Output Gaming Analysis and Chaos Control among Enterprises of Rational Difference in a Two-Level Supply Chain

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Abstract: In supply chain, the output decision making of the manufacturers and the ordering decision making of the distributors are closely connected. This paper studies the cooperation and gaming between manufacturers and distributors of rational difference with a model based on system dynamics and an analysis of the Nash equilibrium point. Numerical simulations show that under the circumstances of information asymmetry, the manufacturers' tendency toward cooperative gaming gives rise to chaos in output decision making, while the boundly rational decision making of the distributors is relatively stable. It is also revealed that the speed of the adjustment coefficient of output decision making exerts great impact on the stability of the system. This paper argues that the application of coefficient control creates a time-lag in the chaos of the manufacturers' output decision making and therefore allows for output adjustment to eliminate the negative effects of chaos.

Key–Words: systematic dynamics, rational difference, chaos control, repeated games

1 Literature review

In the recent years, the evolution of supply chain management has recognized that use tools from game theory and economics to understand, predict, and help managers to make strategic operational decisions in complex multi-agent supply chain systems. There is a broad division of game theory into two approaches: the cooperative and the non-cooperative approach. The cooperative approach suits the current situation better and therefore is of greater potential than the non-cooperative one. Mahesh Nagarajand et al [1] have detailed the evolution of the cooperative game theory and its prospect. Xinyan Zhang and George Q. Huang [2] have constructed a dynamic game model along the supply chain consisting one manufacturer and multiple cooperative suppliers. It is found that a supply chain with cooperative suppliers is more effective by using the lot-for-lot policy and more competitive by accommodating higher product variety. Yugang Yu and George Q. Huang [3] have investigated how a manufacturer and its retailers interact with each other to optimize their product marketing strategies, platform product configuration and inventory policies in a VMI (Vendor Managed Inventory) supply chain. Chenxi Zhou, Ruiqing Zhao and Wansheng Tang [4] have explored a two-echelon supply chain games in a fuzzy environment, and discussed the strategies adopted by the manufacturers who behave as a Stackelberg leader and dominate the supply chain. Expected value models as well as chance-constrained programming models are developed to determine the pricing strategies for the retailer and the manufacturer. M. Esmaeili, Mir-Bahador Aryanezhad, and P. Zeephongsekul [5] have considered separately the case when the seller is the leader and also when the buyer is the leader. Pareto efficient solutions will be provided for the cooperative game model. Numerical examples presented in this paper, including sensitivity analysis of some key parameters, will compare the results between different models considered. Yingxue Zhao, Shouyang Wang, T.C.E. Cheng, Xiaoqi Yang, Zhimin Huang [6] have discussed the coordination of supply chains by option contracts with a cooperative game theory approach. It is found that compared with the benchmark based on the wholesale price mechanism, option contracts can coordinate the supply chain and achieve Pareto-improvement. Zhou Min, Deng Feiqi, Wu Sai [7] have constructed a cooperative game model to analyze the cooperation among the manufacturers in the cluster supply chain. They also provide the development of cluster supply chains with an effective forecasting and Pareto optimizing method.

Focus on the cooperative game among the manufacturers and distributors, the researches reviewed above are based on the premise that the decision making of the two types of players is entirely rational. However, in the reality of economic life, they are unlikely to have complete information. Therefore, their decision making is not entirely rational, but boundly rational [8-13]. This paper constructs a cooperative game model of manufacturers and distributors of rational difference and analyzes the Nash equilibrium point and its stability. It then discusses the impact of the adjustment coefficient of output decision making on the stability of the system. Numerical simulations are conducted to reveal the dynamic behavior of the system, and chaos control is exercised on the chaotic phenomenon in the system.

2 Model constructions

In a two-level supply chain, the cooperative game consists of two phases: the ordering decision making of the distributors and the output decision making of the manufacturers. Assume the manufacturers are all boundly rational manufacturers and their output decision making is based on the ordering volume of the downstream distributors. And the distributors are all thin rational distributors and their ordering decision making is based on marginal profit. For the convenience of the study, assume the output of the manufacturers equals the ordering volume of the distributors.

$q_{i,t}$ denotes the output decision making of enterprise i in period t.

$P_{i,t} = \alpha_i + \beta_i q_{i,t} - \gamma_i q_{i,t}^2$

$P_{i,t}$ denotes the inverse demand function for enterprise i in period t.

$C_{i,t} = a_i + b_i q_{i,t} + c_i q_{i,t}^2$

$C_{i,t}$ denotes the cost function for enterprise i in period t.

$\pi_{i,t} = P_{i,t} q_{i,t} - C_{i,t}$

$\pi_{i,t}$ denotes the pro?t of enterprise i in period t.

Due to the bounded rationality and information asymmetry among the enterprises along the supply chain as well as the excessively high cost involved in attaining complete information, when it comes to output decision making, the enterprises tend to increase the output until the maximum pro?t for marginal product is attained. Therefore,

$$\frac{\partial \pi_{i,t}}{\partial q_{i,t}} = -3\gamma_i q_{i,t}^2 + 2(\beta_i - c_i) q_{i,t} + \alpha_i - b_i \tag{1}$$

Since the decision making process is long, repetitive and dynamic, it is characterized by adaptability and long-term memory effect. In most cases, it is rationally bounded. When the enterprises realize that the results achieved in period t is satisfactory, they will follow the same strategy in period $t+1$. The aim of the enterprises is profit maximization. Marginal product is one of the strategies that they adopt in the game. If the marginal product in period t is positive, then they will continue their output adjustment strategy in period $t+1$. The model can be constructed as follow:

$$q_{i,t+1} = q_{i,t} + k_i q_{i,t} \frac{\partial \pi_{i,t}}{\partial q_{i,t}} \tag{2}$$

where k_i is the output adjustment coefficient for enterprise i which reflects the sensitivity of the enterprise for marginal profit. The more sensitive the enterprise, the greater k_i. Thus, the manufacturer can decide its output with an estimate of the marginal profit, even if it does not have the complete information on the demand function.

