Motion Control of a Mobile Robot Based on Double-Scroll Chaotic Circuits

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Abstract: - In this paper, the motion control of a mobile robot, which is based on a specific type of chaotic systems, is studied. These systems produce the well-known from the nonlinear theory double-scroll chaotic attractors. So, the proposed motion control strategy of the mobile robot is based on a chaotic path planning generator in order to cover a terrain fast and with unpredictable way. For choosing the system with the best appearance in regard of the terrain coverage, three different nonlinear dynamical systems with double-scroll chaotic behavior, the Chua oscillator, the Lorenz system, and a circuit with a nonlinear resistor having an i-v characteristic based on a saturation function, were used. The comparative study of the proposed double-scroll systems shows that the third system has significantly greater terrain coverage, among the three used dynamical systems.

Key-Words: - Mobile robot, motion control, chaos, Chua circuit, Lorenz system, double-scroll chaotic attractors.

1 Introduction

In the last decades the research field of autonomous mobile robots has become a topic of great interest because of its ever-increasing applications in various activities. Industrial transportation, floor-cleaning devices and fire fighting devices have been developed accenting autonomous mobile robots as very useful tools in industrial and civil life [1-3]. Also, many military activities, which put human integrity in risk, such as the surveillance of terrains, the terrain exploration for explosives or dangerous materials and the patrolling for intrusion in military facilities, have driven to the development of intelligent robotic systems [4-6].

Especially, in these military missions robotic systems must have some very important features such as the perception and identification of the target, the positioning of the robot on the terrain and the updating of the terrain's map. However, the most useful feature, determining the success of these military missions, is the path planning. For this reason many research teams trying to find out the way to generate a trajectory, which will guarantee the surveillance of the entire terrain or the finding of the explosives. Furthermore, in the case of patrolling for intrusion, the path of the robot must be as much difficult to be predicted by the intruder as possible. So, the mission of patrolling a terrain with a mobile robot is an issue that has to do with finding a plan which must satisfy three major targets: the unpredictability of the trajectory, the scan of the entire terrain and the fast scanning of the robot's workplace. These are the basic requirements for selecting the most suitable autonomous mobile robots for the specific kind of missions.

These characteristics were the beginning of using nonlinear dynamical systems in the development of autonomous mobile robots, especially in the last decade [6-8]. As it is known, nonlinear systems have a very rich dynamic behavior, showing a variety of chaotic phenomena. This chaotic behavior is the reason for which nonlinear systems have been used in many other engineering fields such as communications, cryptography, random bits generators and neural networks [10-14].

The aim of using nonlinear systems in autonomous robots is achieved by designing controllers, which ensure chaotic motion. Signals, which are produced by chaotic systems or circuits, are used to guide autonomous robots for exploration of a terrain for vigilance, search or de-mining tasks. The main feature of chaotic systems, which is the unpredictability, is a necessary condition in the previous mentioned tasks. In the literature very known chaotic systems, such as Arnold dynamical system, Standard or Taylor-Chirikov map, Lorenz system and Chua circuit, have been used [4,5,15-18].

In this work, the motion control strategy of a mobile robot is studied, in order to generate the most unpredictable trajectory. This is implemented by using different chaotic path planning generators. The common feature of the used chaotic generators is the production of double-scroll chaotic attractors. The comparative study accents the chaotic system with the better results in regard to unpredictability of the trajectory and the coverage rate of the robot's workplace.

The rest of the paper is organized as follows. In the next section the basic features of chaotic systems are presented. The mathematical models of the nonlinear systems, which are adopted as robot's driver and the proposed model for the robot are described in Section 3. The simulation results and their analysis are presented in Section 4. Finally, Section 5 includes the conclusions of this work.

2 Chaotic Systems

The basic components of the chaotic robotic systems, which are developed nowadays, are microcontrollers or CPUs for controlling their chaotic motion [5]. Many researchers are trying to impart the main feature of chaotic systems to robots, which is the great sensitivity on initial conditions, so as to show unpredictable trajectories.

Nevertheless a nonlinear dynamical system, in order to be considered as chaotic, must fulfill the following three conditions [19].

