The term structure model of corporate bond yields

JIE-MIN HUANG\textsuperscript{1}, SU-SHENG WANG\textsuperscript{1}, JIE-YONG HUANG\textsuperscript{2}

\textsuperscript{1}Shenzhen Graduate School
Harbin Institute of Technology
Shenzhen University Town in Shenzhen City
People’s Republic of China, 086-518055
\textsuperscript{2}Kaifeng city, Henan Province

jieminhuang0819@gmail.com; wangsusheng@gmail.com; jieyong1987@126.com

Abstract: We build the term structure of corporate bond yields with N-factor affine model, and we estimate the parameters by using Kalman filtering. We choose weekly average corporate bond yields data in Shanghai Stock Exchange and Shenzhen Stock Exchange. We find the one-factor model and two-factor model could do one-step forward forecasting well, but the three-factor model could fit the observable data well.

Key-Words: corporate bond; yields; term structure; Kalman filtering

1 Introduction

Many scholars research on term structure affine models of bonds. The literatures are as below. Some scholars find the three factor model fits observable data well. Dai, Singleton(2000) [1] analyzes the structural differences and goodness-of-fits of affine term structure models. Some models are good at modeling the conditional correlation, some are good at modeling volatilities of the risk factors. He extends N-factor affine model into N+1-factor affine model. Vasicek (1977) Cox, Ingersoll, and Ross (1985) [2,3] assume instantaneous short rate \( r(t) \) is the equation of N-factor state variable \( Y(t) \), and

\[ r(t) = \alpha_{\theta} + \alpha_{\gamma} Y(t), \]

and \( Y(t) \) follows Gaussian and square root diffusions. Some scholars extend Markov one factor short rate model, and add in a stochastic long-run mean \( \theta(t) \) and a volatility \( \nu(t) \) of \( r(t) \), \[ dr(t) = (\theta - r(t))dt + \nu dB(t) \]. These models come from bond pricing and interest-rate derivatives. Duffee(2002)[4] considers affine model can’t forecast treasury yields. He thinks assuming yields follow stochastic random walk and forecasting results are good. He considers the models failure for the reason that variation of risk compensation is related with interest-rate volatility. He raises essential affine model, and the model keeps the advantage of standard model, but it makes interest-rate variation independent from interest-rate volatility, and this is important for forecasting future yield. Jong(2000) [22] analyzes term structure affine model combining with time series and cross-section information, and he uses discretization continuous time to do Kalman filtering. He finds the three factors model could fit cross-section and dynamic term structure model. Duffie, Kan(1996) [5] finds yields with fixed maturity follow stochastic volatility multi-parameters Markov diffusion process by using continuous no arbitrage multi-factor model of interest-rate term structure. He uses jump-diffusion to solve interest-rate term structure model. Longstaff and Schwartz(1995) [6] evaluate corporate bonds value which have default risk and interest-rate risk by using simple methods. He finds the relation between default risk and interest-rate has important effect on credit spread. Also, he finds credit spread correlates with interest-rate negatively, and the risky bond duration depends on interest rate. He uses \( V \) to represent corporate total asset value, and it follows dynamic variation below:

\[ dv = \mu V dt + \sigma V dz_1, \]

and \( \sigma \) is constant, and \( z_1 \) is
standard Wiener process. He uses \( r \) to represent risk-free interest rate, and \( dt = (\varepsilon - \beta r) dt + \alpha \, dz_2 \), and \( \varepsilon, \beta \text{ and } \alpha \) are constant, and \( z_2 \) is standard Wiener process, and the correlation of \( dz_1 \) and \( dz_2 \) is \( \rho dt \).

Dai and Singleton (2003) [18] observe dynamic term structure model, and it fits on treasury and swap yield curve, and default factor follows diffusion, jump diffusion. Duffie and Lando (2001) [19] study on corporate bond credit spread term structure with imperfect information. He assumes bond investors can't observe the assets of bond issuers, and they only get the imperfect accounting reports. He considers corporate assets follow Geometric Brownian Motion, and the credit spread has accounting information character.


