Deriving the Optimal Production-Shipment Policy with Imperfect Quality and an Amending Delivery Plan using Algebraic Method

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Abstract: - This paper derives the optimal production-shipment policy for a vendor-buyer integrated system with imperfect quality and an amending product delivery plan using an algebraic method. It is assumed that the system may randomly produce certain portion of nonconforming products during each production run. Quality assurance in terms of inspection and rework has been given effect to the defective items during and after the production run. A (n+1) finished goods distribution policy is used in the proposed vendor-buyer integrated system, such a delivery plan aims at reducing inventory holding costs for both producer and buyer. Unlike the conventional approach using the differential calculus on the long-run average production-inventory-delivery cost function with the need to prove optimality before solving the optimal replenishment-delivery problem, this paper adopts a simplified solution procedure using algebraic derivations to deal with the production-shipment decision makings. The result demonstrates that the optimal replenishment lot-size and optimal number of deliveries can be derived without derivatives. Such an alternative approach enables practitioners who with little knowledge of calculus to understand the real world vendor-buyer integrated systems with ease.

Key-Words: - Replenishment lot size, Multi-deliveries, Supply chain optimization, Vendor-buyer integration, Rework, Scrap

1 Introduction

This paper presents an algebraic method for deriving the optimal production-shipment policy for a vendor buyer integrated system with imperfect quality and an amending product delivery plan. The economic manufacturing quantity (EMQ) model (also known as the finite production rate (FPR) model [1]) is commonly used by the manufacturing firms to assist them in making routine decisions on “when to start a production run?” and “how many to be replenished in each production run?” [2-6]. Classic EMQ model assumes that all items produced are of perfect quality. However, in real world production environment, due to various controllable and/or uncontrollable factors, production of nonconforming items is inevitable. Many studies have been carried out to address the imperfection issue in production systems [7-16]. Shih [8] examined two inventory models where the proportion of defective units in the accepted lot is a random variable with known probability distribution. Optimal solutions to the modified systems were developed and comparisons with the traditional models were also presented via numerical examples. de Kok [9] studied a lost-sales production/inventory control model with two adjustable production rates to meet demand. The practical approximations were obtained for optimal switch-over levels to such a model under the service level constraints. Cheung and Hausman [11] proposed an analytical model of preventive maintenance (PM) and safety stock (SS) strategies in a production environment subject to random machine breakdowns. They illustrated the trade-off between investing in the two options (PM and SS) and provided optimality conditions under which either one or both strategies should be implemented to minimize associated cost function. Both the deterministic and exponential repair time distributions are analyzed in detail in their study. Boone et al. [12] studied the impact of imperfect processes on the production run time. They built a model in an attempt to provide managers with guidelines to choose the appropriate production run times to cope with both the defective items and stoppages occurring due to machine breakdowns.

The defective items sometimes can be reworked and total production-inventory costs will be reduced [17-24]. For instance, production processes in plastic injection molding, or in printed circuit board (PCB) assembly, sometimes uses rework as an acceptable process to increase level of quality as well as to cut overall production-inventory costs. Yum and McDowell [17] treated the allocation of inspection effort problem for serial systems as a 0-1 mixed integer linear programming problem (MILP). Their formulation permitted any combination of scrap, rework, or repair at each station and allowed the problem to be solved using standard MILP software packages. An additional advantage of their approach when compared with the traditional dynamic programming approach was the ease with which the basic model may be modified. Gopalan and Kannan [18] considered the manufacturing, inspections and rework activities as a two-stage transfer-line production system. They analyzed some of the transient state characteristics of such a two-stage production system subject to an initial buffer of infinite capacity, inspection at both the inter- and end-stages and rework. A stochastic model was developed to investigate the system. Explicit analytical expressions for some of the system characteristics have been obtained using the state-space method and regeneration point technique. Chern and Yang [19] considered a threshold control policy for an imperfect production system with only a work center handling both regular and rework jobs. The imperfect production system generates defect jobs by factors other than machine failures. A threshold control policy sets the guideline for a work center to switch between regular and rework jobs. They assumed the outcome of each completed regular job is an independent Bernoulli trial with three possibilities: good, rework, or scrap. Once the work center accumulates more than a threshold of rework jobs, it finishes the last batch of regular jobs and switches to rework jobs. The objective of their research was to find a threshold ω and a lot size s that maximize the average long-term profit. Inderfurth et al. [20] studied a deterministic problem of planning the production of new and recovering defective items of the same product manufactured on the same facility. The processing of a batch includes two stages: the regular production and the rework process. While waiting for rework, defective items deteriorate and there is a given deterioration time limit. Deterioration results in an increase in time and cost for performing rework processes. The objective of their study was to find batch sizes and positions of items to be reworked such that overall production-inventory costs are minimized. A polynomial dynamic programming algorithm was presented to solve this problem. Chiu et al. [24] examined a finite production rate model with scrap, rework and stochastic machine breakdown. Stochastic breakdown rate and random defective rate along with the reworking of nonconforming items were assumed in their study. The objective was to derive the optimal production run time that minimize the long run average production cost.