1. It must be topologically mixing,
2. its chaotic orbits must be dense and
3. it must be very sensitive on initial conditions.

Firstly, the term *topologically mixing* means that the chaotic dynamical system, especially chaotic mobile robot, will move over time so that each designated area of the trajectory will eventually cover part of any particular region. This property of chaotic systems guarantees a complete scan of the entire working space environment.

The second feature of chaotic systems is that its chaotic orbits have to be dense. This means that, the trajectory of a dynamical system is dense, if it comes arbitrarily close to any point in the domain.

Finally, the most important feature of chaotic systems, as it is mentioned, is the sensitivity on initial conditions. This means that a small variation on a system's initial conditions will produce a totally different chaotic trajectory. This is the feature, which is contributed to the desired robot's unpredictable trajectory and makes the long-term prediction of this trajectory, based on finite-time measurements, practically impossible.

Therefore, based on these features of chaotic systems, a chaotic trajectory, from the perspective of an intruder, presents a complicated behavior, that does not exhibit any recurrent pattern and seems to be completely random. Nevertheless, these two approaches, chaotic and random, have a major and substantial difference. The chaotic motion in contrary to the random one, is based on determinism, which in the case of mobile robots is an advantage. This happens because the behavior of a robot can be predicted in advance only by the system designer. So, an autonomous chaotic mobile robot, with such characteristics, may be used successfully in many missions such as a patrol robot or as a de-mining device.

3 The Mobile Robot Model

Many works on kinematic control of chaotic robots is based on a typical differential motion with two degrees of freedom, composed by two active, parallel and independent wheels and a third passive wheel [4,20]. The active wheels are independently controlled on velocity and rotation sense. A well-known commercial model of this type of mobile robots is the mini-robot Khepera (Fig.1).

The above mentioned mechanism has been used to the kinematic control of the robot of this work. So, the proposed mobile robot's motion is described by the linear velocity $v(t)$ [m/s], the angle $\theta(t)$ [rad] describing the orientation of the robot, and the angular velocity $\omega(t)$ [rad/s]. The linear velocity provides a linear motion of the medium point of the wheels axis, while the direction velocity provides a rotational motion of the robot's over the same point. In Fig.2 the description of the robot motion on a plane is shown. The robot's motion control is

Fig.1. The mobile robot Khepera.

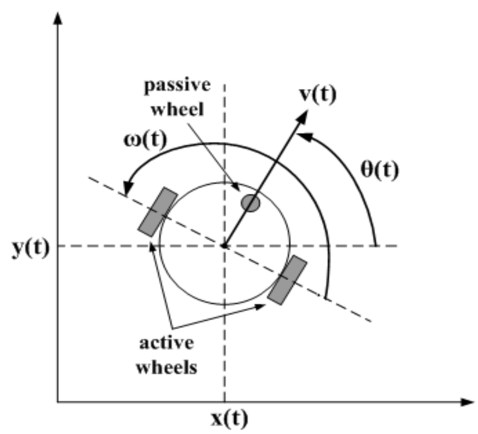


Fig.2. The description of the robot motion on a plane.

described by the following system equation (1).

$$\begin{pmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\theta}(t) \end{pmatrix} = \begin{pmatrix} \cos\theta(t) & 0 \\ \sin\theta(t) & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v(t) \\ \omega(t) \end{pmatrix} \qquad (1)$$

where, $\{x(t), y(t)\}$ is the robot's position on the plane and $\theta(t)$ is the robot's orientation. Also, it must be mentioned that in the case in which the robot reaches the borders of the terrain, the robot stops and waits the next direction order to move.

Furthermore, in many cases, robots move in spaces with boundaries like walls or obstacles. So, many robots have sensors, like sonar or infrared devices (like Khepera), which provide the capability to detect the presence of obstacles or even more the recognition of the searched objects or intruders. In this work, for a better understanding of the robot's kinematic behavior we assume that the robot, by using the proposed control scheme, works in a smooth state space without any sensor. So, when the robot reaches to a boundary stops and waits the next motion order from the chaotic generator.