As the decision making of the enterprises along the two-level supply chain is not entirely rational, the output decision making reaches the point of Nash Equilibrium only after multiple gaming. The output adjustment coefficient k_i is of great importance to the system. It can be controlled by the enterprises through internal and external measures. Due to the limitation of the space, the discussion about the effect of output adjustment coefficient k_i on the model will be detailed in another research paper.

Then, the dynamic adjustment of the output of the upstream enterprise can be written as follow:

$$\begin{aligned} q_{1,t+1} = q_{1,t} + k_1 q_{1,t}(&-3\gamma_1 q_{1,t}^2 + 2(\beta_1 - c_1) q_{1,t} \\ &+\alpha_1 - b_1) \end{aligned} \tag{3}$$

Likewise, the dynamic adjustment of the output of enterprise can be written as follow:

$$\begin{aligned} q_{i,t+1} = q_{i,t} + k_i q_{i,t}(&-3\gamma_i q_{i,t}^2 + 2(\beta_i - c_i) q_{i,t} \\ &+\alpha_i - b_i) \end{aligned} \tag{4}$$

Hence, the output game model can be represented by an n-dimensional nonlinear map.

$$\begin{cases} {x_1}' = x_1 + k_1x_1[-3\gamma_1{x_1}^2 + 2(\beta_1 - c_1)x_1 \\ +\alpha_1 - b_1] \\ {x_2}' = x_2 + k_2x_2[-3\gamma_2{x_2}^2 + 2(\beta_2 - c_2)x_2 \\ +\alpha_2 - b_2] \\ \vdots \\ {x_i}' = x_i + k_ix_i[-3\gamma_i{x_i}^2 + 2(\beta_i - c_i)x_i \\ +\alpha_i - b_i] \\ \vdots \\ {x_n}' = x_n + k_nx_n[-3\gamma_n{x_n}^2 + 2(\beta_n - c_n)x_n \\ +\alpha_n - b_n] \end{cases} \tag{5}$$

We can interpret this multi-dimensional discrete dynamical system through a study on the first map. We consider two types of players along the supply chain: the manufacturers and the distributors. The distributors make adjustment on their ordering strategies in response to the changes in the retailers' ordersand the manufacturers to the changes in the distributors' orders. Then a nonlinear dynamic model is constructed as follows:

Let $q_{1,t} = x, q_{2,t} = y, q_{1,t+1} = x', q_{2,t+1} = y'$, then:

$$\begin{cases} x' = x + k_1y(-3\gamma_1x^2 + 2(\beta_1 - c_1)x + \alpha_1 - b_1) \\ y' = y + k_2y(-3\gamma_2y^2 + 2(\beta_2 - c_2)y + \alpha_2 - b_2) \end{cases} \tag{6}$$

In the dynamic output game model, parameters $\alpha_i, \beta_i, \gamma_i, a_i, b_i, c_i$ are relatively stable, whereas the output adjustment coefficients k_1, k_2 are adjustable. For the convenience of comparison and study, fix the parameters as follows:

$\alpha_1 = 5, \beta_1 = 5, \gamma_1 = 1, b_1 = 5, c_1 = 0.4, \alpha_2 = 5, \beta_2 = 0.5, \gamma_2 = 1, b_2 = 0.4, c_2 = 0.3$,Then this dynamic output game model can be written as follows:

$$\begin{cases} x' = x + k_1y(-3x^2 + 0.2x + 4.5) \\ y' = y + k_2y(-3y^2 + 0.4y + 4.6) \end{cases} \tag{7}$$

3 Model analysis

The fixed points in the model satisfy the following algebraic equations:

$$\begin{cases} k_1x_1[-3\gamma_1x_1^2 + 2(\beta_1 - c_1)x_1 + \alpha_1 - b_1] = 0 \\ k_2x_2[-3\gamma_2x_2^2 + 2(\beta_2 - c_2)x_2 + \alpha_2 - b_2] = 0 \\ \vdots \\ k_ix_i[-3\gamma_ix_i^2 + 2(\beta_i - c_i)x_i + \alpha_i - b_i] = 0 \\ \vdots \\ k_nx_n[-3\gamma_nx_n^2 + 2(\beta_n - c_n)x_n + \alpha_n - b_n] = 0 \end{cases} \tag{8}$$

Note that the solution of the algebraic equations is independent of parameters $k_1, k_2 \ldots, k_n$. For models in economics, only non-negative equilibrium solution makes sense. By simple computation of the above algebraic system, an equilibrium point is found as follows:

$$\Omega_1 = \sqrt{(\beta_1 - c_1)^2 + 3\gamma_1(\alpha_1 - b_1)}$$

$$\Omega_2 = \sqrt{(\beta_2 - c_2)^2 + 3\gamma_2(\alpha_2 - b_2)}$$

$$\Omega_n = \sqrt{(\beta_n - c_n)^2 + 3\gamma_n(\alpha_n - b_n)}$$

The Jacobian matrix at Nash equilibrium point can be represented by the following form:

$$J(\Omega) = \begin{bmatrix} 1 + k_1\omega_1 & 0 & \cdots & 0 \\ 0 & 1 + k_2\omega_2 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 + k_n\omega_n \end{bmatrix}$$

where $\omega_i = -9\gamma_ix^2 + 4(\beta_i - c_i)x_i + \alpha_i - b_i$,The above is a diagonal matrix and its values can be expressed as follows: $\lambda_1 = 1 - k_1\omega_1, \lambda_2 = 1 - k_2\omega_2, \cdots, \lambda_n = 1 - k_n\omega_n$,Consider the manufacturers and the distributors, and the algebraic equations can be written as follows:

$$\begin{cases} k_1y[-3\gamma_1x^2 + 2(\beta_1 - c_1)x + \alpha_1 - b_1] = 0 \\ k_2y[-3\gamma_2y^2 + 2(\beta_2 - c_2)y + \alpha_2 - b_2] = 0 \end{cases} \tag{9}$$