In China, Fan longzhen and Zhang guoqing (2005) [25] analyze time continuous two-factor generalized Gaussian affine model by using Kalman filtering. The model could reflect cross-section characteristic of interest rate term structure, but it can’t reflect the time series character. Wang
xiaofang, Liu fenggen and Hanlong (2005) [26] build interest rate term structure curve by using cubic spline function. Fan longzhe (2005) [27] estimates bond interest rate by using term structure of yields with three-factor Gaussian essential affine model. Fan longzhen (2003) [28] estimates treasury time continuous two factor Vasicek model by using Kalman filtering. There are many literatures on interest rate term structure model, the abroad research focuses on commodity futures, corporate bond pricing, and some of corporate bond spread and bond yield. In China, they are mainly about treasury term structure and few of corporate bond term structure. We research on corporate bond yield term structure in Shanghai and Shenzhen Exchange by using Kalman filtering, and few scholars has ever researched on it by using the method, and also we plan to research on the complex factors on corporate bond spread in Shanghai and Shenzhen Exchange.

2. Data description
We choose corporate bond yields in Shanghai Exchange and Shenzhen Exchange. We choose bonds with more than 1 year to maturity, because bonds with less than 1 year to maturity are very sensitive to interest rate. We choose corporate bonds weekly average returns with 3 years, 5 years, 7 years and 10 years maturity from January 1st 2012 to December 31st 2012. The data descriptive statistics are in table1. We can see the long term bonds have lower weekly average yields than short term bonds. According to JB values, only 7 years bonds do n't follow normal distribution, and others follow normal distribution.

Table1 descriptive statistics

<table>
<thead>
<tr>
<th></th>
<th>Y1</th>
<th>Y2</th>
<th>Y3</th>
<th>Y4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>5.6770</td>
<td>5.6300</td>
<td>5.8959</td>
<td>4.2039</td>
</tr>
<tr>
<td>Median</td>
<td>5.4073</td>
<td>5.3979</td>
<td>5.7922</td>
<td>4.6498</td>
</tr>
<tr>
<td>Max</td>
<td>6.8343</td>
<td>6.8026</td>
<td>6.7393</td>
<td>5.4585</td>
</tr>
<tr>
<td>Min</td>
<td>4.8176</td>
<td>4.6666</td>
<td>5.1545</td>
<td>1.4359</td>
</tr>
<tr>
<td>Std.d</td>
<td>0.6288</td>
<td>0.6465</td>
<td>0.4826</td>
<td>1.0270</td>
</tr>
<tr>
<td>skewness</td>
<td>0.6356</td>
<td>0.5716</td>
<td>0.3868</td>
<td>-1.481</td>
</tr>
<tr>
<td>kurtosis</td>
<td>2.0264</td>
<td>1.8824</td>
<td>1.9321</td>
<td>4.1194</td>
</tr>
<tr>
<td>JB</td>
<td>5.4480</td>
<td>5.4311</td>
<td>3.6929</td>
<td>21.301</td>
</tr>
<tr>
<td>P</td>
<td>0.0656</td>
<td>0.0662</td>
<td>0.1578</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

3. Term structure affine model
Vasicek (1977) and Cox, Ingersoll and Ross(1985) assume instantaneous short term interest rate r(t) is the affine equation of N-factor state vector Y(t). We assume the equation of r(t) and Y(t) as below:

\[
r_t = \alpha_0 + \delta_1 f_{1t} + \delta_2 f_{2t} + \delta_3 f_{3t} + \cdots + \delta_n f_{nt} \quad (1)
\]

\[
r_t \text{ is short term interest rate, } \alpha_0 \text{ is constant and}
\]

\[
f_{1t}, f_{2t}, f_{3t}, \ldots f_{nt} \text{ are the N-state variables which decide interest rate value. According to short term interest rate model of Longstaff and Schwartz (1995), state variables follow mean reversion in the condition of risk neutral probability.}
\]

\[
dF_t = KF_t dt + \sigma dW_t \quad (2)
\]