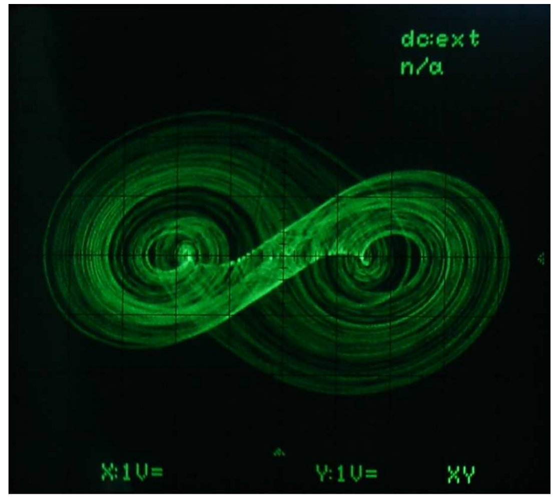


Fig.3. Experimental double-scroll chaotic attractor.

4 The Double-Scroll Systems

It has been known, that many nonlinear functions can generate chaos. Typical examples are piecewise-linear functions (e.g. Chua's circuit) [21-26], the smooth cubic function (e.g. the Duffing oscillator) [27-29], the smooth quadratic function (e.g. the Lorenz system) and the piecewise-quadratic function [30]. Among them Chua's circuit is a paradigm for chaos, being a simple nonlinear electrical circuit that exhibits double-scroll chaotic attractors (Fig.3). An alternative way for generating double-scroll like behavior was proposed by means of a simple circuit implementation [31].

Since there are many chaotic systems, which pattern is a good candidate to be chosen for use as a chaotic path planning generator? In general the criterion in many works is the largest possible coverage area through computer simulation.

In this work three of the most well-known double-scroll systems are used. These are the Chua and Lorenz systems, because they were used in previous works, as such a double-scroll chaotic system based on a saturated function proposed by Volos, Kyprianidis and Stouboulos (VKS) [32].

4.1 Chua Circuit

As it is mentioned, the first one is the dynamical system (2), which describes the most studied nonlinear circuit, the Chua circuit (Fig.4). This nonlinear circuit has been studied both theoretically and experimentally and a variety of phenomena, which are related with Chaos theory, such as the dependence of a system on initial conditions, the crisis of chaotic attractors and the

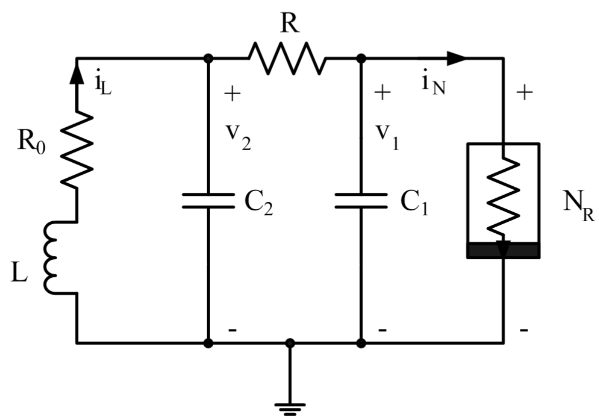


Fig.4. The Chua's circuit.

route to chaos through the period doubling, are observed [23-25].

Chua circuit has three equilibrium points. One of these equilibria is the origin, the other two usually referred as P^+ and P^{--}. These two latter points are located at the center of the two holes in Fig.3. A typical trajectory of the attractor rotates around one of these equilibrium points, getting further from it after each rotation until either it goes back to a point closer to the equilibrium or either repeats the process or directs toward the other equilibrium point and repeats a similar process, but around the other equilibrium point. In both cases the number of rotations is random. This unpredictability of double-scroll chaotic attractors is one of the peculiarities of deterministic chaos.

