Unidirectional Chaotic Synchronization of Rossler Circuit and Its Application for Secure Communication

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Abstract: - Synchronization is considered as the complete coincidence of the states of individual systems. Such a regime can result from an interaction between systems or subsystems, as well as from the influence of external noisy or regular fields. In this paper, we have performed the design and numerical simulation of the synchronization between two identical coupled Rossler circuits and applied to a security system of communication. We have demonstrated in simulations that chaotic systems can be synchronized and this technique can be applied to signal masking communications by using MATLAB and MultiSIM programs. All simulations results performed on Rossler system, verify the applicable of secure communication

Key-Words: - Chaos, Chaotic synchronization, Unidirectional coupling, Rossler circuit, Secure communication, Numerical simulation.

1 Introduction

Chaotic behaviour has been found in many typical iterated maps such as the logistic map [1] and Henon map etc[2]. In various electronic circuit systems, including the Duffing oscillator [3], Double-Bell circuit [4] and hyper chaotic Rossler system [5]. Chaos has been widely applied to many scientific disciplines: biology [6], ecology [7], financial markets [8], psychology [9] and robotics [10].

One of the pioneers of synchronization is probably the Dutch scientist Christiaan Huygens. In the 17th century he described an observation of two pendulum clocks, both attached to the same beam that was supported by two chairs, that always end up swinging in opposite direction independent of their starting positions, Even when he applied a disturbance the two clocks showed *anti-phase* synchronized motion within half an hour [11-12].

Besides synchronization of pendulum clocks, a vast number of examples of synchronization of coupled oscillators can be found in nature, especially amongst living animals. Great examples are the simultaneous chirping of crickets and the synchronous flashing of fireflies on banks of rivers in Malaysia, Thailand and New Guinea. With this flashing in unison male fireflies try to attract female species on the other side of the river. Synchronization does also occur in brain dynamics where individual neurons are firing their action potentials at the same time [11].

Synchronization of chaotic oscillators in particular became popular when Pecora and Carroll published their observations of synchronization in unidirectionally coupled chaotic systems [13]. Their results were remarkable since chaos can be seen as a form of instability while synchronization implies stability of the error dynamics [12]. The research in synchronization of couple chaotic circuits is carried out intensively and some interesting applications such as cryptography, communications with chaos have come out of that research [14].

In this paper, a simple electronic system of two coupled circuits in the development scheme of chaos-based secure communication system has been used. First, we examine separately each oscillator circuit to study the dynamic behaviour when varying one parameter, which it has been done before. Furthermore, the unidirectional coupling method is applied to synchronize Rossler circuit. Finally, chaotic masking communication circuits and their simulations of the Rossler circuit are realized also MATLAB and MultiSIM.

2 Mathematical Model of Chaotic Rossler Circuit

One of the most well-known autonomous nonlinear systems [15-18]. The one nonlinearity in the circuit is a piecewise linear function made from op amp U4A with diode, 3 resistors and a diode.

The Rossler electronic circuits are describe by the following equations [15]:

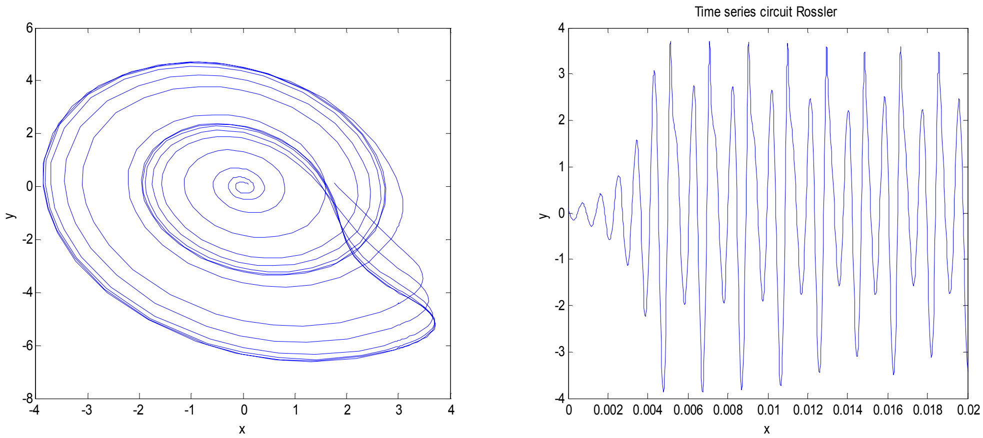
$$\left.\begin{aligned}\frac{dx}{dt} &= -\alpha(\Gamma x + \beta y + \lambda z)\\ \frac{dy}{dt} &= -\alpha(-x - \gamma y + 0.02z)\\ \frac{dz}{dt} &= -\alpha(-g(x) + z)\end{aligned}\right\} \qquad (1)$$