Computation of the above algebraic system shows that $p_1(0, 0)$, $p_2(1.2585, 1.3067)$, $p_3(-1.1919, -1.1734)$, $p_4(1.2585, -1.1734)$, $p_5(-1.1919, 1.3067)$. It is obvious that the equilibrium does not depend on the parameters k_1, k_2. When the output is negative, the system is meaningless.p_2 is the Nash equilibrium point of the system. The Jacobian matrix at p_2 has the following form:

$$J = \begin{bmatrix} 1 - 9.6056k_1 & 0.0002k_1 \\ 9.7221k_2 & 1 - 9.7218k_2 \end{bmatrix}$$

The characteristic equation is $|\lambda E - J| = \lambda^2 + A\lambda + B = 0$, where $A = -2 + 9.6056k_1 + 9.7218k_2$, $B = 1 - 9.6056k_1 - 9.7218k_2 + 93.3818k_1k_2$. An analysis of this characteristic equation reveals that to make the equilibrium point stable, the moduli of the roots of the characteristic equation should be both less than 1, i.e.$-A - 1 < B < 1$, Make $k_2 = 0.03$ and $0 \leq k_1 \leq 1$, then $k_1 \in (0, 0.2082)$, and the system is stable.

In the dynamic output game model, parameters $\alpha_i, \beta_i, \gamma_i, a_i, b_i, c_i$ are relatively fixed whereas the output adjustment coefficients k_1, k_2 are not. For the convenience of the study, make the parameters fixed as follows:

$\alpha_1 = 5, \beta_1 = 5, \gamma_1 = 1, b_1 = 5, c_1 = 0.4, \alpha_2 = 5, \beta_2 = 0.5, \gamma_2 = 1, b_2 = 0.4, c_2 = 0.3$,Then the dynamic output game model can be written as:

$$\begin{cases} x' = x + k_1 y[-3x^2 + 0.2x + 4.5] \\ y' = y + k_2 y[-3y^2 + 0.4y + 4.6] \end{cases} \tag{10}$$

4 Numerical simulations

For a better understanding of the dynamics in the system, numerical simulations are conducted. Take into consideration one instance, that is coefficient k_1 is free and coefficient k_2 is fixed. When $k_2 = 0.03$, simulate the impact of the change in the manufacturers' output adjustment speed k_1 on the system's complexity change. The changes in the output decision making of the manufacturers and the distributors is first illustrated by bifurcation diagrams. Then the relationship between the complexity of the system and k_1 is studied by using Lyapunov exponents, attractors, sequence diagrams and sensitive dependence on initial conditions. The simulation results are interpreted with a perspective from supply chain. Figure 1 and Figure 2 are the bifurcation diagrams showing that when the manufacturers' output adjustment coefficient $k_1 \in (0, 0.2082)$, the output decision making is stable at Nash equilibrium point $p_2(1.2585, 1.3067, 1.3557)$. However, with the increase in k_1, when the first bifurcation occurs at $k_1 = 0.2082$ and the second at $k_1 = 0.2661$, the third at $k_1 = 0.2785, \ldots$,, chaos occurs. In contrast, the ordering volume of the distributors remains stable. Therefore, it can be concluded that the greater the speed of output adjustment becomes, the faster the output decision making responds to the change in market demand, and the more likely that chaos will occur in the market. Complexity will be caused in the manufacturer's output and production management. In contrast, the distributors are stable as they are not involved in production.

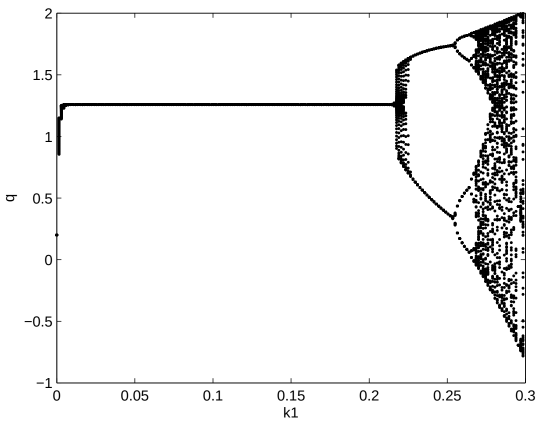

Figure 1: Bifurcation diagram of manufacturer x

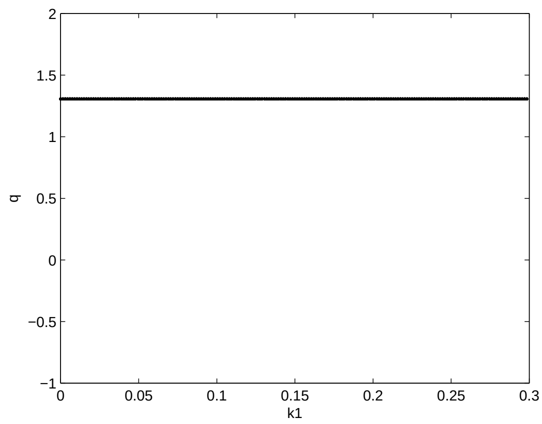

Figure 2: Bifurcation diagram of distributor y

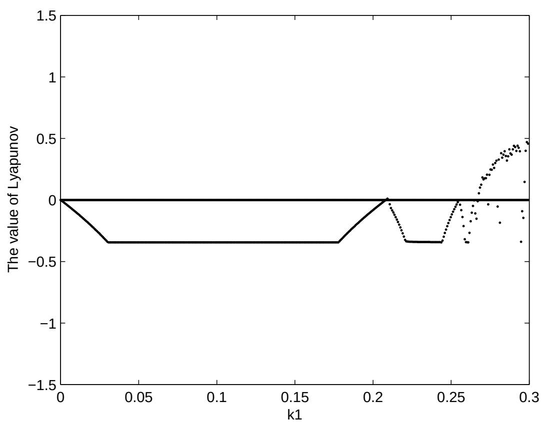

Figure 3: Lyapunov exponents of manufacturer x

The Lyapunov exponent is one of the most useful quantitative measures of chaos. A positive largest Lyapunov exponent indicates chaos. For detecting the type of dynamics, Lyapunov exponents are also

