# Outage Analysis in Multi-user and Multi-relay Cognitive AF Relaying Networks Using MRC 

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Abstract: This paper investigates the outage performance in multi-user and multi-relay cognitive amplify-andforward relaying networks for MinW relay selection scheme over independent non-identically distributed Rayleigh fading channels, where the eavesdropper node may intercept the source message. To guarantee the quality-ofservice of the primary user (PU), both the maximum tolerable peak interference power at the PU and maximum allowable transmit power at secondary users are considered. Relay selection is employed in this paper to improve the secure transmission from secondary source to secondary destination by minimizing instantaneous signal-tonoise ratio at the eavesdropper. For the considered relay selection scheme, we analyze its outage performance and obtain the closed-form lower and upper bounds as well as asymptotic expressions for reliability outage probability (OP) with maximal ratio combining utilized. From the asymptotic expressions, it can be observed that the diversity gain of OP equals to $N+1$, where $N$ is the number of secondary destinations. Finally, Monte-Carlo simulations are presented to validate our analysis results.

Key-Words: Outage probability, amplify-and-forward, cognitive relaying networks, relay selection, maximal ratio combining.

## 1 Introduction

Cognitive radio networks (CRNs) with spectrum sharing are regarded as a promising solution to improve spectral efficiency and solve the problem of spectrum scarcity [1]. In underlay cognitive networks, the secondary users (SUs) can simultaneously access the licensed spectrum of the primary user (PU) without reducing PU's quality-of-service (QoS). Thus, to protect PU from harmful interference, the transmit power constraint at SUs must be considered. Due to the transmit power constraint and the fading nature of wireless channels, the performance of the SU may be degraded considerably. In recent years, to further improve the performance of the secondary networks, incorporating cooperative relaying into cognitive networks has gained extensive attention [2-4].

On the other hand, due to the distributed nature of the broadcasting channel, wireless communication has become more and more vulnerable to serious security threats such as the wiretap of the eavesdroppers, especially in CRNs where many unknown wireless devices can opportunistically access the licensed
spectrum [5]. Physical-layer security of CRNs against eavesdropping attacks have received more and more attention [6-8]. To improve the physical-layer security of wireless transmissions, selection technique has been widely used in CRNs $[6,9,10]$. For example, the multi-relay cognitive DF relaying networks have been considered in [6] and the relay selection scheme has been proposed to schedule the best relay by maximizing the achievable secrecy rate without harming the primary user. In [11], we analyzed the intercept probability for a relay selection scheme, namely MinW scheme, which can minimize the instantaneous SNR at the eavesdropper, however, reliability performance was not studied. As well known, reliability and safety are two important performance metrics.

In this paper, we investigate the reliability outage performance for MinW scheme in multi-user and multi-relay cognitive AF relaying networks in the p resence of eavesdropping attacks with maximal ratio combining (MRC) utilized. Closed-form lower and upper bounds for reliability OP over independent nonidentically distributed (i.n.i.d.) Rayleigh fading chan-
nels are obtained. Furthermore, in order to provide further insights, asymptotic analysis for the corresponding bounds of OP are also presented from which it can be observed that the diversity gain of OP equals to $N+1$, where $N$ is the number of secondary destinations. Specially, to guarantee the QoS of the PU, both the maximum tolerable peak interference power at the PU and maximum allowable transmit power at secondary users (SUs) are considered in this paper.

## 2 System Model and Relay Selection Scheme

We consider a dual-hop cognitive AF relaying network, where multiple SUs can access the same frequency spectrum licensed to the PU simultaneously. Specifically, the considered network is composed of one SU source $S$, $K$ SU relays $\left\{R_{k} \mid k=1,2, \ldots, K\right\}$, $N$ SU destination $\left\{D_{n} \mid n=1,2, \ldots, N\right\}$, one PU receiver PU and one eavesdropper EVE, as shown in Fig. 1. Assume that all terminals in the network are equipped with single antenna, and operating in a halfduplex mode; all links undergo i.n.i.d. Rayleigh fading and all the noise components are additive white Gaussian noise (AWGN) with zero mean and variance $N_{0}$. To guarantee the communication quality at the primary network, total accumulated interference at PU should not exceed the maximum tolerable interference power $\mathcal{Q}$. Thus, the transmit powers at $S$ and $R_{k}$ are strictly governed by $\overline{P_{S}}=\min \left(\mathcal{Q} /\left|h_{S P}\right|^{2}, P_{S}\right)$ and $\overline{P_{R_{k}}}=\min \left(\mathcal{Q} /\left|h_{R_{k} P}\right|^{2}, P_{R_{k}}\right)$, where $P_{S}$ and $P_{R_{k}}$ are the maximum transmit power at $S$ and $R_{k}$, respectively, and $h_{M T}$ is channel coefficient with $M$ and $T$ denoting two arbitrary nodes.