The equation is as below:

\[
\begin{pmatrix}
    f_{1t} \\
    f_{2t} \\
    \vdots \\
    f_{nt}
\end{pmatrix}
\begin{pmatrix}
    -k_1 & 0 & 0 & 0 \\
    0 & -k_2 & 0 & 0 \\
    \vdots & \vdots & \ddots & \vdots \\
    0 & 0 & 0 & -k_n
\end{pmatrix}
\begin{pmatrix}
    f_{1t} \\
    f_{2t} \\
    \vdots \\
    f_{nt}
\end{pmatrix}
= dt +
\begin{pmatrix}
    dW_{1t} \\
    dW_{2t} \\
    \vdots \\
    dW_{nt}
\end{pmatrix}
\quad (3)
\]

Parameters \( k_1, k_2, k_3, \ldots, k_n \) indicate state variables, and \( f_{1t}, f_{2t}, f_{3t}, \ldots f_{nt} \) indicate mean reversion rate, and \( \sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_n \) indicate state variables volatility, and \( w_{1t}, w_{2t}, w_{3t}, \ldots, w_{nt} \) indicate \( N \) independent Standard Brown Motions. In risk neutral probability, the unconditional mean of state variable is 0. \( \alpha_0 \) denotes long term mean of short term interest rate in risk neutral probability. In real probability \( P \), the state variables change as below:

\[
\begin{pmatrix}
    f_{1t} \\
    f_{2t} \\
    \vdots \\
    f_{nt}
\end{pmatrix}
\begin{pmatrix}
    -k_1 & 0 & 0 & 0 \\
    0 & -k_2 & 0 & 0 \\
    \vdots & \vdots & \ddots & \vdots \\
    0 & 0 & 0 & -k_n
\end{pmatrix}
\begin{pmatrix}
    f_{1t} \\
    f_{2t} \\
    \vdots \\
    f_{nt}
\end{pmatrix}
= dt +
\begin{pmatrix}
    dW_{1t} \\
    dW_{2t} \\
    \vdots \\
    dW_{nt}
\end{pmatrix}
+ \begin{pmatrix}
    \beta_{10} \\
    \beta_{20} \\
    \vdots \\
    \beta_{n0}
\end{pmatrix}
\quad (4)
\]

\[
\begin{pmatrix}
    \sigma_1 \\
    \sigma_2 \\
    \vdots \\
    \sigma_n
\end{pmatrix}
\begin{pmatrix}
    dW_{1t} \\
    dW_{2t} \\
    \vdots \\
    dW_{nt}
\end{pmatrix} + \begin{pmatrix}
    \beta_{10} \\
    \beta_{20} \\
    \vdots \\
    \beta_{n0}
\end{pmatrix}
\quad (5)
\]

\[
\begin{pmatrix}
    \sigma_1 \\
    \sigma_2 \\
    \vdots \\
    \sigma_n
\end{pmatrix}
\begin{pmatrix}
    dW_{1t} \\
    dW_{2t} \\
    \vdots \\
    dW_{nt}
\end{pmatrix} \quad (6)
\]

\[
\begin{pmatrix}
    \sigma_1 \\
    \sigma_2 \\
    \vdots \\
    \sigma_n
\end{pmatrix}
\begin{pmatrix}
    dW_{1t} \\
    dW_{2t} \\
    \vdots \\
    dW_{nt}
\end{pmatrix} + \begin{pmatrix}
    \beta_{10} \\
    \beta_{20} \\
    \vdots \\
    \beta_{n0}
\end{pmatrix}
\quad (7)
\]