During past decades, many studies have focused on optimization of supply chains and vendor-buyer integrated production-inventory-delivery systems. Multiple or periodic deliveries of finished products are commonly and practically assumed rather than the continuous issuing policy as was assumed by classic EMQ model. Schwarz [25] considered a one-warehouse N-retailer inventory system with the objective of determining optimal stocking policy that minimizes average system cost. He derived some necessary properties for the optimal policy as well as the optimal solutions. Heuristic solutions were also provided for the general problem and tested against analytical lower bounds. Studies have since been carried out to address various aspects of supply chains optimization [26-39]. Selected papers are surveyed as follows. Goyal [26] proposed a method that is typically applicable to those inventory problems where a product is procured by a single customer from a single supplier, and an example was provided to illustrate his proposed method. Schwarz et al. [27] studied fill-rate of a one-warehouse N-identical retailer distribution system. Approximation model was adopted from a prior study to maximize system fill-rate subject to a constraint on system safety stock. As results, properties of fill-rate policy were suggested to provide management when looking into system optimization. Banerjee and Banerjee [28] developed an analytical model for a coordinated, orderless inventory system for the single product, single vendor, multiple purchasers case. Such a system was made practical in electronic data interchange at the time, for the exchange of information between trading partners. On the basis of the potential benefits of this technology, they proposed a common cycle replenishment approach, where the supplier alone makes all replenishment decisions, without ordering on the part of the customers. Their model and concepts were demonstrated by a simple numerical example and concluded that EDI-based inventory control can be attractive from economic, as well as other standpoints. Sarker and Khan [30] considered a manufacturing system that procures raw materials from suppliers in a lot and processes them to convert into finished product. An ordering policy is proposed for raw materials to meet the requirements of a production facility which, in turn, must deliver finished products demanded by outside buyers at fixed interval points in time. They first formulated a general cost model considering both raw materials and finished products, then by using this model they developed a simple procedure to determine an optimal ordering policy for procurement of raw materials, and the manufacturing batch size to minimize the total cost for meeting the customer demand in time. Goyal and Nebebe [31] considered the problem of determining economic production and shipment policy of a product supplied by a vendor to a single buyer. The objective of their study was to minimize the total joint annual costs incurred by the vendor and the buyer. Sarmah et al. [33] considered coordination between two different business entities is an important way to gain competitive advantage as it lowers supply chain cost, so they reviewed literature dealing with buyer vendor coordination models that have used quantity discount as coordination mechanism under deterministic environment and classified the various models. An effort was also made to identify critical issues and scope of future research. Sarker and Diponegoro [35] considered an optimal policy for production and procurement in a supply-chain system with multiple non-competing suppliers, a manufacturer and multiple non-identical buyers. They assumed that the manufacturer procures raw materials from suppliers, converts them to finished products and ships the products to each buyer at a fixed-interval of time over a finite planning horizon. The demand of finished product is given by buyers and the shipment size to each buyer is fixed. Their objective was to determine the production start time, the initial and ending inventory, the cycle beginning and ending time, the number of orders of raw materials in each cycle, and the number of cycles for a finite planning horizon so as to minimize the system cost. A surrogate network representation of the problem developed to obtain an efficient, optimal solution to determine the production cycle and cycle costs with predetermined shipment schedules in the planning horizon. This research prescribed the optimal policies for a multi-stage production and procurements for all shipments scheduled over the planning horizon. Chiu et al. [36] incorporated a multi-delivery policy and quality assurance into an imperfect economic production quantity (EPQ) model with scrap and rework. They assumed the reworking of repairable defective items in each production run and the finished items can only be delivered to customers if the whole lot is quality assured after rework. The expected integrated cost function per unit time was derived. A closed-form optimal batch size solution to the problem was obtained. Chiu et al. [40] determined jointly the optimal replenishment lot size and optimal shipment policy for an extended EPQ model with failure in rework and multi-delivery policy. A closed-form optimal production-shipment decision was obtained for such a specific vendor-buyer integrated system.