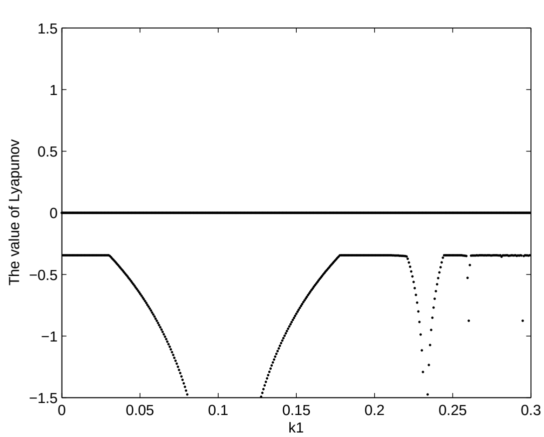

Figure 4: Lyapunov exponents of distributor y

calculated in [14]. According to Lyapunov exponent Figure 3 and Figure 4, the system is stable when $k_1 \in (0, 0.2082)$. When $k_1 < 0.2785$, the system undergoes period doubling bifurcation, and the oscillation is periodic. Therefore, the maximum Lyapunov exponent always has a value less than zero. It becomes equal to zero exactly at the bifurcation point. When $k_1 > 0.2785$, the majority Lyapunov exponents have a value greater than zero, indicating that the system is chaotic. The maximum Lyapunov exponent is less than zero only within a narrow range which corresponds to a periodic window in chaos. The Lyapunov exponents of distributor y always have a value less than zero, implying that the system is not chaotic. The analysis on the Lyapunov exponents is in conformity to the bifurcation diagrams. Figure 5 and Figure

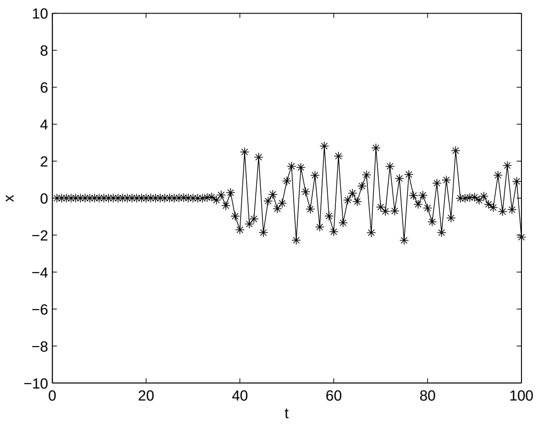

Figure 5: Sensitive dependency on initial conditions of output decision making of manufacturer x

6 are the sequence diagrams showing the changes in the manufacturers' output decision making and distributors' ordering decision making with time passing

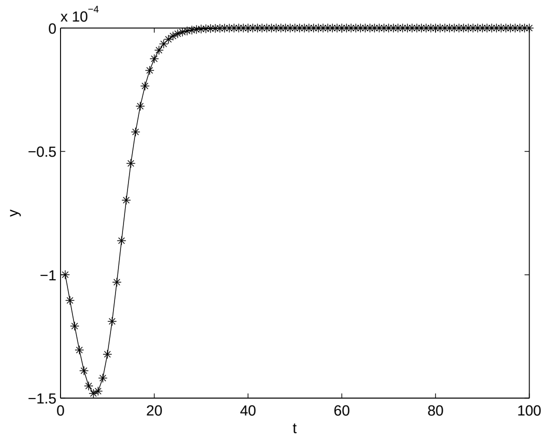

Figure 6: Sensitive dependency on initial conditions of output decision making of order volum of distributor y

by. As the chaotic motion is sensitively dependent on the initial conditions, trivial differences in the initial conditions will lead to the departure of two adjacent trajectories. Replace the initial conditions (0.2,0.4) with (0.2,0.4001), as shown in Figure 3. It is found that the difference at the initial stage is close to zero. With the passing of time, the differences in the manufacturers' output decision making increase after 40 iterations, leading the adjacent trajectories into different domains of attraction. In contrast, the distributors in Figure 4 tend to stabilize after 30 iterations. These further support the analysis made above.

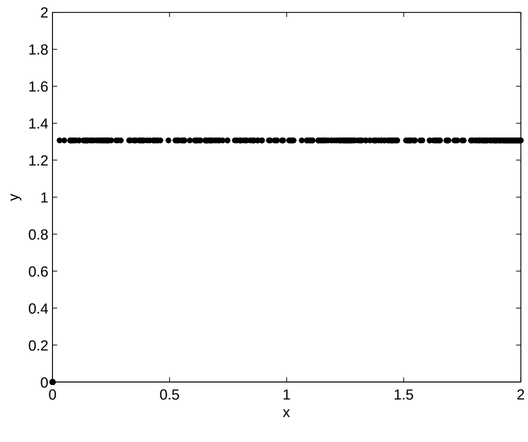

Figure 7: attractors of the system when $k_1 = 0.3, k_2 = 0.03$

The chaotic attractors in Figure 7 are peculiar in that they form a dashed line, which is caused by the fact that in the system only the manufacturers' output decision making is in chaos, whereas the distributors' ordering decision making is stable. Figure 8 is the