The piecewise linear function $g(x)$is defined by:

$$g(x) = \begin{cases} 0 & x \le 3 \\ \mu(x-3) & x > 3 \end{cases} \qquad (2)$$

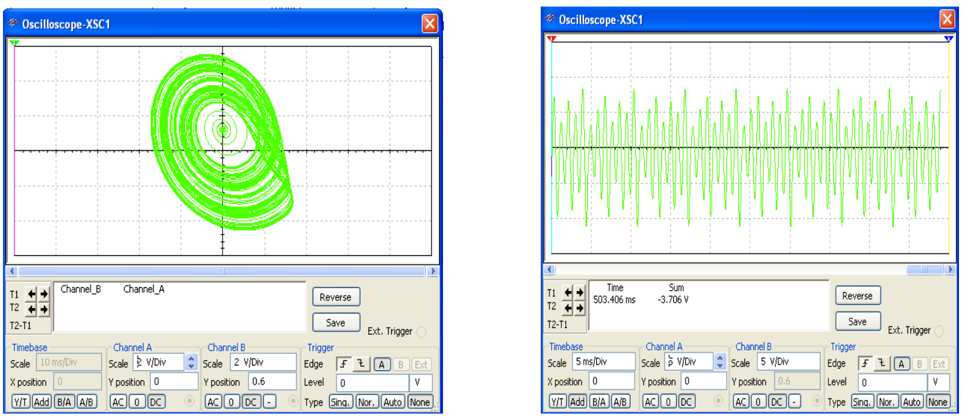
Where time factor α is $10^4\ s^{-1}$, Γ is 0.05, β is 0.5, λ is 1, μ is 15 and the circuit contains a variable resistor that can be used to change the value of γ. The relation between the value R_0 of the variable resistor and γ is R/R_0, with R=10kΩ. R_0=R_6 is a control parameter which exhibit bifurcation and chaotic dynamics.

The complete implementation of the Rossler chaotic circuit design using MultiSIM software is shown in Fig. 3. The function of nonlinear resistor as see in Fig. 3, are implemented with the analog operational amplifier. By comparing Fig. 1, and Fig. 2 a good qualitative agreement between the numerical integration of (1) and (2) by using MATLAB, and the circuits simulation by using MultiSIM, can be concluded.



(a) Phase Portrait of y versus x (b) Time-series of signal y

Fig.1 Numerical simulation results for Rc = 50 kΩ, with MATLAB



(a) Phase Portrait of y versus x (b) Time-series of signal y

Fig.2 Numerical simulation results for Rc = 50 kΩ, with MultiSIM

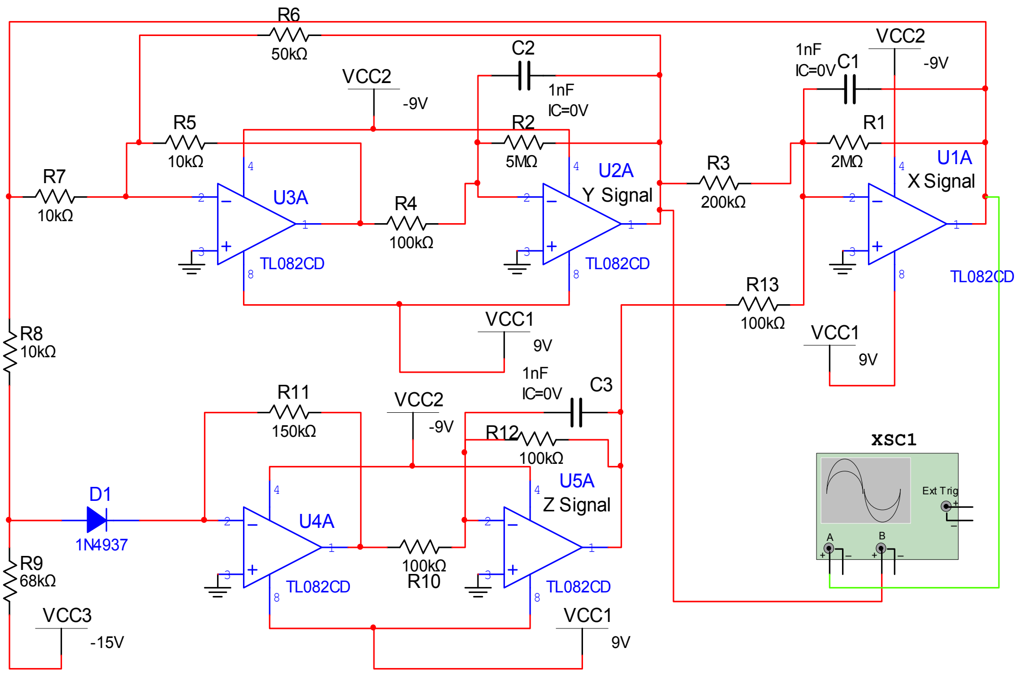


Fig. 3. Schematic of the proposed Rossler circuit

3 Unidirectional Chaotic Synchronization and Circuit's Analysis

There are two main forms of coupling. In the case of unidirectional or Master-Slave scheme, the master is the guide or reference system and the slave is driven system which is dependent on the master. In the case of bidirectional coupling two systems interact and are coupled with each other creating a mutual synchronization.

In this work, the overall system consists of two subsystems coupled by a configuration master-slave type. This implies that the slave system behaviour depends on the behaviour of the master, while the latter is not influenced by the behaviour of the slave system. As a result, the slave system is forced to follow the dynamics (or a specific function of the dynamics) of the master. In other words, when evolution of one of the two systems is not altered by coupling the resulting configuration is a unidirectional coupling.