A recent article, Grubbström and Erdem [41] proposed algebraic approaches for solving the economic order quantity (EOQ) model with backlogging without reference to the use of derivatives, neither applying the first-order nor second-order differentiations. A few studies used the same algebraic approach to derive various specific production lot size problems [42-45].

This paper uses the similar algebraic approach to reexamine the problem studied by Chiu et al. [45]. We incorporate the scrap rate in regular production process and failure rate in rework process (see Fig. 2) in Chiu's et al. model, and demonstrate that the optimal production-shipment policy to such an extended model can be derived with derivatives.

It would also be interesting, for comparisons, to model supply chain problems by using artificial intelligence techniques like agents which proved to be applicable to other complex domains such as financial markets [46].

2 The Problem and Formulations

Consider a product can be produced at a constant production rate P and it has a flat annual demand rate λ. Its production process may generate x portion of random nonconforming items at a rate d, where $d=Px$. It is also assumed that under regular operating schedule, P is larger than the sum of λ and d. That is: $(P-d-\lambda)>0$. As stated in [45], to assure the product quality, all manufactured items are screened and the inspection cost is included in unit production cost C. Among the defective products, a θ portion (where $0<=\theta<1$) is scrap and the other $(1-\theta)$ portion can be reworked at a rate P_1, right after regular production each cycle. Further, there is a failure in rework rate θ_1 (where $0<=\theta_1<1$) during the rework process.

Let d_1 denote production rate of scrap items during rework, then d_1 can be expressed as $d_1=P_1\theta_1$.

An amending product distribution policy is considered for the purpose of lowering stock holding costs for both vendor and buyer. Under such a specific delivery plan, the first installment of finished products is distributed to buyer to satisfy product demand during vendor's uptime t_1 and rework time t_2 (see Figure 1). Then, when the whole lot is quality assured (in the end of rework process), fixed quantity n installments of the rest of finished items are shipped to customer at a fixed interval of time during the production downtime t_3.

Figure 1 illustrates the on-hand inventory level of vendor's perfect quality products in the proposed model. It is noted that a significant difference in total stock holding when comparison with that in [40].

Parameters used in the modeling and formulations include the setup cost K per production cycle, unit manufacturing cost C, unit holding cost h, unit holding cost h_1 for reworked items, unit reworking cost C_R, disposal cost for scrap each item C_S, and the fixed delivery cost K_1 per shipment, and the variable distribution cost C_T per finished product delivered.

Additional notation is listed as follows.

T = time between initiate a production run,

t = the production time needed for producing enough finished products to satisfy buyer's product demand during t_1 and t_2,

t_1 = the regular production uptime,

t_2 = rework time,

t_3 = production downtime, time for delivering the rest of quality assured finished items,