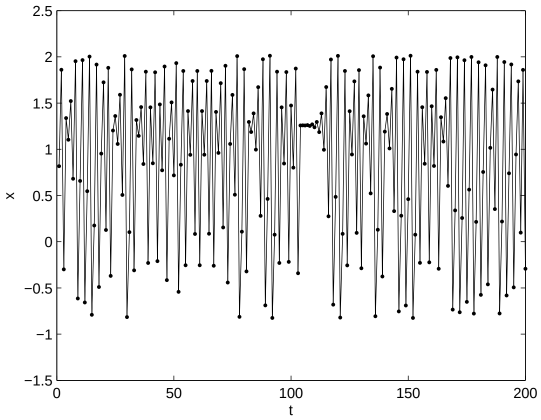

Figure 8: sequence diagrams of the changes of manufacturer x

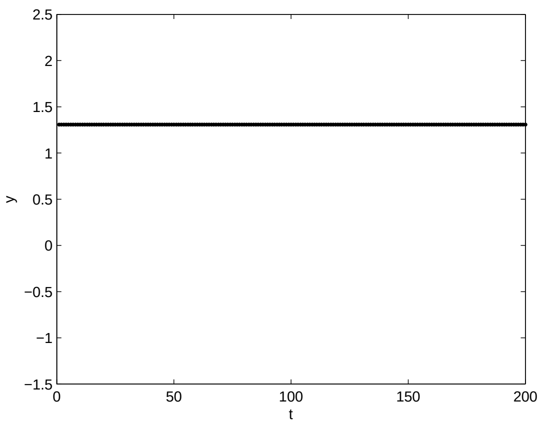

Figure 9: sequence diagrams of the changes of distributor y

sequence diagram of manufacturer x and distributor y. The two figures support the analysis made above: manufacturers' output decision making is in chaos, whereas the distributors' ordering decision making is stable.

The analysis shows that the cooperation among enterprises of rational difference in a two-level supply chain presents dynamical complexities after multiple games. When chaos appears in cooperation, even minor changes in the initial parameters along the two-level supply chain will exert great impact on the final game result. This will make the cooperation between the manufacturers and the distributors unpredictable and therefore exerts negative impact on the manufacturers' production and the distributors' sales. Such situation should be brought under control.

Assume that the initial condition of the model is the even distribution of $[0, 1] \times [0, 1]$, and every manufacturer can decide his output between 0 and max value. The output adjustment coefficient of these three manufacturers reaches a stable density state and is fixed at an equilibrium $(0.8457, 0.8377)$, as is shown in Fig.10, after multiple games when it satisfies the condition $k_1 = k_2 = 0.26$. When the output adjustment coefficient satisfies the condition $k_1 = k_2 = 0.266$, the density distribution of these three manufacturers circulates between $(0.7961, 0.7865)$ and $(0.8150, 0.8148)$ after 2,000 games, which exhibits double period of density, as is shown in Fig.11. And when the output adjustment coefficient satisfies the condition $k_1 = k_2 = 0.2673$, the output's density distribution is no longer stable, and exhibits quadraple period of density, as is shown in Fig.12. If we continue increasing the adjustment coefficient of the output till $k_1 = k_2 = 0.26736$, then we can find the octo-periodic phenomenon, which means that we can

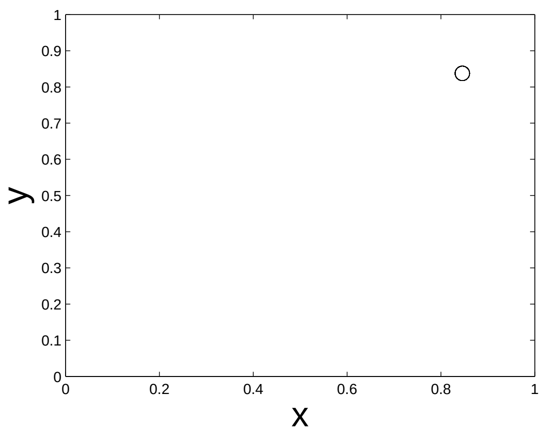

Figure 10: Output density distribution after 2,000 games when $k_1 = k_2 = 0.26$

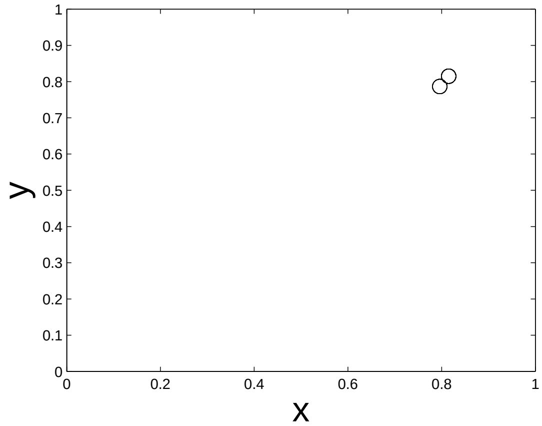

Figure 11: Output density distribution after 2,000 games when $k_1 = k_2 = 0.266$