Based on systems (3) and (4), a master systems in function of (x_1,y_1,z_1) and a slave systems in function of (x_2,y_2,z_2), can be designed. The slave systems, once it is coupled, it will be in function of (x_2, y_2, z_2, x_1). The state equations that describe the systems master and slave, both of them coupled, are

$$\left.\begin{aligned} &\textit{Master}\\ &\frac{dx_1}{dt} = -\alpha\left(\Gamma x_1 + \beta y_1 + \lambda z_1\right)\\ &\frac{dy_1}{dt} = -\alpha\left(-x_1 - \gamma y_1 + 0.02\, z_1\right)\\ &\frac{dz_1}{dt} = -\alpha\left(-g(x_1) + z_1\right) \end{aligned}\right\} \tag{3}$$

$$g(x_1) = \begin{cases} 0 & x_1 \le 3 \\ \mu(x_1 - 3) & x_1 > 3 \end{cases}$$

$$\left.\begin{aligned} &\textit{Slave}\\ &\frac{dx_2}{dt} = -\alpha(\Gamma x_2 + \beta y_2 + \lambda z_2) + g_c(x_1 - x_2)\\ &\frac{dy_2}{dt} = -\alpha(-x_2 - \gamma y_2 + 0.02 z_2)\\ &\frac{dz_2}{dt} = -\alpha(-g(x_2) + z_2) \end{aligned}\right\} \tag{4}$$

$$g(x_2) = \begin{cases} 0 & x_2 \le 3 \\ \mu(x_2 - 3) & x_2 > 3 \end{cases}$$

where $g_c = 1/Rc.C$ is the coupling strength , $Rc = R_1$ is the variable resistor and C is Capacitance in x signal , (Fig. 5). The asymptotic synchronized situation is defined as:

$$\lim_{t\to\infty} \left| x_1(t) - x_2(t) \right| = 0$$

The dynamic error system is defined as follows:

$$\left.\begin{aligned} e_x &= x_1 - x_2 \\ e_y &= y_1 - y_2 \\ e_z &= z_1 - z_2 \end{aligned}\right\} \quad (5)$$

The time derivative of this error signal is

$$\left.\begin{aligned} \dot{e}_x &= \dot{x}_1 - \dot{x}_2 \\ \dot{e}_y &= \dot{y}_1 - \dot{y}_2 \\ \dot{e}_z &= \dot{z}_1 - \dot{z}_2 \end{aligned}\right\} \quad (6)$$

By substituting (3) and (4) into (6), we have the following error dynamics:

$$\begin{aligned} \dot{e}_x &= -\alpha(\Gamma x_1 + \beta y_1 + \lambda z_1) - (-\alpha(\Gamma x_2 + \beta y_2 + \lambda z_2)) + g_c(x_1 - x_2) \\ &= -\alpha\Gamma x_1 - \alpha\beta y_1 - \alpha\lambda z_1 + \alpha\Gamma x_2 + \alpha\beta y_2 + \alpha\lambda z_2)) + g_c(x_1 - x_2) \\ &= -\alpha\Gamma(x_1 - x_2) - \alpha\beta(y_1 - y_2) - \alpha\lambda(z_1 - z_2) + g_c(x_1 - x_2) \\ &= (-\alpha\Gamma + g_c)e_x - \alpha\beta e_y - \alpha\lambda e_z \end{aligned} \quad (7)$$

$$\begin{aligned} \dot{e}_y &= -\alpha(-x_1 - \gamma y_1 + 0.02 z_1) - (-\alpha(-x_2 - \gamma y_2 + 0.02 z_2)) \\ &= \alpha x_1 + \alpha\gamma y_1 - \alpha 0.02 z_1 - \alpha x_2 - \alpha\gamma y_2 + \alpha 0.02 z_2 \\ &= \alpha e_x + \alpha\gamma e_y - \alpha 0.02 e_z \end{aligned}$$

$$\begin{aligned} \dot{e}_z &= -\alpha(-g(x_1) + z_1) - (-\alpha(-g(x_2) + z_2)) \\ &= \alpha g(x_1) - \alpha z_1 - \alpha g(x_2) + \alpha z_2)) \\ &= -\alpha e_z + \alpha e_g \end{aligned}$$

Where $e_g = g(x_1) - g(x_2)$

With the objective to demonstrate synchronization, we analyze the stability of dynamic error system. Thus, we propose the following candidate function to Lyapunov function

$$V = \frac{1}{2}(e_x^{\ 2} + e_y^{\ 2} + e_z^{\ 2}) \quad (8)$$

Derive equation (8), is obtained:

$$\begin{aligned} \dot{V} &= e_x\dot{e}_x + e_y\dot{e}_y + e_z\dot{e}_z) \\ &= (-\alpha\Gamma + g_c)e_x - \alpha\beta e_y - \alpha\lambda e_z)e_x \\ &\quad + (\alpha e_x + \alpha\gamma e_y - \alpha 0.02 e_z)e_y \\ &\quad (-\alpha e_z + \alpha e_g)e_z \end{aligned}$$

$$\dot{V} = \begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix}^T \begin{bmatrix} \alpha\Gamma - g_c & \alpha\beta & \alpha\lambda \\ -\alpha & -\alpha\gamma & 0.02\alpha \\ -\alpha\mu & 0 & \alpha \end{bmatrix} \begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix}$$