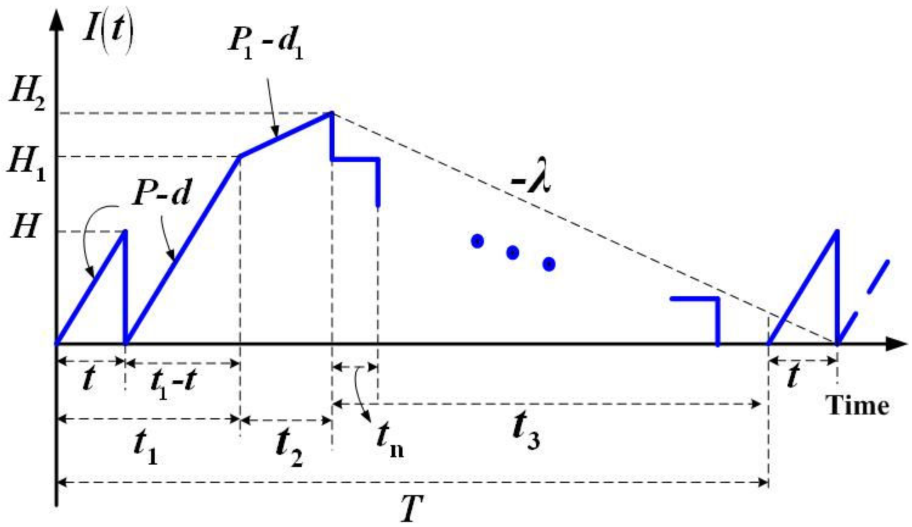

Fig. 1: The on-hand inventory level of vendor's perfect quality products in the proposed model

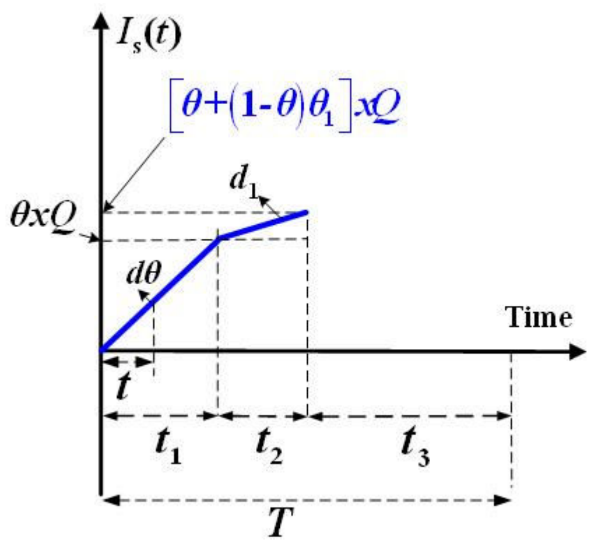

Fig. 2: The on-hand inventory of scrap items in the proposed model.

t_n = a fixed interval of time between each installment of products delivered in t_3,
H = the level of enough inventory to satisfy product demand during t_1 and t_2,
H_1 = the level of on-hand inventory when regular production ends,
H_2 = the level of on-hand inventory when the rework process ends,
Q = production lot size, one of the decision variables,
n = number of fixed quantity installments of the remaining finished items to be delivered to buyer during t_3, the other decision variable,
$I(t)$ = the level of on-hand inventory of perfect quality items at time t,
$I_s(t)$ = the level of on-hand inventory of scrap items at time t (see Fig. 2),
$TC(Q,n)$ = total production-distribution costs per cycle for the proposed model,
E[$TCU(Q,n)$] = the long-run average production-inventory-distribution costs per unit time for the proposed model.

Total production-distribution costs per cycle for the proposed model, $TC(Q,n)$ consists of the setup cost; variable production costs; variable rework costs; variable disposal costs for scrap items generated during t_1 and t_2; the fixed and variable delivery costs; vendor's inventory holding costs in periods t_1, t_2, and t_3; variable holding costs for reworked items; and the stock holding costs in buyer's end. Therefore, total production-distribution costs per cycle for the proposed model, $TC(Q,n)$ is

$$\begin{aligned}TC(Q,n) &= K + CQ + C_R\left[(1-\theta)xQ\right] + C_S(x\varphi Q)\\ &+(n+1)K_1 + C_T\left[Q(1-\varphi x)\right]\\ &+h\begin{bmatrix}\frac{H}{2}(t)+\frac{H_1}{2}(t_1-t)+\frac{dt_1}{2}(t_1)\\ +\frac{H_1+H_2}{2}(t_2)+\left(\frac{n-1}{2n}\right)H_2t_3\end{bmatrix}\\ &+h_1\left[\frac{dt_1(1-\theta)}{2}\right](t_2)\\ &+h_2\left[\frac{H(t_1+t_2)}{2}+n\left(\frac{D+2I}{2}\right)t_n\right]\end{aligned} \tag{1}$$