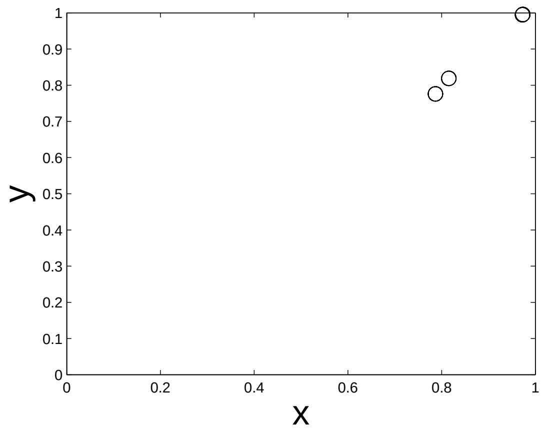

Figure 12: Output density distribution after 2,000 games when $k_1 = k_2 = 0.2673$

find eight nash equilibrium solution, and according to the conclusion of Li-York, at the third period of this phenomenon, the chaos appears and all of the system is in chaos, as shown in Fig.13. And at this time, even a tiny change of the adjustment coefficient of the output can cause a huge difference of the distribution of the density. And with the increasing of the adjustment coefficient of the output to $k_1 = k_2 = 0.26737$, the density's sixteen-periodic phenomenon and the whole system is in the condition of chaos, as shown in Fig.14. The density distribution diagrams show that

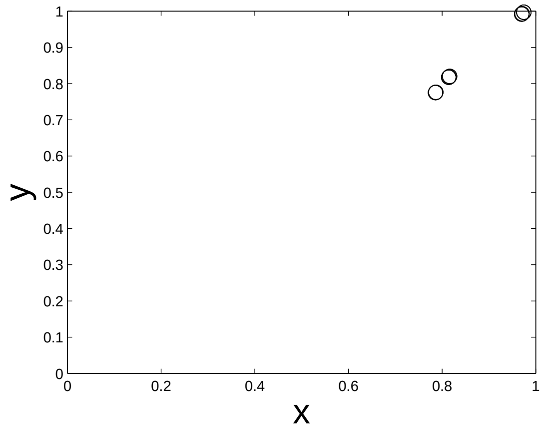

Figure 13: Output density distribution after 2,000 games when $k_1 = k_2 = 0.26736$

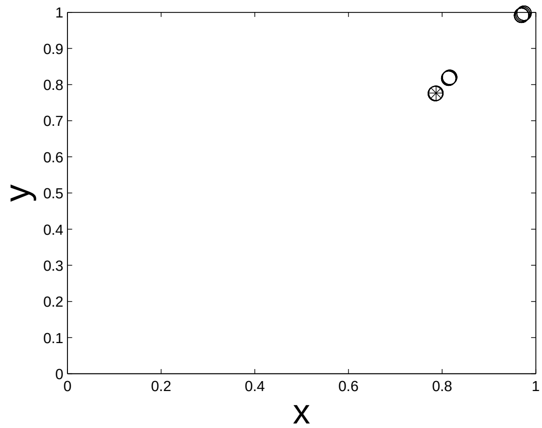

Figure 14: Output density distribution after 2,000 games when $k_1 = k_2 = 0.26737$

when the decision making is stable, the manufacturers fix the output at 0.8457, and the distributors fix the ordering volume at 0.8377. The output of the manufacturers is always larger than the ordering volume of the distributors which is larger than that of the retailers. This is result of reverse logistics from retailers to distributors and then to manufacturers.

5 Chaos control

Chaos control aims at altering the chaotic motions in non-linear dynamical systems to display periodic dynamics. Methods of chaos control include OGY method [8], continuous feedback control method [9], variable structure control and impulse control [10], and etc. The ground that these methods share in common is that the Lyapunov exponent is adjusted from positive to negative so that stability is achieved in the originally unstable system. In other words, for the purpose of chaos control, control signal is fed to the system for bifurcation control so that the chaos is eliminated or delayed.

Assume the manufacturers and distributors in the supply chain are boundly rational decision makers. Feed control signal μ to the system, then the response of enterprise at level i satisfies the condition $\mu_i \in [0,1]$. Assume all the enterprises share the same control rule, then the output decision making function of enterprise at level i can be written as:

$$\begin{aligned} x_i' = \mu_i(x_i + k_i x_i(-3\gamma_i {x_i}^2 + 2(\beta_i - c_i)x_i \\ +\alpha_i - b_i)) + (1-\mu_i)x_i \end{aligned} \tag{11}$$

Therefore, the output game model of the system-controlled enterprise can be written as:

$$\left\{\begin{aligned} x_1' &= \mu_1(x_1 + k_1 x_1(-3\gamma_1 x_1^2 + 2(\beta_1 - c_1)x_1 \\ &\quad +\alpha_1 - b_1)) + (1-\mu_1)x_1 \\ x_2' &= \mu_2(x_2 + k_2 x_2(-3\gamma_2 x_2^2 + 2(\beta_2 - c_2)x_2 \\ &\quad +\alpha_2 - b_2)) + (1-\mu_2)x_2 \\ &\vdots \\ x_i' &= \mu_i(x_i + k_i x_i(-3\gamma_i x_i^2 + 2(\beta_i - c_i)x_i \\ &\quad +\alpha_i - b_i)) + (1-\mu_i)x_i \\ &\vdots \\ x_n' &= \mu_n(x_n + k_n x_n(-3\gamma_n x_n^2 + 2(\beta_n - c_n)x_n \\ &\quad +\alpha_n - b_n)) + (1-\mu_n)x_n \end{aligned}\right. \tag{12}$$

The analysis on the equilibrium point of the model is not elaborated here, because it follows the same train of thought. As the solution for the mathematical formula group is independent of parameter , the Nash equilibrium solution is fixed.

When $\mu = 1$, the system degenerates into the original system. Select the appropriate adjustment parameter so that the equilibrium will remain stable in a scale larger than the original system and the bifurcation will be delayed.

Feed in the parameter mentioned above, and the controlled model can be written as:

$$\left\{\begin{aligned} x' &= \mu(x + k_1 y(-3x^2 + 0.2x + 4.5)) + (1-\mu)x \\ y' &= \mu(y + k_2 y(-3y^2 + 0.4y + 4.6)) + (1-\mu)y \end{aligned}\right. \tag{13}$$

Numerical simulations and analysis on the model are provided below by using Lyapunov exponents, attractors, sequence diagrams and sensitive dependence on initial conditions. Chaos control parameter μ is adjusted for the study of the control effect on the chaotic output decision making of the manufacturers. As the distributors' ordering decision making is stable, no control is exercised on it.