Before data transmission, the best secondary destination $D_{n^{*}}$ is firstly selected based on the channel quality of the direct links, i.e., $n^{*}=\arg \max _{n \in N}\left(\gamma_{S D_{n}}\right)$, where $\gamma_{S D_{n}}=\min \left(\mathcal{Q} /\left|h_{S P}\right|^{2}, P_{S}\right)\left(d_{S D_{n}}^{-\rho}\left|h_{S D_{n}}\right|^{2} / N_{0}\right)$ denotes SNR at $D_{n}$ when $S$ transmits a signal directly to $D_{n}, d_{S D_{n}}$ represents the distance between $S$ and $D_{n}$, and $\rho$ represents the path loss coefficient. After $D_{n^{*}}$ is selected, the communication from $S$ to $D_{n^{*}}$ completes in two phases. In phase I, the SU source broadcasts its information to both $R_{k}$ and $D_{n^{*}}$, while simultaneously the EVE intercepts the information. In phase II, $R_{k}$ amplifies the received signal from the SU source and forwards it to $D_{n^{*}}$, while the EVE also can wiretap the signal transmitted from $R_{k}$. Based upon this procedure, the received SNR at EVE, i.e.,


Fig. 1: System model.
$\gamma_{S R_{k} E}$, and at $D_{n^{*}}$, i.e., $\gamma_{S R_{k} D_{n^{*}}}$, during the relay transmission process, can be expressed as

$$
\begin{align*}
\gamma_{S R_{k} E} & =\frac{\gamma_{S R_{k}} \gamma_{R_{k} E}}{\gamma_{S R_{k}}+\gamma_{R_{k} E}+1},  \tag{1}\\
\gamma_{S R_{k} D_{n^{*}}} & =\frac{\gamma_{S R_{k}} \gamma_{R_{k} D_{n^{*}}}}{\gamma_{S R_{k}}+\gamma_{R_{k} D_{n^{*}}}+1}, \tag{2}
\end{align*}
$$

respectively, where

$$
\begin{gathered}
\gamma_{S R_{k}}=\min \left(\mathcal{Q} /\left|h_{S P}\right|^{2}, P_{S}\right) \frac{d_{S R_{k}}^{-\rho}\left|h_{S R_{k}}\right|^{2}}{N_{0}}, \\
\gamma_{R_{k} E}=\min \left(\mathcal{Q} /\left|h_{R_{k} P}\right|^{2}, P_{R_{k}}\right) \frac{d_{R_{k} E}^{-\rho}\left|h_{R_{k} E}\right|^{2}}{N_{0}}, \\
\gamma_{R_{k} D_{n^{*}}}=\min \left(\mathcal{Q} /\left|h_{R_{k} P}\right|^{2}, P_{R_{k}}\right) \frac{d_{R_{k} D_{n^{*}}}^{-\rho}\left|h_{R_{k} D_{n^{*}}}\right|^{2}}{N_{0}} .
\end{gathered}
$$

In order improve the physical-layer security, from (1), one relay, i.e., $R_{k^{*}}$, is selected following MinW scheme [10, 11]

$$
\begin{equation*}
k^{*}=\arg \min _{k \in K}\left\{\gamma_{S R_{k} E}\right\} \tag{3}
\end{equation*}
$$

Using maximum ratio combing (MRC), the selected destination $D_{n^{*}}$ combines the received data from the SU source and the selected relay $R_{k^{*}}$. Hence, the end-to-end SNR at $D_{n^{*}}$ can be given as

$$
\begin{equation*}
\gamma_{D_{n^{*}}}=\gamma_{S R_{k^{*}} D_{n^{*}}}+\gamma_{S D_{n^{*}}} \tag{4}
\end{equation*}
$$