One notes that since defective rate x is a random variable with a known probability density function, one could use the expected values of x in the related cost analysis. With further derivation, the long-run average production-inventory-distribution costs per unit time for the proposed model E[$TCU(Q,n)$] is as follows.

$$\begin{aligned}E\left[TCU(Q,n)\right] &= \frac{E\left[TC(Q,n)\right]}{E(T)}\\ &=\lambda\begin{Bmatrix}CE_3+\frac{\left[(n+1)K_1+K\right]}{Q}E_3+\left[C_R(1-\theta)E_4\right]\\ +(C_S\varphi E_4)+C_T\end{Bmatrix}\\ &+\frac{hQ}{2}\begin{Bmatrix}\frac{2\lambda^3E_0}{P^3}+\frac{4\lambda^3(1-\theta)E_1}{P^2P_1}+\frac{2\lambda^3(1-\theta)^2E_2}{PP_1^2}\\ -\frac{\lambda(1-\theta)(1-\varphi)E_5}{P_1}+\frac{1}{E_3}-\frac{\lambda\left[1-2\varphi E(x)\right]E_3}{P}\end{Bmatrix}\\ &+\frac{Q(h_2-h)}{2n}\begin{bmatrix}\frac{1}{E_3}-\frac{2\lambda}{P}-\frac{2\lambda E(x)(1-\theta)}{P_1}\\ +\frac{2\lambda^2(1-\theta)E_4}{PP_1}+\frac{\lambda^2E_3}{P^2}+\frac{\lambda^2(1-\theta)^2E_5}{P_1^2}\end{bmatrix}\\ &+h_2Q\lambda^2\begin{bmatrix}\frac{E_6}{P^2}-\frac{\lambda E_0}{P^3}-\frac{2\lambda(1-\theta)E_1}{P^2P_1}+\frac{\lambda(1-\theta)^2E_2}{PP_1^2}\\ +\frac{(1-\theta)E_7}{PP_1}\end{bmatrix}\\ &+\frac{Q\lambda^2(h_2-h)}{2}\left[\frac{E_3}{P^2}+\frac{2(1-\theta)E_4}{PP_1}+\frac{(1-\theta)^2E_5}{P_1^2}\right]\\ &+\frac{h_1Q\lambda(1-\theta)^2}{2P_1}E_5\end{aligned} \tag{2}$$

where

$$E_0=\frac{E\left(\frac{1}{1-x}\right)}{\left[1-\varphi E(x)\right]};\ E_1=\frac{E\left(\frac{x}{1-x}\right)}{\left[1-\varphi E(x)\right]};\ E_2=\frac{E\left(\frac{x^2}{1-x}\right)}{\left[1-\varphi E(x)\right]};$$
$$E_3=\frac{1}{\left[1-\varphi E(x)\right]};\ E_4=\frac{E(x)}{\left[1-\varphi E(x)\right]};\ E_5=\frac{\left[E(x)\right]^2}{\left[1-\varphi E(x)\right]};$$
$$E_6=E\left(\frac{1}{1-x}\right);\quad E_7=E\left(\frac{x}{1-x}\right) \tag{3}$$

3 Derivations of the Optimal Production-Shipment Policy

A simplified algebraic solution procedure is adopted in this section instead of using differential calculus on the long-run average production-inventory-delivery cost function E[$TCU(Q,n)$]. It is noted that Eq. (2) has two decision variables Q and n, and they are in terms of coefficients associated with nQ^{-1}, Q^{-1}, Q, and Qn^{-1}.

Let β_1, β_2, β_3, β_4 and β_5 denote the following:

$$\beta_1=\lambda\left\{CE_3+\left[C_R(1-\theta)E_4\right]+(C_S\varphi E_4)+C_T\right\} \tag{4}$$