The state equations describing the Chua's circuit are as follows:

$$\begin{cases} \dfrac{dv_1}{dt} = \dfrac{1}{C_1} \cdot \left[\dfrac{1}{R} \cdot (v_2 - v_1) - g(v_1) \right] \\ \dfrac{dv_2}{dt} = \dfrac{1}{C_2} \cdot \left[\dfrac{1}{R} \cdot (v_1 - v_2) + i_L \right] \\ \dfrac{di_L}{dt} = \dfrac{1}{L} \cdot \left[-v_2 - R_0 \cdot i_L \right] \end{cases} \quad (2)$$

where, $g(v_1)$ is a piecewise-linear function. In this paper the nonlinear element N_R of this circuit implements the cubic function of the form:

$$g(v_1) = i_N = -k_1 \cdot v_1 + k_3 \cdot v_1^3 \quad (3)$$

where, k_1, $k_3 > 0$. The practical circuit for realizing the cubic polynomial of Eq.(3) is shown in Fig. 5 [26]. The two terminal nonlinear resistor N_R consists of one Op Amp (LF411), two analog multipliers (AD633JN) and five resistors.

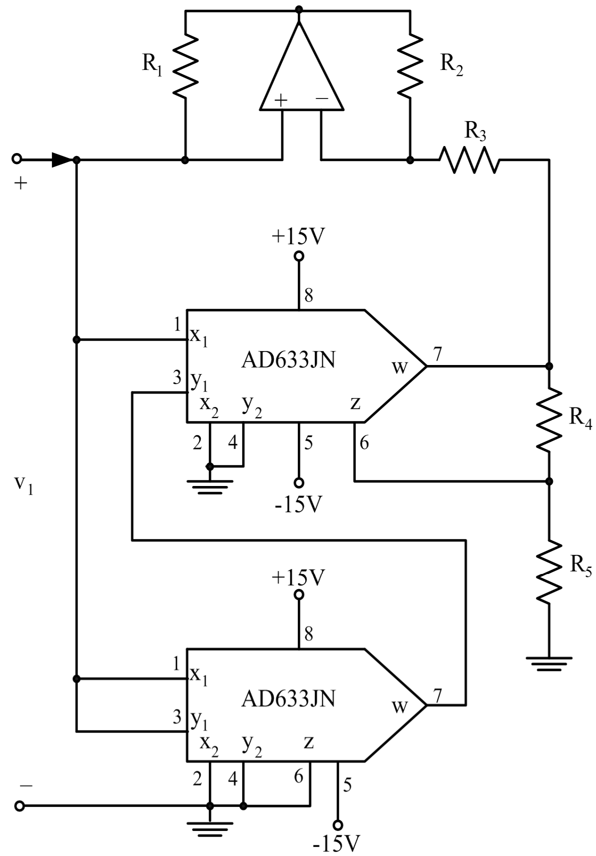


Fig.5. Practical circuit for realizing the cubic polynomial v-i characteristic.

Each multiplier implements the function:

$$w = \frac{(x_1 - x_2) \cdot (y_1 - y_2)}{10V} + z \quad (4)$$

where the factor 10V is an inherent scaling voltage in the multiplier. The connections of the Op-Amp and the resistors R_1, R_2 and R_3 form an equivalent negative resistor R_e, when $R_1 = R_2$ and the Op-Amp operates in its linear region, in order to obtain the desired coefficients k_1 and k_2. The voltages of the positive and negative electrical sources are ±15V. The driving point v-i characteristic of N_R is as below:

$$i_N = -\frac{1}{R_3} \cdot v_1 + \frac{R_4 + R_5}{R_3 \cdot R_4} \cdot \frac{1}{10V} \cdot \frac{1}{10V} \cdot v_1^3 \quad (5)$$

where,

$$k_1 = \frac{1}{R_3} \text{ and } k_3 = \frac{R_4 + R_5}{R_3 \cdot R_4} \cdot \frac{1}{10V} \cdot \frac{1}{10V}.$$