## 3 Outage Probability Analysis

The outage probability, $P_{\text {out }}\left(\gamma_{\text {th }}\right)$, is defined as the probability that the end-to-end received SNR at the selected secondary destination, i.e., $\gamma_{D_{n^{*}}}$, falls below a specified SNR threshold $\gamma_{\text {th }}$, i.e., $P_{\text {out }}\left(\gamma_{\text {th }}\right)=$ $\operatorname{Pr}\left\{\gamma_{D_{n^{*}}} \leqslant \gamma_{\text {th }}\right\}$. Without any loss of generality, it will be assumed that $P_{R_{k}}=P_{S}=$ $P_{t}$. Define $\beta_{S P} \triangleq 1 / E\left\{\left|h_{S P}\right|^{2}\right\}, \beta_{R_{k} P} \triangleq$ $1 / E\left\{\left|h_{R_{k} P}\right|^{2}\right\}, \beta_{M T}^{P} \triangleq 1 / E\left\{P_{t} d_{M T}^{-\rho}\left|h_{M T}\right|^{2} / N_{0}\right\}$, $\beta_{M T}^{Q} \triangleq 1 / E\left\{\mathcal{Q} d_{M T}^{-\rho}\left|h_{M T}\right|^{2} / N_{0}\right\}$ with $M \in\left\{S, R_{k}\right\}$ and $T \in\left\{R_{k}, D_{n}\right\}$.

In the following, we will investigate the reliability outage performance of the considered network for MinW scheme.

### 3.1 Bounds Analysis for OP

Based on the probability theory, we know that $\operatorname{Pr}\{X+$ $Y \leq c\}<\operatorname{Pr}\{X \leq c\} \operatorname{Pr}\{Y \leq c\}$ if $X>0, Y>0$ and $c>0$, thus, using (4), the upper bound of reliability OP can be obtained by

$$
\begin{align*}
& P_{\text {out,ub }}\left(\gamma_{\text {th }}\right)=\operatorname{Pr}\left\{\max _{n \in N}\left(\gamma_{S D_{n}}\right) \leq \gamma_{\text {th }}\right\} \\
& \quad \times \operatorname{Pr}\left\{\frac{\gamma_{S R_{k^{*}}} \gamma_{R_{k^{*}} D_{n^{*}}}}{\gamma_{S R_{k^{*}}}+\gamma_{R_{k^{*}} D_{n^{*}}}+1} \leq \gamma_{\text {th }}\right\} \tag{5}
\end{align*}
$$

Then, the upper bound of conditional OP can be expressed as

$$
\begin{align*}
& P_{\text {out, ub }}\left(\gamma_{\text {th }} \mid X, Y\right)=\overbrace{\operatorname{Pr}\left\{\max _{n \in N}\left(\gamma_{S D_{n}}\right) \leq \gamma_{\text {th }} \mid X\right\}}^{\mathcal{T}_{1}} \\
& \times \underbrace{\operatorname{Pr}\left\{\left.\frac{\gamma_{S R_{k^{*}}} \gamma_{R_{k^{*}} D_{n^{*}}}}{\gamma_{S R_{k^{*}}}+\gamma_{R_{k^{*}} D_{n^{*}}}+1} \leq \gamma_{\text {th }} \right\rvert\, X, Y\right\}}_{\mathcal{T}_{2}} . \tag{6}
\end{align*}
$$

Since all the links from secondary $S$ to $D_{n^{*}}$ experience Rayleigh fading, $\mathcal{T}_{1}$ can be expressed as

$$
\mathcal{T}_{1}=\prod_{n=1}^{N} F_{\gamma_{S D_{n}}}\left(\gamma_{\mathrm{th}} \mid X\right)=\prod_{n=1}^{N}\left[1-\exp \left(\gamma_{\mathrm{th}} \beta_{S D_{n}}\right)\right]
$$

where $\beta_{M T}=1 / E\left\{\gamma_{M T}\right\}$ with $M \in\left\{S, R_{k}\right\}$ and $T \in\left\{R_{k}, D_{n}, E\right\}$. And $\mathcal{T}_{2}$ in (6) can be written as