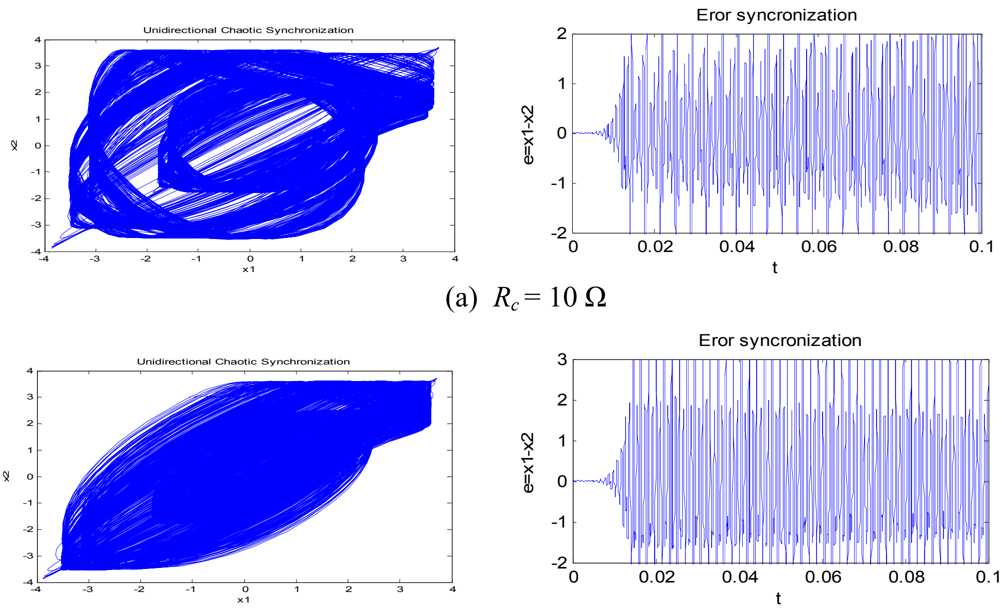
$$\dot{V} = -e^T A e < 0$$

Which is a negative definite function. It means that the dynamic error system (7) is asymptotically stable and, therefore, each one of synchronization errors, e_x, e_y and e_z, tends to zero as t tends to infinite. If synchronization errors tends to zero, then the states from slave system tend to those from master system, which means that they synchronize.

4 Numerical Simulations

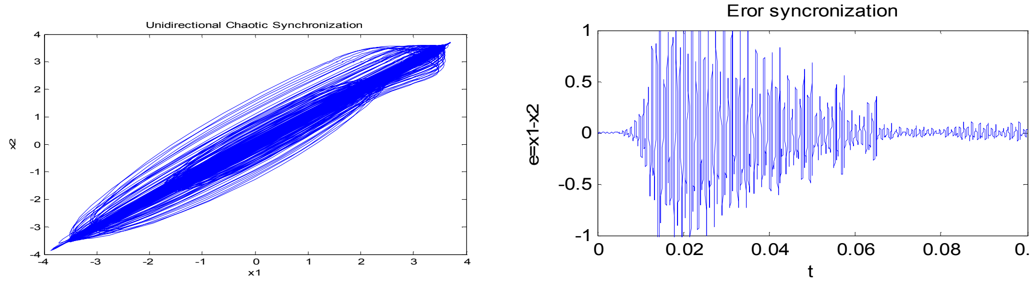
4.1 Simulation in MATLAB

First synchronization between identical systems is considered. We consider coupling through g_c = $1/R_c$.C It can be seen in Fig. 4. That synchronization occurs if R_c does not exceed 10 mΩ.

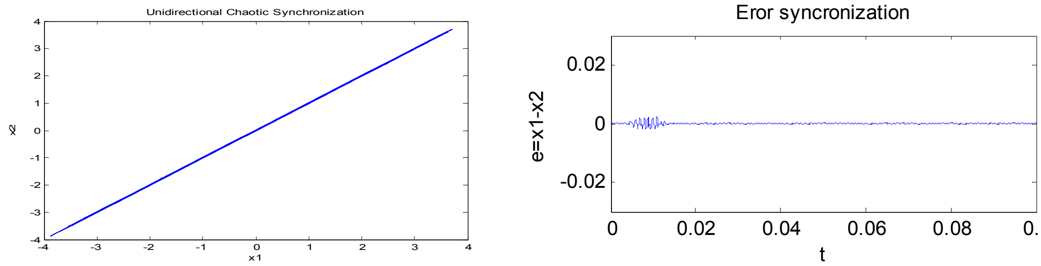


(a) $R_c = 10\ \Omega$

(b) $R_c = 1\ \Omega$



(c) R_c = 100 mΩ



(d) R_c =10 mΩ

Fig. 4. Numerical results of unidirectional coupling, with MATLAB

Synchronization numerically appears for a coupling strength $R_c \leq 10$ mΩ as shown in Fig. 4(d). For different initial condition, if the resistance coupling strength $R_c > 10$ mΩ, the synchronization cannot occur as shown in Fig. 4 (a)-(c). The synchronization occurs when $R_c \leq 10$ mΩ with errors $e_x = x_1 - x_2 \to 0$ which implies the complete synchronization for this resistance coupling strength as shown in Fig. 4 (d).

4.2 Analog Circuit Simulation in MultiSIM

Simulation results show that the two systems synchronize well. Fig. 5 shows the circuit schematic for implementing the unidirectional synchronization of coupled Rossler systems. We use 741 op-amps, appropriate valued resistors, one diode and capacitors for MultiSIM simulations. Fig. 6 also shows MultiSIM simulation results of this circuit.