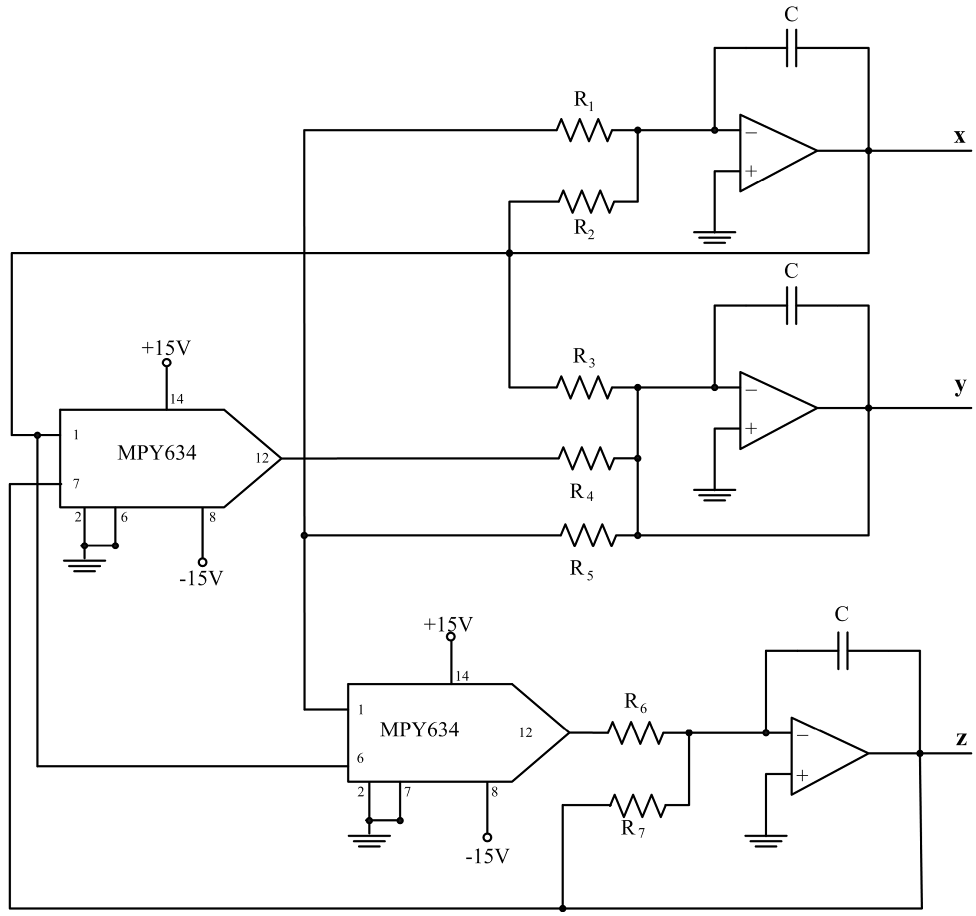


Fig.6. The Lorenz circuit.

4.2 Lorenz System

In 1963 the meteorologist Edward Lorenz published his famous set of coupled nonlinear first-order ordinary differential equations. They are relatively simple, but the resulting behavior is wonderfully complex. In this system the control parameters are the Rayleigh number r, the Prandtl number σ, and the geometric factor b. The equations are:

$$\begin{cases} \dfrac{dx}{d\tau} = \sigma \cdot (-x + y) \\ \dfrac{dy}{d\tau} = r \cdot x - y - x \cdot z \\ \dfrac{dz}{d\tau} = -b \cdot z + x \cdot y \end{cases} \qquad (6)$$

Since today a great number of different implementations of Lorenz equation have been proposed. In this work a simple implementation with just 3 Op-Amps (LF411) and two analog multipliers (MPY634) have been used (Fig.6).

4.3 VKS Circuit

Finally, the third dynamical circuit (Fig.7) is described by the dimensionless system (7).

$$\begin{cases} \dfrac{dx}{d\tau} = y \\ \dfrac{dy}{d\tau} = z \\ \dfrac{dz}{d\tau} = -\alpha \cdot (x + y + z) + b \cdot f(x) \end{cases} \qquad (7)$$

where, α and b are the circuit parameters and are defined as follows:

$$\alpha = (R \cdot C)^{-1}, b = (R_X \cdot C)^{-1} \qquad (8)$$

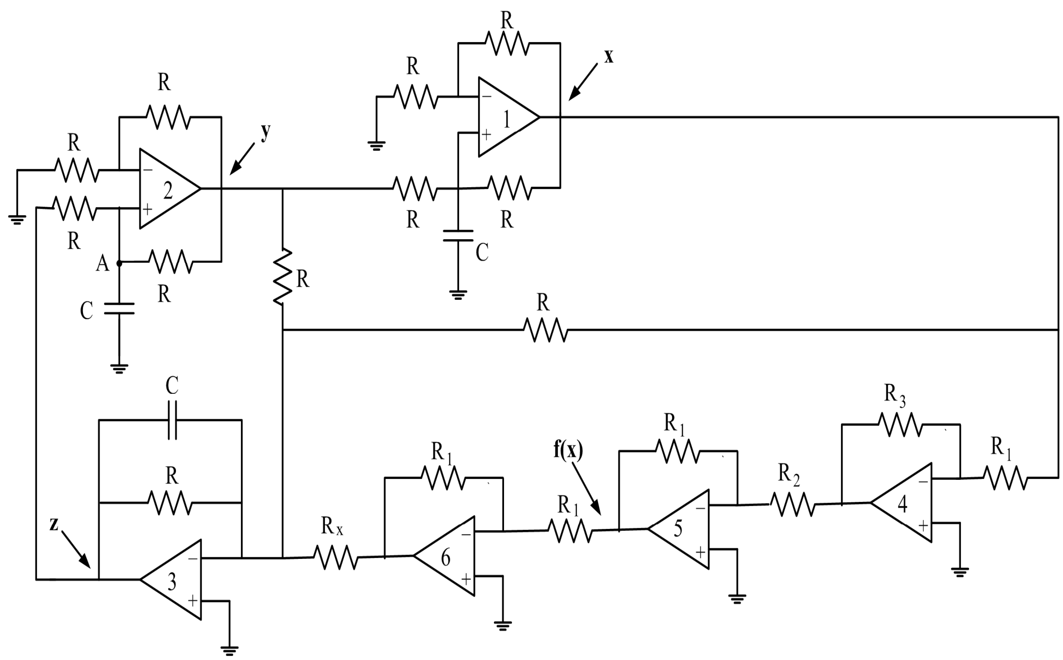


Fig.7. The VKS circuit.

The state parameters x, y, and z represent the voltages at the outputs of the operational amplifiers numbered as “1”, “2” and “3” respectively, as shown in Fig.7.

Function f(x) (Eq.9) is a saturation function, which is calculated in such a way that the saturation plateaus are ±1 and the slope of the intermediate linear region is $k = R_3/R_2$.

$$f(x) = \begin{cases} 1, & \text{if} \quad x > \dfrac{R_2}{R_3}\cdot 1V \\ \dfrac{R_3}{R_2}\cdot x, & \text{if} \quad -\dfrac{R_2}{R_3}\cdot 1V \le x \le \dfrac{R_2}{R_3}\cdot 1V \\ -1, & \text{if} \quad x < -\dfrac{R_2}{R_3}\cdot 1V \end{cases} \quad (9)$$

Furthermore, all the Op-Amps were of the type LF411 and the voltages of the positive and negative power supplies were set ±15V.

5 Numerical Simulations

In this work for integrating the three dynamical systems into the proposed robot’s controller a known strategy is used. In all systems the parameter z will be the angular position θ.

So, the angular velocity of the robot will be:

$$\omega = \frac{d\theta}{d\tau} = \frac{dz}{d\tau} \quad (10)$$

Also, by adding into systems (2), (6) and (9) the two following equations (11), which correspond to mobile robot’s motion, three five-dimension systems are created.

$$\begin{cases} \dfrac{dX}{d\tau} = v\cdot \cos(n\cdot z) \\ \dfrac{dY}{d\tau} = v\cdot \sin(n\cdot z) \end{cases} \quad (11)$$

In the above system (X, Y) are the coordinates of the robot’s position on terrain and v is a constant velocity of the mobile robot. Furthermore, n is a factor of normalization so parameter z of each system has the same magnitude. With this technique the three control dynamical systems give comparable results.

To test the proposed control strategy of a mobile robot the results of the numerical simulations are presented in details in this paragraph. For this reason the terrain coverage, using the known coverage rate (C), which represents the effectiveness, as the amount of the total surface covered by the robot running the algorithm, is used.