$$
\begin{align*}
\mathcal{T}_{2}= & \sum_{n=1}^{N} \operatorname{Pr}\left(n^{*}=n\right) \sum_{k=1}^{K} \operatorname{Pr}\left(k^{*}=k\right) \\
& \times \underbrace{\operatorname{Pr}\left\{\left.\frac{\gamma_{S R_{k}} \gamma_{R_{k} D_{n}}}{\gamma_{S R_{k}}+\gamma_{R_{k} D_{n}}+1} \leq \gamma_{\mathrm{th}} \right\rvert\, X, Y\right\}}_{\psi} \tag{8}
\end{align*}
$$

where

$$
\begin{aligned}
\operatorname{Pr}\left(n^{*}=\right. & n)=1+\sum_{q=1}^{N-1} \\
& \times \sum_{A_{q} \subseteq\{1, \ldots, n-1, n+1, \ldots, N\}}^{\left|A_{q}\right|=q}< \\
& \frac{(-1)^{q} \beta_{S D_{n}}}{\beta_{S D_{n}}+\sum_{j \in A_{q}} \beta_{S D_{j}}} .
\end{aligned}
$$

Utilizing [12, eq. (19)], $\psi$ in (8) can be obtained by

$$
\begin{align*}
\psi=1 & -\beta_{S R_{k}} \exp \left[-\gamma_{\mathrm{th}}\left(\beta_{S R_{k}}+\beta_{R_{k} D_{n}}\right)\right] \\
\times & 2 \sqrt{\frac{\gamma_{\mathrm{th}}\left(\gamma_{\mathrm{th}}+1\right) \beta_{R_{k} D_{n}}}{\beta_{S R_{k}}}} \\
& \times K_{1}\left(2 \sqrt{\gamma_{\mathrm{th}}\left(\gamma_{\mathrm{th}}+1\right) \beta_{S R_{k}} \beta_{R_{k} D_{n}}}\right) \tag{9}
\end{align*}
$$

where $K_{1}(\cdot)$ denotes the first-order modified Bessel function of the second kind [13, eq. (8.432)]. Then, $\operatorname{Pr}\left(k^{*}=k\right)$ can be formulated as

$$
\begin{align*}
& \operatorname{Pr}\left(k^{*}=k\right)=\operatorname{Pr}\left(\bigcap_{\substack{\ell=1 \\
\ell \neq k}}^{K}\left(\gamma_{S R_{k} E}<\gamma_{S R_{\ell} E}\right)\right) \\
& =\int_{0}^{\infty} \underbrace{\left[\prod_{\substack{\ell=1 \\
\ell \neq k}}^{K} \operatorname{Pr}\left(y<\gamma_{S R_{\ell} E}\right)\right]}_{\mathcal{L}} f_{\gamma_{S R_{k} E}}(y) d y . \tag{10}
\end{align*}
$$

Utilizing [12, eq. 19] and recalling $\lim _{x \rightarrow 0} K_{1}(x)=1 / x$, $\mathcal{L}$ in (10) can be obtained,

$$
\begin{align*}
\mathcal{L}= & \prod_{\substack{\ell=1 \\
\ell \neq k}}^{K}\left[1-\operatorname{Pr}\left(\gamma_{S R_{\ell} E} \leq y\right)\right]=\prod_{\substack{\ell=1 \\
\ell \neq k}}^{K}\left[1-F_{\gamma S R_{\ell} E}(y)\right] \\
& =\exp \left[-y \sum_{\substack{\ell=1 \\
\ell \neq k}}^{K}\left(\beta_{S R_{\ell}}+\beta_{R_{\ell} E}\right)\right] \tag{11}
\end{align*}
$$

By substituting (11) into (10) and performing the required integral, a closed-form expression for $\operatorname{Pr}\left(k^{*}=\right.$ $k$ ) is attained as

$$
\operatorname{Pr}\left(k^{*}=k\right)=\frac{\beta_{S R_{k}}+\beta_{R_{k} E}}{\beta_{S R_{k}}+\beta_{R_{k} E}+\sum_{\substack{\ell=1 \\ \ell \neq k}}^{K}\left(\beta_{S R_{\ell}}+\beta_{R_{\ell} E}\right)} .
$$