\[
\begin{pmatrix}
    \sigma_1 \\
    \sigma_2 \\
    \vdots \\
    \sigma_n
\end{pmatrix}
\begin{pmatrix}
    dW_{1t} \\
    dW_{2t} \\
    \vdots \\
    dW_{nt}
\end{pmatrix} + \begin{pmatrix}
    \beta_{10} \\
    \beta_{20} \\
    \vdots \\
    \beta_{n0}
\end{pmatrix}
\quad (8)
\]
\[
\begin{bmatrix}
\beta_{11} & 0 & 0 & 0 & 0 \\
0 & \beta_{22} & 0 & 0 & 0 \\
0 & 0 & \beta_{33} & 0 & 0 \\
0 & 0 & 0 & \beta_{n} & 0 \\
0 & 0 & 0 & 0 & \beta_{nn}
\end{bmatrix}
\begin{bmatrix}
f_{1t} \\
f_{2t} \\
f_{3t} \\
f_{nt}
\end{bmatrix}
\, dt
\]
\[
= \begin{bmatrix}
k_1 - \beta_{11} & 0 & 0 & 0 & 0 \\
0 & k_2 - \beta_{22} & 0 & 0 & 0 \\
0 & 0 & k_3 - \beta_{33} & 0 & 0 \\
0 & 0 & 0 & k_n - \beta_{nn}
\end{bmatrix}^{-1}
\begin{bmatrix}
\beta_{10} \\
\beta_{20} \\
\beta_{30} \\
\beta_{n0}
\end{bmatrix}
\times
\begin{bmatrix}
\sigma_1 & 0 & 0 & 0 & 0 \\
0 & \sigma_2 & 0 & 0 & 0 \\
0 & 0 & \sigma_3 & 0 & 0 \\
0 & 0 & 0 & \sigma_n
\end{bmatrix}
\begin{bmatrix}
dw_{1t} \\
dw_{2t} \\
dw_{3t} \\
dw_{nt}
\end{bmatrix}
\tag{4}
\]

\(\beta_{10}, \beta_{20}, \beta_{30}, \ldots, \beta_{n0}\) denote the fixed interest rate risk premium, \(\beta_{11}, \beta_{22}, \beta_{33}, \ldots, \beta_{nn}\) denote the time varying interest rate risk premium.

\[
K = \begin{bmatrix}
k_1 - \beta_{11} & 0 & 0 & 0 & 0 \\
0 & k_2 - \beta_{22} & 0 & 0 & 0 \\
0 & 0 & k_3 - \beta_{33} & 0 & 0 \\
0 & 0 & 0 & k_n - \beta_{nn}
\end{bmatrix},
\]

\[
\theta = \begin{bmatrix}
\beta_{10} \\
\beta_{20} \\
\beta_{30} \\
\beta_{n0}
\end{bmatrix},
\]

\[
\varphi = \begin{bmatrix}
\exp(\beta_{11} - k_1) & 0 & 0 & 0 & 0 \\
0 & \exp(\beta_{22} - k_2) & 0 & 0 & 0 \\
0 & 0 & \exp(\beta_{33} - k_3) & 0 & 0 \\
0 & 0 & 0 & \exp(\beta_{nn} - k_n)
\end{bmatrix}
\]

In the real probability \(P\), state variables mean reversion follow the equation below.

\[
dF_t = K(\theta - F_t)\, dt + \sigma dW_t
\tag{5}
\]

\[
Q = \begin{bmatrix}
\frac{1}{2(k_1 - \beta_{11})} & 0 & 0 & 0 & 0 \\
0 & \frac{1}{2(k_2 - \beta_{22})} & 0 & 0 & 0 \\
0 & 0 & \frac{1}{2(k_3 - \beta_{33})} & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2(k_n - \beta_{nn})}
\end{bmatrix}
\]