The coverage rate (C) is given by the following equation:

$$C = \frac{1}{M} \cdot \sum_{i=1}^{M} I(i) \qquad (12)$$

where, I(i) is the coverage situation for each cell in which the terrain has been divided [33]. This is defined by the following equation

$$I(i) = \begin{cases} 1, & \text{when the cell i is covered} \\ 0, & \text{when the cell i is not covered} \end{cases} \qquad (13)$$

where, i = 1, 2, ..., M. The robot's workplace is supposed to be a square terrain with dimensions $M = 20\text{m} \times 20\text{m} = 400\text{m}^2$ in normalized unit cells. Furthermore, a second interesting evaluation criterion is the coverage time of the system, which is the total time for the system to cover the entire terrain.

In this work, the three proposed dynamical systems were solved numerically by using the fourth order Runge-Kutta algorithm. Searching for sets of optimal parameters for the three dynamical systems for generating the best possible patterns is a very time-consuming task. Therefore, for convenience, we retain their original parameters of these systems as used in the literature. So, the parameters, the initial conditions and the factor n of each system, which were chosen in order to appear double-scroll chaotic attractors, are:

- The values of parameters of Chua's circuit are: R_0 = 30Ω, R = 1960Ω, R_1 = R_2 = 2kΩ, R_3 = 1.671kΩ, R_4 = 3.01kΩ, R_5 = 7.887kΩ, C_1=7.4nF, C_2 = 95.8nF. Also, the set of initial conditions is: (x_0, y_0, z_0) = (0.8, -0.2, 0.4) and n = 4.4.

- The values of parameters of Lorenz's circuit are: R_1 = R_2 = 100kΩ, R_3 = 35.7kΩ, R_4 = 10kΩ, R_5 = 1MΩ, R_6 = 10kΩ, R_7 = 374kΩ C_1 = 0.1μF. So, the set of the normalized parameters and initial conditions are: {σ, r, b} = {5, 28, 0.375},(x_0, y_0, z_0) = (1, -1, 2), and n = 1.

- The values of parameters of VKS's circuit are: R = 20kΩ, R_1 = 1kΩ, R_2 = 14.3kΩ, R_3 = 28.6kΩ, R_X = 10kΩ and C = 1nF. So, the set of the normalized parameters and initial conditions are: {α, b, k} = {0.5, 1, 2}, (x_0, y_0, z_0) = (0.5, 0.2, 0.1) and n = 14.7.

Table 1. Coverage rate of each system.

System	Coverage Rate
Chua	11.5 %
Lorenz	23.0 %
VKS	40.0 %

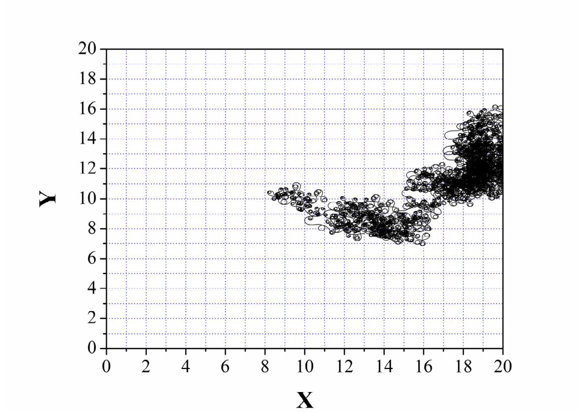
Also, in this work the initial position and velocity of the robot were chosen to be: (X, Y) = (10, 10) and v = 1m/s. Duration for run-time for simulation in this paper was 1000s.

Comparison results of the three proposed dynamical systems, are shown in Figs.8(a)-(c). In these figures simulations of the mobile robot's behavior of each system are presented. Also, in Table 1 the coverage rate for the three systems are compared. As it is shown, Chua circuit appears the smaller value of coverage rate among the systems. Lorenz system has two times the coverage rate of Chua circuit. Finally, VKS system shows significantly higher value of coverage rate as regards to the other systems. So, from the three proposed dynamical systems, VKS has the better performance, which is obvious in Fig.4(c) where the 40% of the terrain shows to be covered by the robot. This happens because the VKS system produces a mobile robot's orbit which is constituted by spiral curves that abstain longer distances concerning the other two systems, as it appears in Fig.8(c).