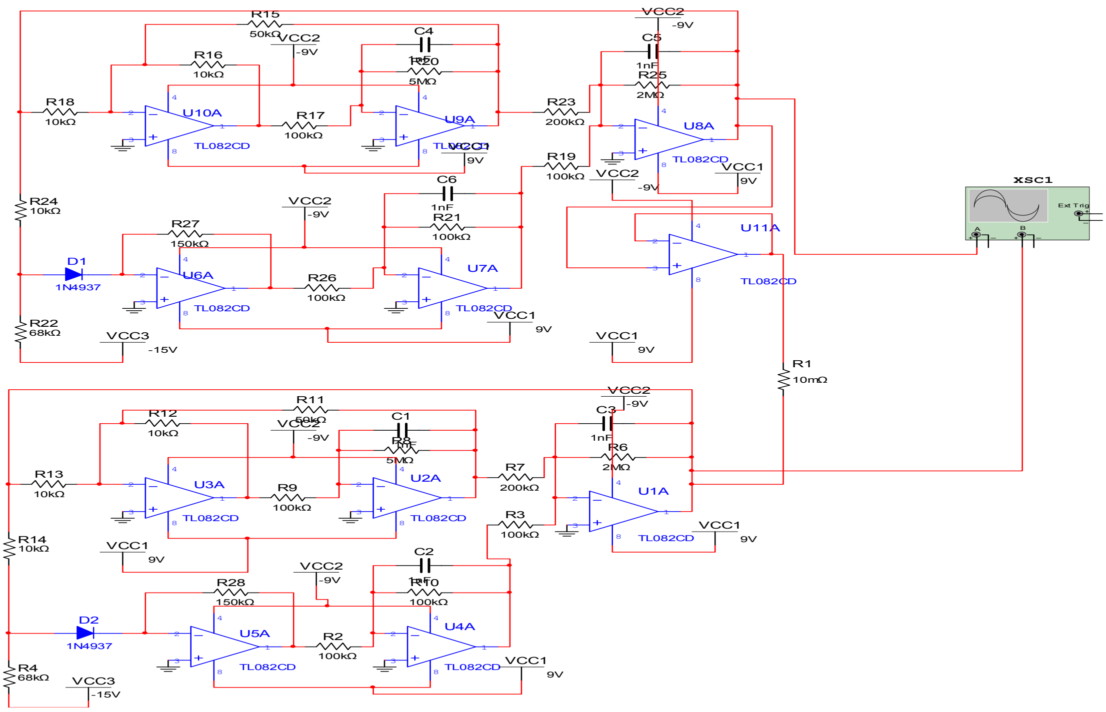
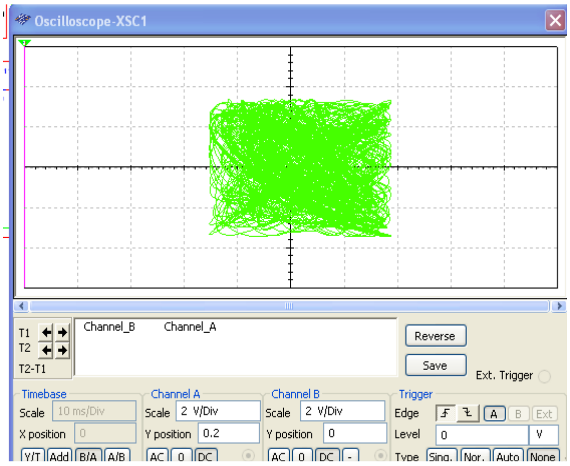
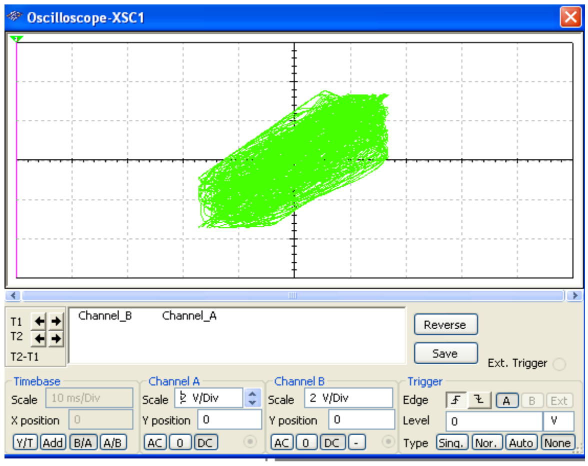
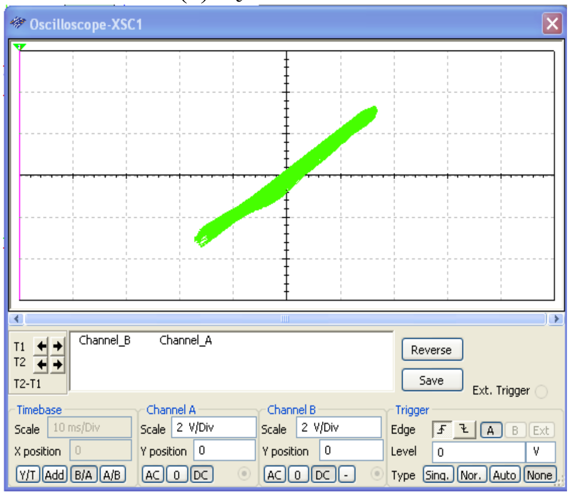
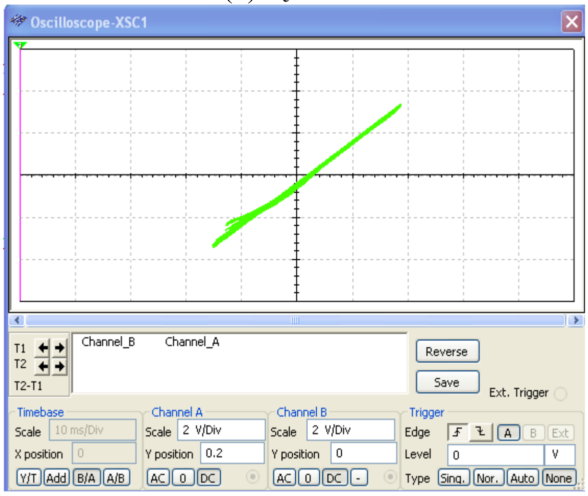