$$\beta_2=\lambda K_1E_3 \tag{5}$$

$$\beta_3=\lambda(K_1+K)E_3 \tag{6}$$

$$\beta_4=\frac{h}{2}\left\{\begin{array}{l}\frac{2\lambda^3E_0}{P^3}+\frac{4\lambda^3(1-\theta)E_1}{P^2P_1}+\frac{2\lambda^3(1-\theta)^2E_2}{PP_1^2}\\-\frac{\lambda(1-\theta)(1-\varphi)E_5}{P_1}+\frac{1}{E_3}-\frac{\lambda\left[1-2\varphi E(x)\right]E_3}{P}\end{array}\right\}$$
$$+h_2\lambda^2\left[\begin{array}{l}\frac{E_6}{P^2}-\frac{\lambda E_0}{P^3}-\frac{2\lambda(1-\theta)E_1}{P^2P_1}\\+\frac{\lambda(1-\theta)^2E_2}{PP_1^2}+\frac{(1-\theta)E_7}{PP_1}\end{array}\right]+\frac{h_1\lambda(1-\theta)^2}{2P_1}E_5 \quad (7)$$
$$+\frac{\lambda^2(h_2-h)}{2}\left[\frac{E_3}{P^2}+\frac{2(1-\theta)E_4}{PP_1}+\frac{(1-\theta)^2E_5}{P_1^2}\right]$$

$$\beta_5=\frac{(h_2-h)}{2}\left[\begin{array}{l}\frac{1}{E_3}-\frac{2\lambda}{P}-\frac{2\lambda E(x)(1-\theta)}{P_1}\\+\frac{2\lambda^2(1-\theta)E_4}{PP_1}+\frac{\lambda^2E_3}{P^2}+\frac{\lambda^2(1-\theta)^2E_5}{P_1^2}\end{array}\right] \quad (8)$$

Therefore, Eq. (2) becomes

$$\begin{aligned}E\left[TCU(Q,n)\right]&=\beta_1+\beta_2\left(nQ^{-1}\right)+\beta_3\left(Q^{-1}\right)\\&+\beta_4(Q)+\beta_5\left(Qn^{-1}\right)\end{aligned} \quad (9)$$

With rearrangement, Eq. (9) becomes

$$\begin{aligned}E\left[TCU(Q,n)\right]&=\beta_1+\left(Q^{-1}\right)\left[\beta_4Q^2+\beta_3\right]\\&+\left(Qn^{-1}\right)\left[\beta_2\left(nQ^{-1}\right)^2+\beta_5\right]\end{aligned} \quad (10)$$

or

$$\begin{aligned}&E\left[TCU(Q,n)\right]=\beta_1\\&+\left(Q^{-1}\right)\left[\begin{array}{l}\left(Q\sqrt{\beta_4}\right)^2+\left(\sqrt{\beta_3}\right)^2-2\left(Q\sqrt{\beta_4}\right)\left(\sqrt{\beta_3}\right)\\+2\left(Q\sqrt{\beta_4}\right)\left(\sqrt{\beta_3}\right)\end{array}\right]\\&+\left(Qn^{-1}\right)\left[\begin{array}{l}\left(nQ^{-1}\sqrt{\beta_2}\right)^2+\left(\sqrt{\beta_5}\right)^2-2\left(nQ^{-1}\sqrt{\beta_2}\right)\left(\sqrt{\beta_5}\right)\\+2\left(nQ^{-1}\sqrt{\beta_2}\right)\left(\sqrt{\beta_5}\right)\end{array}\right]\end{aligned} \quad (11)$$

Hence, one has

$$\begin{aligned}E\left[TCU(Q,n)\right]&=\beta_1+\left(Q^{-1}\right)\left[\left(Q\sqrt{\beta_4}\right)-\left(\sqrt{\beta_3}\right)\right]^2\\&+\left(Qn^{-1}\right)\left[\left(nQ^{-1}\sqrt{\beta_2}\right)-\left(\sqrt{\beta_5}\right)\right]^2\\&+2\left(\sqrt{\beta_4}\cdot\sqrt{\beta_3}\right)+2\left(\sqrt{\beta_2}\cdot\sqrt{\beta_5}\right)\end{aligned} \quad (12)$$

One notes that E[$TCU(Q,n)$] is minimized if the second and the third square terms in Eq. (12) equal zeros. That is

$$\left[\left(Q\sqrt{\beta_4}\right)-\left(\sqrt{\beta_3}\right)\right]=0 \quad (13)$$

and

$$\left[\left(nQ^{-1}\sqrt{\beta_2}\right)-\left(\sqrt{\beta_5}\right)\right]=0 \quad (14)$$