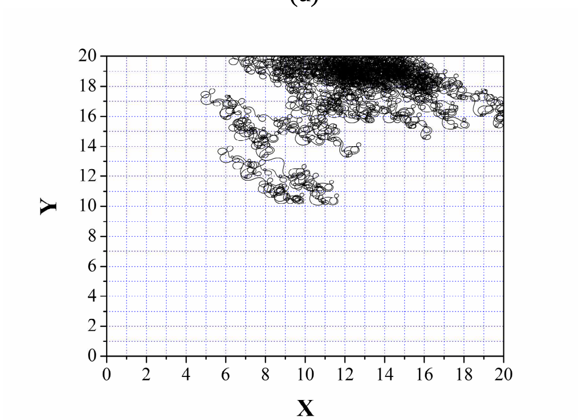
Finally, by choosing the system with the best results, that is VKS system, we run the simulation until the mobile robot covers the whole terrain. Also, the set of initial conditions have been changed, (x_0, y_0, z_0) = (0.8, -0.2, 0.4), so as to be proved that the chaotic motion of the robot is different but independent of the choice of systems initial conditions. The mobile's robot motion path is shown in Fig.9. As it is obvious the terrain has been covered by the robot in 99.25% (only three cells have not been covered) in time of 5000s.

6 Conclusion

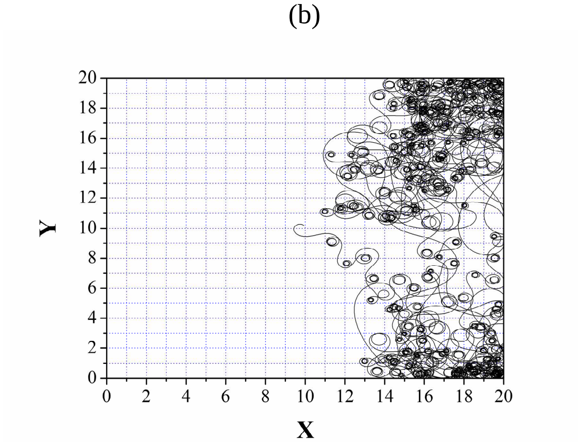
In this work a comparative study of three different chaotic circuits which are used for the driving strategy of a mobile robot was presented. For the aim of this approach the most well-known double-scroll chaotic systems were chosen. The first one



(a)



(b)



(c)

Fig.8. The mobile robot’s motion path with (a) Chua, (b) Lorenz and (c) VKS system.

was the Chua circuit while the second was a circuit which implements the Lorenz equation. The third one was a circuit (VSK) which its nonlinear element has a saturation function for i-v charactreristic.

This approach is followed in order to generate the most unpredictable trajectory, as well as the trajectory with the higher coverage rate of a specific terrain. The results of the comparative study show that the VSK system has significantly higher terrain

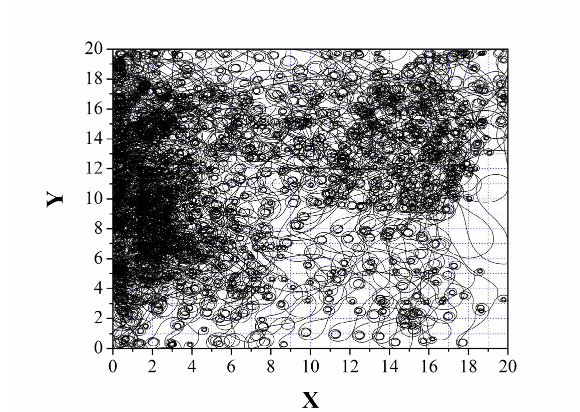


Fig.9. The mobile robot’s motion path with VKS system, after 5000s.

coverage, which is the criterion of success of such robot’s mission, among the proposed dynamical systems. This result was produced because of the nature of the double-scroll chaotic attractors, which were produced by the VSK system. Finally, the coverage of the whole terrain in a very satisfactory time, with the VSK system, was proved.

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