In real probability \(P\), the conditional expectation and variance of state variables are below:

\[
E(F_{t+1} | F_t) = \theta + \varphi[F_t - \theta]
\tag{6}
\]

\[
\text{var}(F_{t+1} | F_t) = Q
\tag{7}
\]

When short term interest rate and state variable are certain, bond price and long term interest rate will be determined by short term interest rate in risk neutral probability. According to literatures, the bond with maturity at time \(T\) and par value \(1\$\), its pricing model is as below.

\[
P_{t}^{(T-t)} = E^{Q}_{t}\left[ \exp\left( -\int_{t}^{T} r_s \, ds \right) \right] =
\]

\[
E^{Q}_{t}\left[ \exp\left( -\int_{t}^{T} (u_0 + \delta_{11} f_{1s} + \delta_{21} f_{2s} + \delta_{31} f_{3s} + \ldots + \delta_{n1} f_{ns}) \, ds \right) \right]
\tag{8}
\]
After derivation, the bond with term $\tau$, at time $t$, the spot interest rate is below:

$$y_t = a_0 - \frac{\sigma_1^2}{k_1} \cdot \frac{\sigma_2^2}{k_2} \cdot \frac{\sigma_3^2}{k_3} \cdot \ldots \cdot \frac{\sigma_n^2}{k_n} + \frac{2\sigma_2^2(1 - e^{-k_2\tau})}{k_2} \frac{2\sigma_3^2(1 - e^{-k_3\tau})}{k_3} \frac{2\sigma_n^2(1 - e^{-k_n\tau})}{k_n}$$

$$- \frac{2\sigma_2^2(1 - e^{-k_2\tau})}{2k_2^2\tau} - \frac{2\sigma_3^2(1 - e^{-k_3\tau})}{2k_3^2\tau} - \frac{2\sigma_n^2(1 - e^{-k_n\tau})}{2k_n^2\tau} \ldots \frac{2\sigma_n^2(1 - e^{-k_n\tau})}{2k_n^2\tau}$$

$$+ \frac{2\sigma_2^2(1 - e^{-k_2\tau})}{k_2^2\tau} - \frac{2\sigma_3^2(1 - e^{-k_3\tau})}{k_3^2\tau} - \frac{2\sigma_n^2(1 - e^{-k_n\tau})}{k_n^2\tau} \ldots \frac{2\sigma_n^2(1 - e^{-k_n\tau})}{k_n^2\tau}$$

$$+ \frac{2\sigma_3^2(1 - e^{-k_3\tau})}{k_3^2\tau} - \frac{2\sigma_2^2(1 - e^{-k_2\tau})}{k_2^2\tau} - \frac{2\sigma_n^2(1 - e^{-k_n\tau})}{k_n^2\tau} \ldots \frac{2\sigma_n^2(1 - e^{-k_n\tau})}{k_n^2\tau}$$

$$+ \frac{2\sigma_n^2(1 - e^{-k_n\tau})}{k_n^2\tau}$$

$$= a_0 - \frac{\sigma_1^2}{k_1} - \frac{\sigma_2^2}{k_2} - \frac{\sigma_3^2}{k_3} - \ldots - \frac{\sigma_n^2}{k_n} + \frac{2\sigma_2^2(1 - e^{-k_2\tau})}{k_2} \frac{2\sigma_3^2(1 - e^{-k_3\tau})}{k_3} \frac{2\sigma_n^2(1 - e^{-k_n\tau})}{k_n}$$

$$- \frac{2\sigma_2^2(1 - e^{-k_2\tau})}{2k_2^2\tau} - \frac{2\sigma_3^2(1 - e^{-k_3\tau})}{2k_3^2\tau} - \frac{2\sigma_n^2(1 - e^{-k_n\tau})}{2k_n^2\tau} \ldots \frac{2\sigma_n^2(1 - e^{-k_n\tau})}{2k_n^2\tau}$$