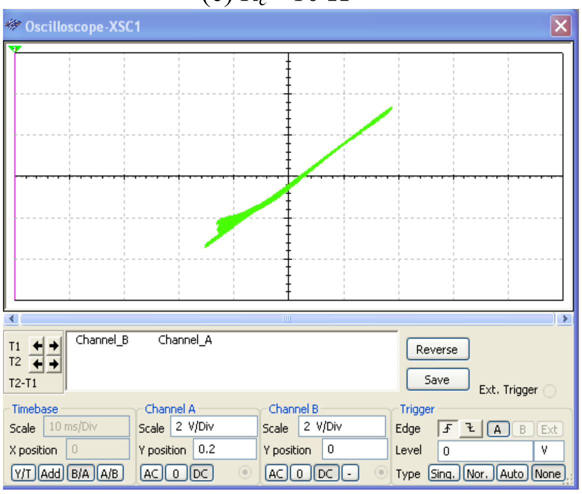
Fig. 5. Unidirectional chaotic synchronization Rossler circuit


(a) R_c=1kΩ


(b) R_c=100 Ω


(c) R_c =10 Ω


(d) R_c =1 Ω


(e) R_c =100 mΩ

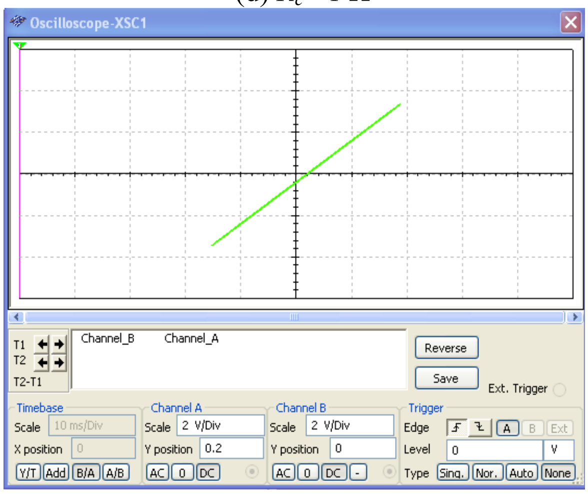

(f) R_c =10 mΩ

Fig. 6. Phase portraits in the case of unidirectional coupling by using MultiSIM

Synchronization with MultiSIM simulation appears for a coupling strength $R_c \leq 10$ mΩ as shown in Fig. 6 (f) For different initial conditions, if the resistance coupling strength $R_c > 10$ mΩ. The synchronization cannot occur as shown in Fig. 6 (a)-(e), the synchronization occurs when $R_c \leq 10$ mΩ with errors $e_x = x_1 - x_2 \to 0$ which implies the complete synchronization for this resistance coupling strength as shown in Fig. 6 (f)

5 Application to Secure Communication Systems

5.1 Simulation in MATLAB

Consider the rescaled Rossler system as the transmitter:

Transmitter

$$\left.\begin{aligned}\frac{dx_1}{dt} &= -\alpha(\Gamma x_1 + \beta y_1 + \lambda z_1)\\ \frac{dy_1}{dt} &= -\alpha(-x_1 - \gamma y_1 + 0.02 z_1)\\ \frac{dz_1}{dt} &= -\alpha(-g(x_1) + z_1)\end{aligned}\right\} \quad (9)$$

$$g(x_1) = \begin{cases} 0 & x_1 \le 3 \\ \mu(x_1 - 3) & x_1 > 3 \end{cases}$$

Receiver

$$\left.\begin{aligned}\frac{dx_2}{dt} &= -\alpha(\Gamma x_2 + \beta y_2 + \lambda z_2) + g_c(x_1 - x_2)\\ \frac{dy_2}{dt} &= -\alpha(-x_2 - \gamma y_2 + 0.02 z_2)\\ \frac{dz_2}{dt} &= -\alpha(-g(x_2) + z_2)\end{aligned}\right\} \quad (10)$$