In addition, it is true that

$$
\min \left(\frac{\mathcal{Q}}{X}, P_{S}\right)= \begin{cases}P_{S}, & \text { if } X \leq \mathcal{Q} / P_{S}  \tag{12}\\ \mathcal{Q} / X, & \text { if } X>\mathcal{Q} / P_{S}\end{cases}
$$

$\min \left(\frac{\mathcal{Q}}{Y}, P_{R_{k}}\right)= \begin{cases}P_{R_{k}}, & \text { if } Y \leq \mathcal{Q} / P_{R_{k}}, \\ \mathcal{Q} / Y, & \text { if } Y>\mathcal{Q} / P_{R_{k}} .\end{cases}$

Then, the upper bound of OP can be expressed as

$$
\begin{align*}
& P_{\text {out }, \text { ub }}\left(\gamma_{\text {th }}\right)=\int_{0}^{\infty} \int_{0}^{\infty} \mathcal{T}_{1} \mathcal{T}_{2} f_{X}(x) f_{Y}(y) \mathrm{d} x \mathrm{~d} y \\
& =\xi_{1}\left(\gamma_{\text {th }}\right)+\xi_{2}\left(\gamma_{\text {th }}\right)+\xi_{3}\left(\gamma_{\text {th }}\right)+\xi_{4}\left(\gamma_{\text {th }}\right) \tag{14}
\end{align*}
$$

where
$\xi_{1}\left(\gamma_{\mathrm{th}}\right)=\Xi\left(0, \frac{\mathcal{Q}}{P_{t}}, 0, \frac{\mathcal{Q}}{P_{t}}\right), \xi_{2}\left(\gamma_{\mathrm{th}}\right)=\Xi\left(0, \frac{\mathcal{Q}}{P_{t}}, \frac{\mathcal{Q}}{P_{t}}, \infty\right)$
$\xi_{3}\left(\gamma_{\mathrm{th}}\right)=\Xi\left(\frac{\mathcal{Q}}{P_{t}}, \infty, 0, \frac{\mathcal{Q}}{P_{t}}\right), \xi_{4}\left(\gamma_{\mathrm{th}}\right)=\Xi\left(\frac{\mathcal{Q}}{P_{t}}, \infty, \frac{\mathcal{Q}}{P_{t}}, \infty\right)$
with $\Xi(a, b, c, d)=\int_{a}^{b} \int_{c}^{d} \mathcal{T}_{1} \mathcal{T}_{2} f_{X}(x) f_{Y}(y) \mathrm{d} x \mathrm{~d} y$. In this sequel, it is assumed that the links from $S$ to $R_{k}$ experience independent identically distributed (i.i.d.) Rayleigh fading, i.e., $\beta_{S R_{k}}=\beta_{S R}, \forall k$. The same assumption can be made for the secondary destinations and the PU receiver, i.e., $\beta_{S D_{n}}=\beta_{S D}, \beta_{R_{k} D_{n}}=$ $\beta_{R D}, \beta_{R_{k} P}=\beta_{R P} \forall k, n$. Note that the channels pertaining to different hops experience distinct fading conditions from each other. Since $\left|h_{S P}\right|^{2}$ and $\left|h_{R P}\right|^{2}$ are Rayleigh distribution, the probability density function (PDF) of $\left|h_{S P}\right|^{2}$ and $\left|h_{R P}\right|^{2}$ can be expressed as

$$
\begin{equation*}
f_{X}(x)=\beta_{S P} e^{-x \beta_{S P}}, f_{Y}(y)=\beta_{R P} e^{-y \beta_{R P}} \tag{15}
\end{equation*}
$$

respectively. Substituting (7), (8) and (15) into (14), and after some manipulations, one can obtain