$$+ \frac{2\sigma_2^2(1 - e^{-k_2\tau})}{k_2^2\tau} - \frac{2\sigma_3^2(1 - e^{-k_3\tau})}{k_3^2\tau} - \frac{2\sigma_n^2(1 - e^{-k_n\tau})}{k_n^2\tau} \ldots \frac{2\sigma_n^2(1 - e^{-k_n\tau})}{k_n^2\tau}$$

$$+ \frac{2\sigma_3^2(1 - e^{-k_3\tau})}{k_3^2\tau} - \frac{2\sigma_2^2(1 - e^{-k_2\tau})}{k_2^2\tau} - \frac{2\sigma_n^2(1 - e^{-k_n\tau})}{k_n^2\tau} \ldots \frac{2\sigma_n^2(1 - e^{-k_n\tau})}{k_n^2\tau}$$

$$+ \frac{2\sigma_n^2(1 - e^{-k_n\tau})}{k_n^2\tau}$$

Equation (9) could be written as below:

$$y_t = C_0 + C_1 f_{1t} + C_2 f_{2t} + C_3 f_{3t} + \ldots + C_n f_{nt}$$

We choose corporate bond yields data from Shanghai Exchange and Shenzhen Exchange and the bonds with maturity of 3 years, 5 years, 7 years and 10 years.

The signal equation is as below:

$$Y_t = \begin{bmatrix} y_t^3 \\ y_t^5 \\ y_t^7 \\ y_t^{10} \end{bmatrix}, \quad C_0 = \begin{bmatrix} C_1^3 \\ C_1^5 \\ C_1^7 \\ C_1^{10} \end{bmatrix}, \quad e_t = \begin{bmatrix} e_t^3 \\ e_t^5 \\ e_t^7 \\ e_t^{10} \end{bmatrix}$$

$$C_1 = \begin{bmatrix} C_1^3 & C_1^5 & C_1^7 & C_1^{10} \\ C_1^3 & C_1^5 & C_1^7 & C_1^{10} \\ C_1^7 & C_1^{10} & C_1^3 & C_1^5 \\ C_1^{10} & C_1^3 & C_1^5 & C_1^7 \end{bmatrix}$$

According to financial theory, interest rate is determined by state variables. The mean value of $\varepsilon_t$ is 0, and it follows the equation below:

$$\text{var}(\varepsilon_t) = \mathbf{M}$$

According to (5), we get the state equation below:

$$F_{t+1} = \theta + \mathbf{O}(F_t - \theta) + \varepsilon_{t+1}$$

The state equation is:

$$\mathbf{F}_t = \theta, \quad \text{vec}(\bar{\mathbf{F}}_0) = [(I - (\mathbf{O} \times 0)]^{-1} \text{vec}(Q)$$
The predicting equation of $F_t$ is below:

$$F_{(t|t-1)} = \theta + \phi(F_{t-1} - \theta)$$  \hspace{1cm} (13)

The conditional variance of predicting value is below:

$$P_{(t|t-1)} = \phi P_{t-1} \phi + Q$$  \hspace{1cm} (14)

and $U_t$ follows normal distribution, so the likelihood equation is below:

$$-2\ln L_t = \ln |V_t| + U_t V_t^{-1} U_t$$  \hspace{1cm} (15)

The parameters meet the condition below:

$$\min_{(parameters)} \sum_t 2 \ln L_t$$

In Kalman filtering analysis,

$$K_t = P_{(t|t-1)} \hat{C}_1 V_t^{-1}, \quad L_t = I - K_t \hat{C}_1$$

Recursive Algorithm is below:

$$\tilde{F}_t = F_{(t|t-1)} + K_t U_t, \quad \tilde{P}_t = L_t P_{(t|t-1)}$$

5. Empirical results analysis

Graph1 observable bonds yields:

Graph1 indicates corporate bond weekly average yields in Shanghai Exchange and Shenzhen Exchange, and Y1 shows corporate bond yields with 3 years maturity, and Y2 shows corporate bond yields with 5 years maturity, and Y3 shows corporate bond yields with 7 years maturity, and Y4 means corporate bonds yields with 10 years maturity. We can see bonds with short term have higher weekly average yields.