$$g(x_2) = \begin{cases} 0 & x_2 \le 3 \\ \mu(x_2 - 3) & x_2 > 3 \end{cases}$$

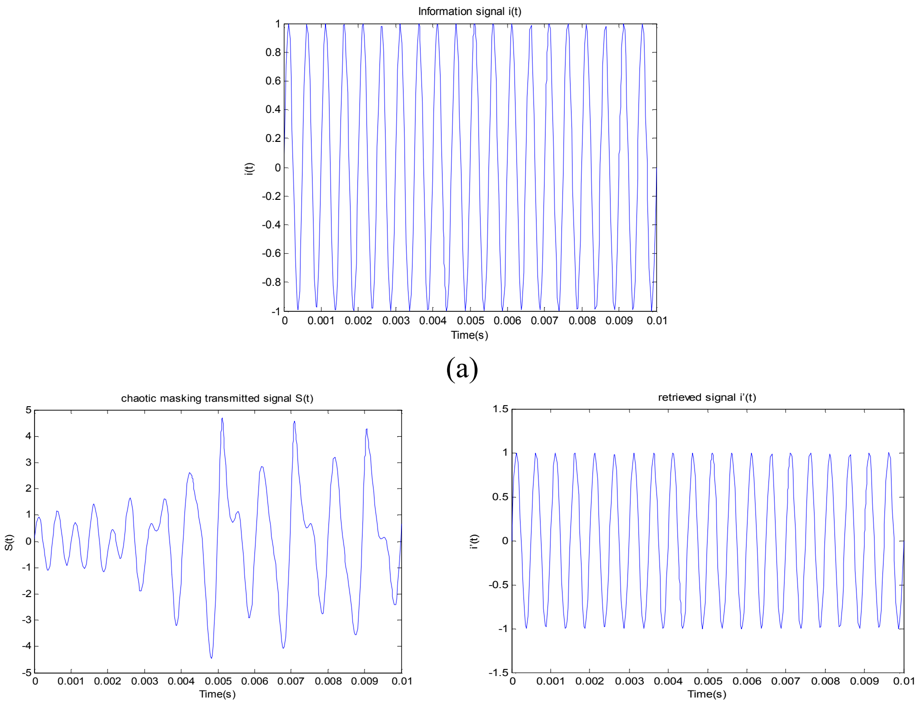
Sinusoidal wave signal recovery. To study the effectiveness of signal masking approach in the Rossler system, we first set the information-bearing signal *i(t)* in the form of sinusoidal wave

$$i(t) = A \sin(2\pi f)t$$

Where A and f are the amplitude and the frequency of the sinusoidal wave signal. Respectively [19].

Due to the fact that output signal can recover input signal, it indicates that it is possible to implement secure communication scheme with the proposed chaotic system. The presence of the chaotic signal between the transmitter and receiver has proposed the use of chaos in secure communication systems [20].

The sinusoidal wave signals of amplitude 1 V and frequency 2 kHz is added to the generated chaotic x signal and the $S(t) = x + i(t)$ is feed into the receiver. The chaotic x signal is regenerated allowing a single subtraction to retrieve the transmitted signal, $[x + i(t)] - xr = i'(t)$, If $x = xr$. Fig. 7 (a-c) shows the MATLAB numerical simulation results for chaotic masking communication



(a)

(b) (c)

Fig. 7 MATLAB simulation of Rossler circuit masking communication system (a) Information signal $i(t)$, (b) Chaotic masking transmitted signal $S(t)$, (c) Retrieved signal $i'(t)$.

5.2 Analog Circuit Simulation

In chaos-based secure communication schemes, information signals are masked or modulated (encrypted) by chaotic signals at the transmitter and the resulting encrypted signals are sent to the corresponding receiver across a public channel (unsafe channel). Perfect chaos synchronization is usually expected to recover the original information signals. In other words, the recovery of the information signals requires the receiver's own copy of the chaotic signals which are synchronized with the transmitter ones. Thus, chaos synchronization is the key technique throughout this whole process [20]. Fig. 8 shows the circuit schematic of implementing the Rossler circuit Chaotic Masking Communication.

MultiSIM simulation results for several different frequencies are shown in Fig. 9. Fig. 9 shows the MultiSIM simulation results for masking signal communication system by varying the input signal's frequency. The red signal describes the wave information signal *i* (t), the green signal describes the transmitted chaotic masking signal *S* (t) and the purple signal describes the retrieved signal *i'*(*t*).

The simulation results shows that circuit autonomous Rossler is an excellent for chaotic masking communication when the frequency information is at intervals of 0.2 kHz – 9kHz. Otherwise, when the frequency information is more than 9 kHz or less than 0.2 kHz, the chaotic masking communication is not occur.