From Eq. (13) one has

$$Q^*=\sqrt{\frac{\beta_3}{\beta_4}} \quad (15)$$

and from Eq. (14) one obtains

$$n^*=\frac{Q\sqrt{\beta_5}}{\sqrt{\beta_2}} \quad (16)$$

From Equations (15) and (16) one obtains

$$n^*=\frac{\sqrt{\beta_3\beta_5}}{\sqrt{\beta_4\beta_2}} \quad (17)$$

3.1 Results with Further Discussion

Upon obtaining Eq. (17), the optimal number of deliveries, one can now substitute β_2, β_3, β_4 and β_5 (from Equations (5)-(8)) in Eq. (17) and obtain n^* as follows.

$$n^*=\sqrt{\frac{(K_1+K)(h_2-h)\left[\begin{array}{l}\frac{1}{E_3}-\frac{2\lambda}{P}-\frac{2\lambda E[x](1-\theta)}{P_1}+\frac{\lambda^2E_3}{P^2}\\+\frac{2\lambda^2(1-\theta)E_4}{PP_1}+\frac{\lambda^2(1-\theta)^2E_5}{P_1^2}\end{array}\right]}{K_1\left\{\begin{array}{l}h\left[-\frac{\lambda(1-\theta)(1-\varphi)E_5}{P_1}+\frac{1}{E_3}-\frac{\lambda\left[1-2\varphi E(x)\right]E_3}{P}\right]\\+2h_2\lambda^2\left[\frac{E_6}{P^2}+\frac{(1-\theta)E_7}{PP_1}\right]+\frac{h_1\lambda(1-\theta)^2E_5}{P_1}\\+\lambda^2(h_2-h)\left[\begin{array}{l}-\frac{2\lambda E_0}{P^3}-\frac{4\lambda(1-\theta)E_1}{P^2P_1}+\frac{(1-\theta)^2E_5}{P_1^2}\\+\frac{2\lambda(1-\theta)^2E_2}{PP_1^2}+\frac{E_3}{P^2}+\frac{2(1-\theta)E_4}{PP_1}\end{array}\right]\end{array}\right\}}} \quad (18)$$

It is noted that n is the number of shipments and it must be an integer number. However, Eq. (18) is likely to result a real number. Let n^+ denote the smallest integer number greater than or equal to n (derived from Eq. (18)) and n^- denote the largest integer number less than or equal to n. Therefore, n^* is determined to be either n^+ or n^-, a known constant.

Therefore, one can now reconsider E[$TCU(Q,n)$] as a cost function with single decision variable Q. Eq. (9) becomes

$$E\left[TCU(Q,n)\right]=\beta_1+Q^{-1}\left(n\beta_2+\beta_3\right)+Q\left(n^{-1}\beta_5+\beta_4\right) \tag{19}$$

Let β_6 and β_7 denote the following:

$$\beta_6 = n\beta_2+\beta_3 \tag{20}$$

$$\beta_7 = n^{-1}\beta_5+\beta_4 \tag{21}$$

Therefore, Eq. (19) becomes

$$\begin{aligned}E\left[TCU(Q,n)\right]&=\beta_1+\beta_6 Q^{-1}+\beta_7 Q\\&=\beta_1+Q^{-1}\begin{bmatrix}\left(Q\sqrt{\beta_7}\right)^2+\left(\sqrt{\beta_6}\right)^2-2\left(Q\sqrt{\beta_7}\right)\left(\sqrt{\beta_6}\right)\\+2\left(Q\sqrt{\beta_7}\right)\left(\sqrt{\beta_6}\right)\end{bmatrix}\\&=\beta_1+Q^{-1}\left[Q\sqrt{\beta_7}-\sqrt{\beta_6}\right]^2+2\sqrt{\beta_7}\cdot\sqrt{\beta_6}\end{aligned} \tag{22}$$