### 5.1 One-factor empirical analysis

With given initial values of parameters, we get parameters in Table 2. From Table 2 we know $a_0$ is significant at 5% level, $\sigma_1$ is significant at 1% confidence level, and it means corporate bond yields fluctuate. $k_1$ is significant at 1% confidence level, and it means bond yields have mean reversion, but they reverse slowly. $\beta_1$ isn’t significant at 1%.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Std</th>
<th>Z</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0$</td>
<td>3.691**</td>
<td>1.581</td>
<td>2.33</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>0.161***</td>
<td>0.036</td>
<td>4.50</td>
</tr>
<tr>
<td>$k_1$</td>
<td>-0.146***</td>
<td>0.018</td>
<td>-8.24</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>-0.265</td>
<td>0.178</td>
<td>-1.48</td>
</tr>
</tbody>
</table>

*** denotes statistical variables are significant at 1% confidence level and ** denotes statistical variables are significant at 5% confidence level.

From graph2 we can see, it’s one-step forward forecasting of corporate bond weekly average yields in Shanghai Exchange and Shenzhen Exchange. The yields curves in graph2 are similar with the yields curves in graph1, so the model fits one-step forward forecasting well. Graph3 indicates the modeling of real yields in graph1, we can see it can’t fit the real curve well. So one-factor Kalman filtering model can’t fit real value well.
**5.2 Two-factor empirical analysis**

From table 3 we can see $a_0$ isn’t significant. $\sigma_1$ is significant at 1% confidence level, and $\sigma_2$ is not significant, and we infer may be they represent default risk and liquidity risk. $k_1$ is significant at 1% confidence level, and $k_2$ is not significant. $\beta_1$ is significant at 1% confidence level, also $\beta_2$ is significant at 1% confidence level, so there are risk premium in both state variable 1 and state variable 2.

<table>
<thead>
<tr>
<th>Table 3 two-factor affine model results</th>
<th>parameters</th>
<th>St.d</th>
<th>Z</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0$</td>
<td>1.087</td>
<td>8.948</td>
<td>0.122</td>
<td>0.9033</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>0.182***</td>
<td>0.0238</td>
<td>7.664</td>
<td>0.0000</td>
</tr>
<tr>
<td>$k_1$</td>
<td>-0.215***</td>
<td>0.006</td>
<td>-38.93</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>-0.215***</td>
<td>0.011</td>
<td>-19.86</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>2.330</td>
<td>6.890</td>
<td>0.338</td>
<td>0.7353</td>
</tr>
<tr>
<td>$k_2$</td>
<td>1.330</td>
<td>2.495</td>
<td>0.533</td>
<td>0.5941</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>-4.484***</td>
<td>1.679</td>
<td>-2.671</td>
<td>0.0076</td>
</tr>
</tbody>
</table>

*** denotes statistical variables are significant on the 1% confidence level.

From graph 4 we can see it’s the one step-forward forecasting of corporate bond weekly average yields in Shanghai Exchange and Shenzhen Exchange, graph 4 is similar with graph 1, and it means Kalman filtering two-factor model could forecast yields well. Graph 5 is modeling the real yields, and we can see graph 5 and graph 1 is very different, so the two-factor Kalman filtering model can’t fit real curve well.
5.3 Three-factor empirical analysis

