{W_{12}} = 0.7595(V_{W_{22}L} - V_{W_{12}L}) \\ \Delta V_{W_{21}} = 0.2489(V_{W_{11}L} - V_{W_{21}L}) \\ \Delta V_{W_{23}} = 0.2489(V_{W_{13}L} - V_{W_{23}L}) \end{cases} \quad (29)$$

So the assembly variation models of M_{12}, M_{22} and M_{23} are changed to Equation (30). The assembly variation of measurement points M_{11} and M_{21} are still equal to the variation in Equation (27).

$$\begin{cases} V_{M_{12}} = -0.0402V_{A_{12}} + 0.5402V_{A_{13}} + 0.2469V_{A_{14}} \\ \quad -0.3164V_{A_{22}} + 0.1266V_{A_{23}} + 0.4429V_{A_{24}} \\ \quad +0.2531V_{W_{22}z} - 0.2531V_{W_{12}z} + V_{M_{12}z} \\ V_{M_{22}} = -0.0747V_{A_{12}} + 0.1742V_{A_{13}} + 0.0498V_{A_{14}} \\ \quad -0.5634V_{A_{22}} + 0.8258V_{A_{23}} + 0.5883V_{A_{24}} \\ \quad +0.1493V_{W_{11}z} - 0.1493V_{W_{21}z} + V_{M_{22}z} \\ V_{M_{23}} = -0.0747V_{A_{12}} + 0.0249V_{A_{13}} + 0.2489V_{A_{14}} \\ \quad -0.5634V_{A_{22}} - 0.0249V_{A_{23}} + 1.3892V_{A_{24}} \\ \quad +0.1493V_{W_{13}z} - 0.1493V_{W_{23}z} + V_{M_{23}z} \end{cases} \quad (30)$$

$$\begin{cases} V_{M_{11}} = V_{M_{11}L} = 0.8333V_{A_{11}} - 0.1667V_{A_{12}} \\ \quad +0.3333V_{A_{13}} + V_{M_{11}z} \\ V_{M_{21}} = V_{M_{21}L} = 0.5V_{A_{22}} - 0.1667V_{A_{23}} \\ \quad +0.6667V_{A_{24}} + V_{M_{21}z} \end{cases} \quad (31)$$

Assume that the manufacturing variation and the fixture locating variation are all distributed normally. So the manufacturing variation of part 1 and part 2 are assumed to follow *N* (0, 0.3333) and *N* (0, 0.5), respectively. And the fixture locating variation for the two parts are all assumed to follow *N* (0, 0.1). The method of Monte Carlo Simulation is used to simulate the assembly process. Then the assembly variation of these measurement points after the weld can be computed as shown in Table 2.

Table 2 Assembly variation results

Name	Assembly Variation
M_{11}	2.0855
M_{12}	2.2580
M_{21}	3.0765
M_{22}	3.1539
M_{23}	3.1828

4.3 Assembly Variation Modeling Method Validation

In this case the locating process variation can be also computed using the method described in [4], and the welding assembly spring-back of welding points can be also computed using the Method of Influence Coefficients [8]. And the results can be compared with the proposed method.

Then the assembly variation models of locating process using the method in [4] are obtained as follows.

$$\begin{cases} V'_{M_{11}L} = 0.8333V_{A_{11}} - 0.1667V_{A_{12}} + 0.3333V_{A_{13}} + V_{M_{11}z} \\ V'_{M_{12}L} = -0.1667V_{A_{12}} + 0.6667V_{A_{13}} + 0.5V_{A_{14}} + V_{M_{12}z} \\ V'_{M_{21}L} = 0.5V_{A_{22}} - 0.1667V_{A_{23}} + 0.6667V_{A_{24}} + V_{M_{21}z} \\ V'_{M_{22}L} = -0.75V_{A_{22}} + V_{A_{23}} + 0.75V_{A_{24}} + V_{M_{22}z} \\ V'_{M_{23}L} = -0.75V_{A_{22}} + 1.75V_{A_{24}} + V_{M_{23}z} \end{cases} \quad (32)$$

And the welding spring-back of the welding points W_{12}, W_{21} and W_{23} can be computed using the Method of Influence Coefficients as follows.

$$\begin{cases} \Delta V'_{W_{12}} = 0.0301V_{W_{11}L} - 0.8495V_{W_{12}L} + 0.0301V_{W_{13}L} \\ \quad +0.2273V_{W_{21}L} + 0.3454V_{W_{22}L} + 0.2273V_{W_{23}L} \\ \Delta V'_{W_{21}} = 0.0822V_{W_{11}L} + 0.0565V_{W_{12}L} + 0.0753V_{W_{13}L} \\ \quad -0.6955V_{W_{21}L} + 0.0907V_{W_{22}L} + 0.3671V_{W_{23}L} \\ \Delta V'_{W_{23}} = 0.0753V_{W_{11}L} + 0.0565V_{W_{12}L} + 0.0822V_{W_{13}L} \\ \quad +0.3671V_{W_{21}L} + 0.0907V_{W_{22}L} - 0.6955V_{W_{23}L} \end{cases} \quad (33)$$

These models show that the welding spring-back is influenced by all the welding point variation, and the coefficients in Equation (33) show the influencing relationship of each welding point to the welding spring-back of one welding point. Generally, there is a little difference between all welding point variation of one part's same welding area. So the coefficients in Equation (33) may be combined, and these welding spring-back models can be changed to:

$$\begin{cases} \Delta V'_{W_{12}} = 0.8V_{W_{22}L} - 0.7893V_{W_{12}L} \\ \Delta V'_{W_{21}} = 0.214V_{W_{11}L} - 0.2377V_{W_{21}L} \\ \Delta V'_{W_{23}} = 0.214V_{W_{13}L} - 0.2377V_{W_{23}L} \end{cases} \quad (34)$$

Compare the equations in Equations. (27) and (29) with those in Equations. (32) and (34), respectively, it can be seen that the obtained results are almost identical to those of the existed achievements. However, the fully rigid assembly variation modeling method in [4] can not solve the assembly under an over-located condition, and the MIC [8] needs much finite element computation. So the proposed method compared with the existed methods combines the rigid and compliant characteristics of sheet metal parts, which can achieve more rapid and accurate assembly variation computation.

5 Conclusion

This paper proposes an assembly variation modeling method of compliant sheet metal parts under an over-located condition. Firstly, an assembly variation modeling method of a rigid part under a 3-2-1 locating layout is given using its spatial motion relationship when some fixture locating variation happens. Secondly, Based on this method, a multi-divided rigid area assembly variation modeling method can be obtained according to part's bending deforming principle during the locating process. Thirdly, a simplified stiffness solving method for the welding spring-back is given, and an assembly variation modeling method for the welding area of parts is presented using the stiffness comparison of welding parts. Then the assembly variation of measurement points in welding area and out of welding area can be obtained according to the proposed method.

The proposed assembly variation modeling method is validated using the existed research achievements. An assembly of compliant sheet metal parts is used to demonstrate the assembly variation analysis process and validate the method. The results show that the proposed method can achieve a rapid and accurate assembly variation analysis compared with the existed rigid and compliant sheet metal assembly variation modeling methods.

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