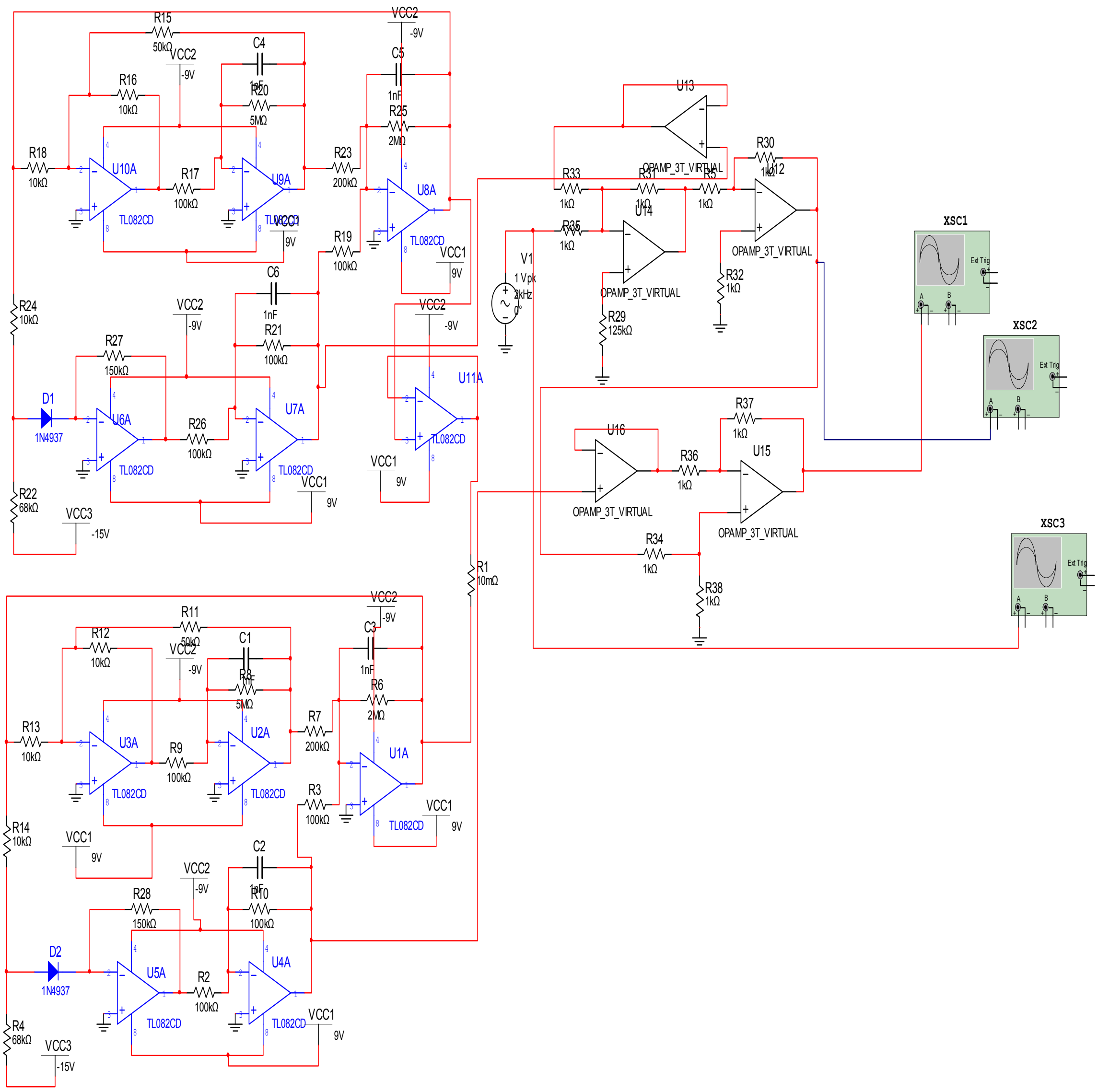
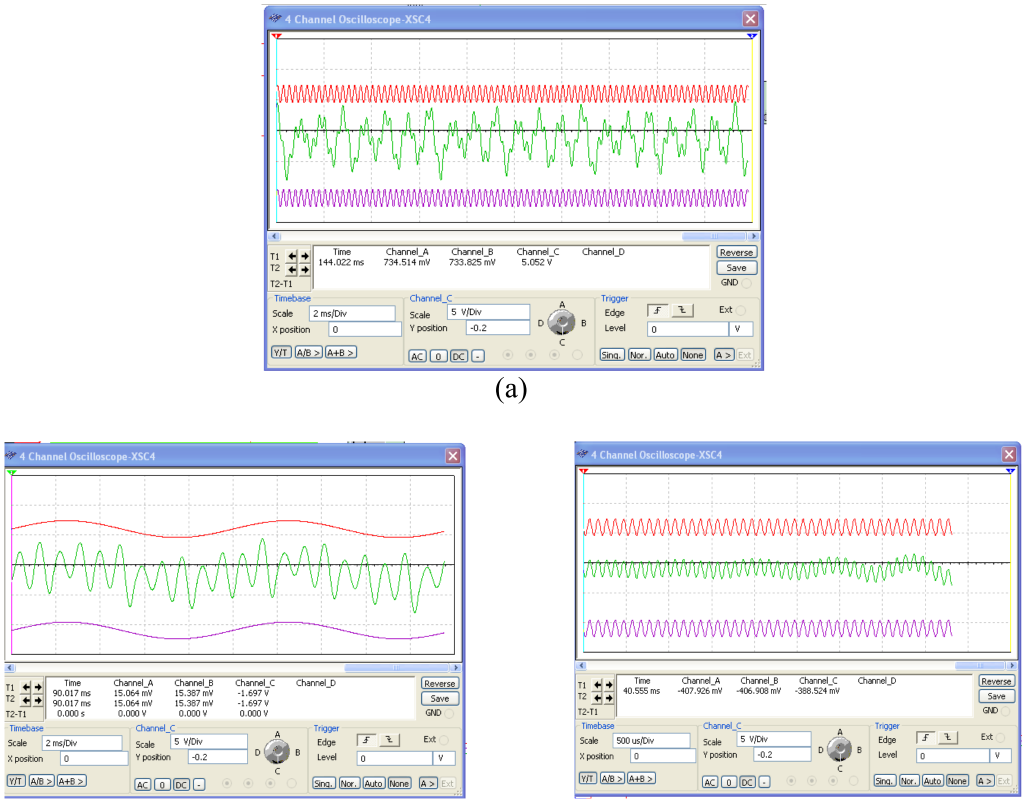


Fig. 8. Rossler circuit masking communication circuit.



(a)

(b) (c)

Fig. 9. MultiSIM outputs of Rossler circuit masking communication systems
(a) Information frequency 4 kHz, (b) Information frequency 0.1 kHz (c) Information frequency 10 kHz

6 Conclusion

We propose a communication scheme for secure communications based on synchronization of chaotic systems. The scheme implies the use of two system variables, the one serves for chaos synchronization and the other is used for signal transmission and recovering. We show that the synchronization error for the novel scheme is smaller when $R_c \leq 10$ mΩ and complete synchronization occurs.

We have demonstrated in simulations that chaotic circuits can be synchronized and applied to secure communication. Chaos synchronization and chaos masking were realized using MultiSIM programs.

In this paper, it has been shown that Rossler circuit can be used in a communication security system at a frequency information interval of 0.2 kHz - 9 kHz. When the frequency information is more than 9 kHz or less than 0.2 kHz. The chaotic masking communication is not occur.

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