One notes that E[$TCU(Q,n)$] is minimized if the second square terms in Eq. (22) equals zero. That is

$$\left[Q\sqrt{\beta_7}-\sqrt{\beta_6}\right]=0 \tag{23}$$

or

$$Q^*=\frac{\sqrt{\beta_6}}{\sqrt{\beta_7}} \tag{24}$$

Substituting Equations (20) and (21) in Eq. (24) one has

$$Q^*=\frac{\sqrt{n\beta_2+\beta_3}}{\sqrt{n^{-1}\beta_5+\beta_4}} \tag{25}$$

Finally, substitute β_2, β_3, β_4 and β_5 (from Equations (5)-(8)) in Eq. (25), and with further derivations, one obtains the optimal replenishment lot size as

$$Q^*=\sqrt{\frac{2\lambda\left[(n+1)K_1+K\right]E_3}{\begin{gathered}h\begin{Bmatrix}\frac{2\lambda^3E_0}{P^3}+\frac{4\lambda^3(1-\theta)E_1}{P^2P_1}+\frac{2\lambda^3(1-\theta)^2E_2}{PP_1^2}\\-\frac{\lambda(1-\theta)(1-\varphi)E_5}{P_1}+\frac{1}{E_3}-\frac{\lambda\left[1-2\varphi E(x)\right]E_3}{P}\end{Bmatrix}\\+\frac{(h_2-h)}{n}\begin{bmatrix}\frac{1}{E_3}-\frac{2\lambda}{P}-\frac{2\lambda E(x)(1-\theta)}{P_1}+\frac{\lambda^2E_3}{P^2}\\+\frac{2\lambda^2(1-\theta)E_4}{PP_1}+\frac{\lambda^2(1-\theta)^2E_5}{P_1^2}\end{bmatrix}\\+2h_2\lambda^2\begin{bmatrix}\frac{E_6}{P^2}-\frac{\lambda E_0}{P^3}-\frac{2\lambda(1-\theta)E_1}{P^2P_1}\\+\frac{\lambda(1-\theta)^2E_2}{PP_1^2}+\frac{(1-\theta)E_7}{PP_1}\end{bmatrix}+\frac{h_1\lambda(1-\theta)^2E_5}{P_1}\\+(h_2-h)\lambda^2\left[\frac{E_3}{P^2}+\frac{2(1-\theta)E_4}{PP_1}+\frac{(1-\theta)^2E_5}{P_1^2}\right]\end{gathered}}} \tag{26}$$

4 Numerical Example

This section uses numerical example to demonstrate the savings in stock holding costs for both vendor and buyer from research results derived by the present study and what was given in Chiu et al. [40]. The values of parameters used are

- P = 60000 products per year,
- λ = 3100 items per year,
- x = defective rate which follows a uniform distribution over the range [0, 0.3],
- P_1 = 3600 units per year,
- θ = 0.1, portion of scrap during regular production process,
- θ_1 = 0.111, portion of scrap during rework process,
- K = \$35000, setup cost per production run,
- C = \$100, unit production cost,
- h = \$25, holding cost per item per year at producer's end,
- C_R = \$60, repaired cost for each item reworked,
- C_S = \$20, disposal cost for each scrap item,
- h_1 = \$60, unit holding cost during rework, per unit time,
- K_1 = \$2500, fixed cost per shipment,
- C_T = \$0.1, variable delivery cost per item,
- h_2 = \$80, holding cost per item per year at the buyer's end.

Applying Equations (15) and (18), one obtains (Q, n)=(2975, 4.5). As stated earlier, because n takes on an integer value only, one compares two adjacent integer number of delivery and its operating production policies (i.e. (Q, n)=(2896, 4) and (Q, n) =(3049, 5)), and obtains E[TCU(2896, 4)]= \$452538 and E[TCU(3049, 5)]=\$452517 respectively (from Eq. (2)).

Hence, the optimal production-shipment policy for proposed model (Q^*,n^*+1)=(3049,6) is derived without derivatives.

Note that the aforementioned approach can be used to solve any real world production-delivery integrated systems of the described characteristics.

5 Concluding Remarks

This paper presents a simplified algebraic solution procedure to reexamines the problem in Chiu et al. [45]. We incorporate the scrap rate in regular production process and failure rate in rework process into Chiu's et al. model, and demonstrate that the optimal production-shipment policy for such an extended model can be derived with derivatives.

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