From table4 we can see $a_0$ is significant. $\sigma_1$ is significant at 1% confidence level, and it means the state variable 1 fluctuates with time, and $\sigma_2$ isn't significant, also $\sigma_3$ isn't significant. $k_1$ is significant at 1% confidence level, and it means state variable 1 follows mean reversion, and $k_2$ is significant at 5% confidence level, and it means state variable 2 follows mean reversion, and $k_2 > k_1$, means state variable 2 reverses more quickly than state variable 1, and $k_3$ is significant at 1% confidence level, and means state variable 3 follows mean reversion, but it reverses more slowly than variable 2. $\beta_{11}$ is significant at 10% confidence level, and means state variable 1 has time varying risk premium, and $\beta_{22}$ is significant at 5% confidence level, and means state variable 2 has time varying risk premium, and also $\beta_{33}$ is significant at 1% confidence level, and it means state variable 3 has time varying risk premium, and state variable 2 has the largest time varying risk premium. $\beta_{10}$ is significant at 10% confidence level, and it means state variable 1 has fixed risk premium, but both $\beta_{20}$ and $\beta_{50}$ aren't significant.

<table>
<thead>
<tr>
<th>parameters</th>
<th>St.d</th>
<th>Z</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0$</td>
<td>3.681</td>
<td>24.374</td>
<td>0.151</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>0.152***</td>
<td>0.058</td>
<td>2.631</td>
</tr>
<tr>
<td>$k_1$</td>
<td>-0.229***</td>
<td>0.072</td>
<td>-3.182</td>
</tr>
<tr>
<td>$\beta_{11}$</td>
<td>-0.635*</td>
<td>0.383</td>
<td>-1.658</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>-0.764</td>
<td>16.977</td>
<td>-0.045</td>
</tr>
<tr>
<td>$k_2$</td>
<td>0.969**</td>
<td>0.388</td>
<td>2.497</td>
</tr>
<tr>
<td>$\beta_{22}$</td>
<td>0.865**</td>
<td>0.434</td>
<td>1.992</td>
</tr>
<tr>
<td>$\sigma_3$</td>
<td>0.103</td>
<td>9.854</td>
<td>0.010</td>
</tr>
<tr>
<td>$k_3$</td>
<td>0.203***</td>
<td>0.054</td>
<td>3.735</td>
</tr>
<tr>
<td>$\beta_{33}$</td>
<td>0.187***</td>
<td>0.056</td>
<td>3.367</td>
</tr>
<tr>
<td>$\beta_{10}$</td>
<td>0.976*</td>
<td>0.547</td>
<td>1.787</td>
</tr>
<tr>
<td>$\beta_{20}$</td>
<td>1.162</td>
<td>4.954</td>
<td>0.234</td>
</tr>
<tr>
<td>$\beta_{30}$</td>
<td>-0.033</td>
<td>2.001</td>
<td>-0.017</td>
</tr>
</tbody>
</table>

*** denotes statistical variables are significant at the 1% confidence level, ** denotes statistical variables are significant at the 5% confidence level, * denotes statistical variables are significant at 10% confidence level.
Kalman filtering to estimate the parameters of one-factor model, two-factor model and three-factor model. The results indicate one-factor model and two-factor model could do one-step forward forecasting well, and they have fixed risk premium, but they can't fit the real data well. Three-factor model can’t forecast well, but it could fit real data well, and we add the time varying risk premium factor into three-factor model, and find they are all significant, so the three state variables have time varying risk premium. But only state variable 1 has the significant fixed risk premium. And the results are similar with other scholars. I would do further research on corporate bond spread by using Kalman filtering.

Reference


6. Conclusion

We analyze corporate bond yields term structure in Shanghai Exchange and Shenzhen Exchange by using Kalman filtering model. We build N-factor affine term structure model, and then we use

Graph 6 Three-factor one-step forward forecasting

Graph 7 modeling real curve of yields

From graph 6 we can see it’s one-step forward forecasting of average weekly corporate bond yields in Shanghai Exchange and Shenzhen Exchange, and it’s very different with graph 1, so the forecasting isn’t good. Graph 7 is the modeling of real curve, and it’s similar with graph 1, so the three-factor model could fit real data well.