$$
\begin{align*}
& \xi_{1}\left(\gamma_{\text {th }}\right)=\prod_{n=1}^{N}\left[1-\exp \left(-\gamma_{\text {th }} \beta_{S D_{n}}^{P}\right)\right] \sum_{n=1}^{N} \operatorname{Pr}\left(n^{*}=n\right) \\
& \times\left(1-\mathcal{A}_{1}\right)\left(1-\mathcal{A}_{2}\right) \sum_{k=1}^{K} \operatorname{Pr}\left(k^{*}=k\right) \\
& \times\left[1-2 \beta_{S R_{k}}^{P} \exp \left[-\gamma_{\text {th }}\left(\beta_{S R_{k}}^{P}+\beta_{R_{k} D_{n}}^{P}\right)\right]\right. \\
& \times \sqrt{\left.\frac{x(x+1) \beta_{R_{k} D_{n}}^{P}}{\beta_{S R_{k}}^{P}} K_{1}\left(2 \sqrt{x(x+1) \beta_{S R_{k}}^{P} \beta_{R_{k} D_{n}}^{P}}\right)\right],} \begin{array}{l}
\xi_{2}\left(\gamma_{\text {th }}\right)=\prod_{n=1}^{N}\left[1-\exp \left(-\gamma_{\text {th }} \beta_{S D_{n}}^{P}\right)\right] \sum_{n=1}^{N} \operatorname{Pr}\left(n^{*}=n\right) \\
\times\left(1-\mathcal{A}_{1}\right) \mathcal{A}_{2} \sum_{k=1}^{K} \operatorname{Pr}\left(k^{*}=k\right) \\
\times\left(1-\frac{\beta_{R P} \exp }{}\left[-\gamma_{\text {th }}\left(\beta_{S R_{k}}^{P}+\frac{\mathcal{Q}}{P_{t}} \beta_{R_{k} D_{n}}^{Q}\right)\right]\right. \\
\gamma_{\text {th }} \beta_{R_{k} D_{n}}^{Q}+\beta_{R P}
\end{array}, \quad(17)  \tag{16}\\
& \xi_{3}\left(\gamma_{\text {th }}\right)=\sum_{m=0}^{N}\binom{N}{m}(-1)^{m} \sum_{n=1}^{N} \operatorname{Pr}\left(n^{*}=n\right) \sum_{k=1}^{K} \\
& \times \operatorname{Pr}\left(k^{*}=k\right) \mathcal{A}_{1}\left(1-\mathcal{A}_{2}\right) \beta_{S P} \\
& \times \exp \left(-\gamma_{\text {th }} m \frac{\mathcal{Q}}{P_{t}} \beta_{S D_{m}}^{Q}\right)\left(\frac{1}{\gamma_{\text {th }} m \beta_{S D_{m}}^{Q}+\beta_{S P}}\right.  \tag{17}\\
& \quad \exp \left[-\gamma_{\text {th }}\left(\frac{\mathcal{Q}}{P_{t}} \beta_{S R_{k}}^{Q}+\beta_{R_{k} D_{m}}^{P}\right)\right] \\
& \left.\quad-\frac{\gamma_{\text {th }}\left(m \beta_{S D_{m}}^{Q}+\beta_{S R_{k}}^{Q}\right)+\beta_{S P}}{}\right),
\end{align*}
$$

$$
\begin{align*}
& \xi_{4}\left(\gamma_{\mathrm{th}}\right)=\sum_{m=0}^{N}\binom{N}{m}(-1)^{m} \sum_{n=1}^{N} \operatorname{Pr}\left(n^{*}=n\right) \sum_{k=1}^{K} \\
& \times \operatorname{Pr}\left(k^{*}=k\right) \mathcal{A}_{1} \mathcal{A}_{2} \beta_{S P} \beta_{R P} \exp \left(-\gamma_{\mathrm{th}} m \frac{\mathcal{Q}}{P_{t}} \beta_{S D_{m}}^{Q}\right) \\
& \times\left\{-\frac{\exp \left[-\gamma_{\mathrm{th}} \frac{\mathcal{Q}}{P_{t}}\left(\beta_{S R_{k}}^{Q}+\beta_{R_{k} D_{m}}^{Q}\right)\right]}{\left(\gamma_{\mathrm{th}}\left(m \beta_{S D_{m}}^{Q}+\beta_{S R_{k}}^{Q}\right)+\beta_{S P}\right)\left(\gamma_{\mathrm{th}} \beta_{R_{k} D_{n}}^{Q}+\beta_{R P}\right)}\right. \\
& \left.\quad+\frac{1}{\beta_{R P}\left(\gamma_{\mathrm{th}} m \beta_{S D_{m}}^{Q}+\beta_{S P}\right)}\right\} . \tag{19}
\end{align*}
$$