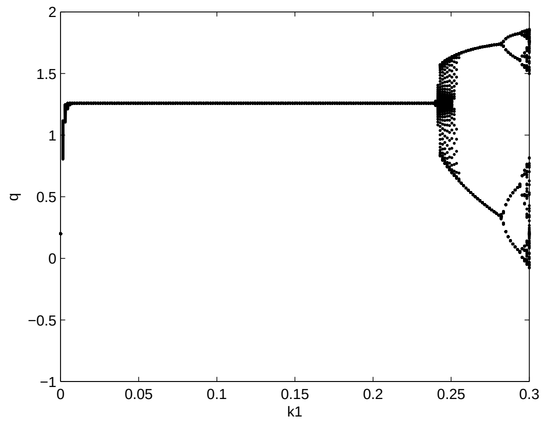

Figure 15: Output bifurcation diagram of manufacturer x when μ=0.9

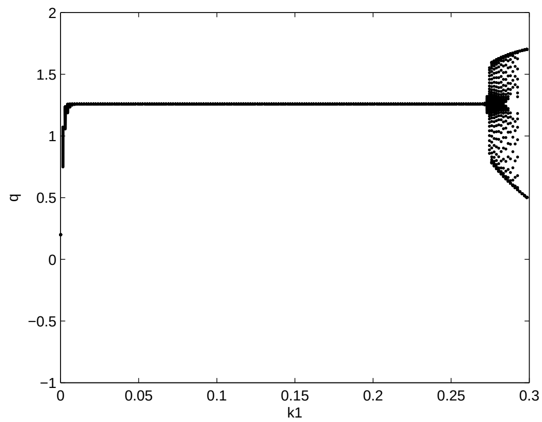

Figure 16: Output bifurcation diagram of manufacturer x when μ=0.8

Figure 15, Figure 16 and Figure 17 are the output bifurcation diagrams of manufacturer x. Make the controlling parameter μ=0.9, then the first bifurcation occurs in the system when k_1=0.2385, the sec-

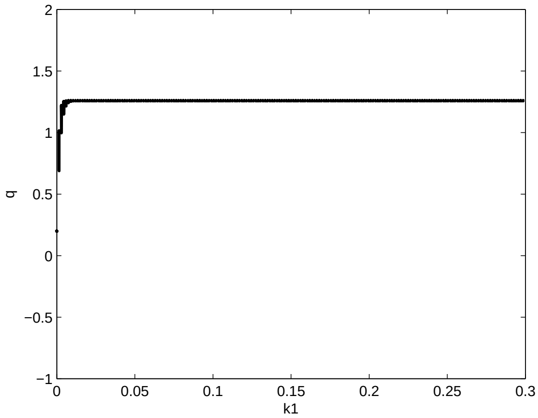

Figure 17: Output bifurcation diagram of manufacturer x when μ=0.7

ond when k_1=0.282, and the third when k_1=0.294,, till chaos occurs. The stable area in Figure 7 is larger than that in Figure 1, into which the controlling parameter μ is not fed. Make the controlling parameter μ=0.8, then the first bifurcation occurs in the system when k_1=0.273. The bifurcation figure of the system shows a dashed line when the controlling parameter μ=0.7, which indicates that the system is stable.

Figure 18,Figure 19 and Figure 20 are the sequence diagrams of manufacturer x with different controlling parameters. Make μ=0.9, the output decision making of the manufacturer changes irregularly. Make μ=0.8, the output decision making of the manufacturer changes regularly. Make μ=0.7, the output decision making of the manufacturer is stable and fixed. Change the initial condition (0.2, 0.4) into (0.2, 0.4001) in Figure 21,Figure 22 and Figure 23. When μ=0.9, the adjacent trajectories of manufacturer x enter into different domains of attraction after 70 iterations, creating a lag of 30 times and reducing the gap between adjacent trajectories compared with Figure 3. When the control parameter μ=0.8 or μ=0.7, manufacturer x is stable.

Based on the analysis above, three typical control solutions are worked out:

(1) Make μ=0.9, the system is chaotic.

(2) Make μ=0.8, the system is in bifurcation.

(3) Make μ=0.7, the system is stable.

Make controlling parameter μ=0.9, the system is chaotic, but the stable area is larger and there is chaos lag. In this case, the effect is noticeable and it is easy to control, but it is not suitable for long-term control. Make the controlling parameter μ=0.8, the chaos is brought under control and the system is in bifurcation. In this case, the maximum Lyapunov exponent is zero, indicating the emergence of chaotic periodi-

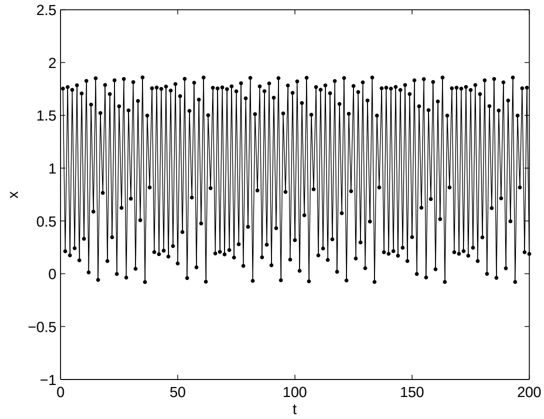

Figure 18: Sequence diagrams of manufacturer x when μ=0.9

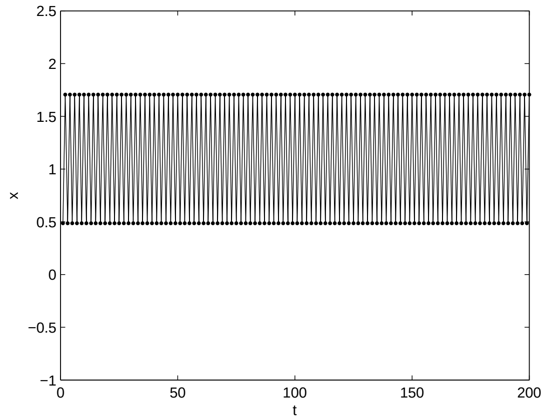

Figure 19: Sequence diagrams of manufacturer x when μ=0.8

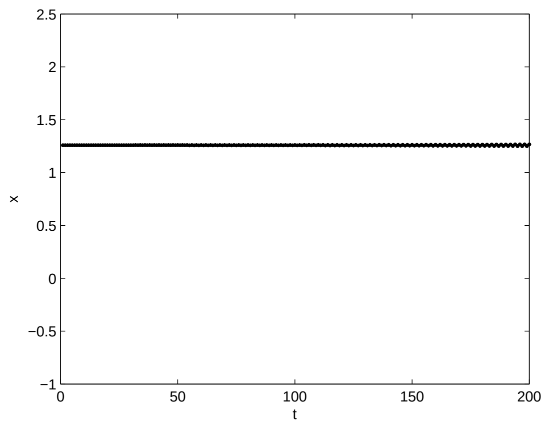

Figure 20: Sequence diagrams of manufacturer x when μ=0.7

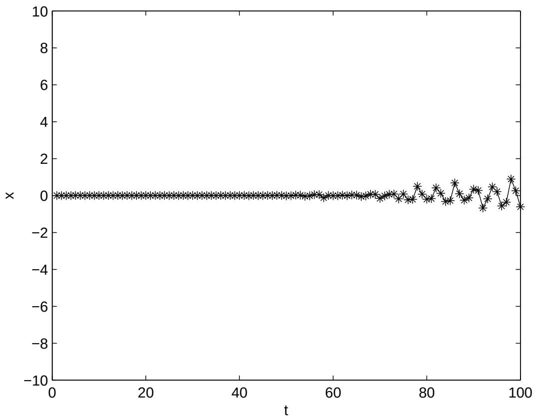

Figure 21: Sensitive dependency on initial conditions of output decision making of manufacturer x when μ=0.9

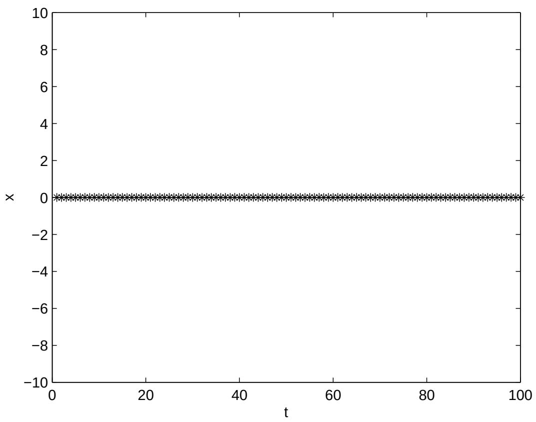

Figure 22: Sensitive dependency on initial conditions of output decision making of manufacturer x when μ=0.8

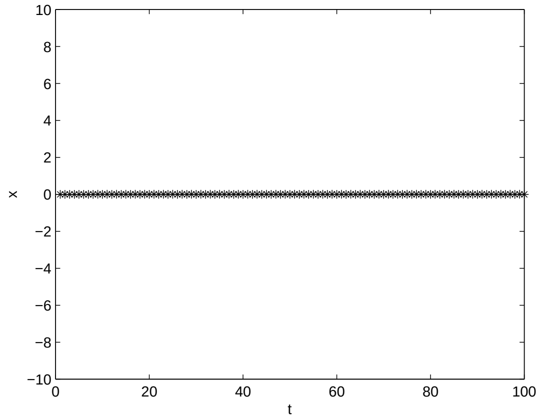

Figure 23: Sensitive dependency on initial conditions of output decision making of manufacturer x when μ=0.7

cal window. This solution will lead to instability and the system might be brought back to chaos due to the interaction between some controlling factors. Make controlling parameter μ=0.7, chaos in the system is brought under complete control. The maximum Lyapunov exponent is less than zero, indicating the system is in stability. However, it is difficult to put this solution into reality and the cost involved is high.

6 Conclusion

This paper first discusses the repetitive gaming in terms of output decision making between manufacturers and distributors when faced with demand fluctuation. It then constructs a nonlinear dynamic model and analyzes its complexity from the perspective of systematic stablity and Lyapunov exponent. The rule of the output gaming evolution between the manufacturers and the distributors is also offered. It is found that the output decision making of the manufacturers is chaotic whereas the ordering decision making of the distributors is stable. In a two-level supply chain, the fluctuation in the distributors' ordering will cause chaos in the manufacturers' output decision making, which is the so-called "bullwhip effect." This effect has negative effect on both the manufacturers and the distributors. For the purpose of chaos control, this paper uses strategies of state variable feedback control and parameter adjustment. Control is exercised on the period doubling bifurcation and unstable periodic trajectory of the manufacturers' output decision making. Explanation from the perspective of economics is also offered. As the chaos combines complexity, randomicity and disorderedness, manufacturers should have a well-defined expectation for chaos control. If the controlling parameter ? is fed into the system, the periodic trajectory could be fixed at the point expected. Therefore, the variable bifurcation point of the system will be delayed and even eliminated completely. Thus, the chaotic motions are delayed or even avoided. This is the theoretical foundation and the guideline for the manufacturers' output decision making.

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