where $\mathcal{A}_{1}=\exp \left(-\frac{\mathcal{Q}}{P_{t}} \beta_{S P}\right)$ and $\mathcal{A}_{2}=\exp \left(-\frac{\mathcal{Q}}{P_{t}} \beta_{R P}\right)$.
It can be observed that $\gamma_{D^{*}}$ in (4) is upper bounded by $\gamma_{D^{*}} \leqslant 2 \max \left\{\gamma_{S D^{*}}, \gamma_{S R_{k^{*}} D^{*}}\right\}$, as a result, the lower bound of OP can be obtained as

$$
\begin{align*}
& P_{\text {out }, \mathrm{lb}}\left(\gamma_{\mathrm{th}}\right)=\operatorname{Pr}\left\{\gamma_{S D_{n^{*}}} \leq \frac{\gamma_{\mathrm{th}}}{2}\right\} \\
& \quad \times \operatorname{Pr}\left\{\frac{\gamma_{S R_{k^{*}}} \gamma_{R_{k^{*}} D_{n^{*}}}}{\gamma_{S R_{k^{*}}}+\gamma_{R_{k^{*}} D_{n^{*}}}+1} \leq \frac{\gamma_{\mathrm{th}}}{2}\right\} \\
& \quad=\xi_{1}\left(\frac{\gamma_{\mathrm{th}}}{2}\right)+\xi_{2}\left(\frac{\gamma_{\mathrm{th}}}{2}\right)+\xi_{3}\left(\frac{\gamma_{\mathrm{th}}}{2}\right)+\xi_{4}\left(\frac{\gamma_{\mathrm{th}}}{2}\right) \tag{20}
\end{align*}
$$

### 3.2 Asymptotic Analysis for OP

To provide further insights into the performance and diversity order of the system, we now investigate the asymptotic expressions for OP in the high-SNR regime. Without loss of generality, define $\bar{\gamma}=1 / N_{0}$ as the average SNR and assume $Q / P_{t}=\mu$, where $\mu$ is a positive constant. Thus, $\beta_{M T}^{P}=1 /\left(\bar{\gamma} E\left\{P_{t} d_{M T}^{-\rho}\left|h_{M T}\right|^{2}\right\}\right), \beta_{M T}^{Q}=$ $1 /\left(\bar{\gamma} E\left\{\mathcal{Q} d_{M T}^{-\rho}\left|h_{M T}\right|^{2}\right\}\right)$ with $M \in\left\{S, R_{k}\right\}$ and $T \in\left\{R_{k}, D_{n}\right\}$. Consider the facts (1) $\lim _{a \rightarrow 0} e^{-a x}=$ $1-a x$; (2) $\lim _{x \rightarrow 0} K_{1}(x)=1 / x$. Using these facts and after some algebraic manipulations, when $\bar{\gamma} \rightarrow \infty$, $\xi_{1}(x), \xi_{2}(x), \xi_{3}(x)$ and $\xi_{4}(x)$ become

$$
\begin{gather*}
\xi_{1}^{\infty}(x) \simeq \prod_{m=1}^{N}\left(x \beta_{S D_{m}}^{P}\right) \sum_{n=1}^{N} \operatorname{Pr}\left(n^{*}=n\right) \sum_{k=1}^{K} \\
\times \operatorname{Pr}\left(k^{*}=k\right)\left(1-e^{-\mu \beta_{S P}}\right)\left(1-e^{-\mu \beta_{R P}}\right) \\
\times\left[x\left(\beta_{S R_{k}}^{P}+\beta_{R_{k} D_{m}}^{P}\right)\right] \propto\left(\frac{1}{\bar{\gamma}}\right)^{N+1}  \tag{21}\\
\xi_{2}^{\infty}(x) \simeq \prod_{m=1}^{N}\left(x \beta_{S D_{m}}^{P}\right) \sum_{n=1}^{N} \operatorname{Pr}\left(n^{*}=n\right) \sum_{k=1}^{K} \\
\times \operatorname{Pr}\left(k^{*}=k\right)\left(1-e^{-\mu \beta_{S P}}\right) e^{-\mu \beta_{R P}} \\
\times\left[x\left(\beta_{S R_{k}}^{P}+\mu \beta_{R_{k} D_{m}}^{Q}\right)\right] \propto\left(\frac{1}{\bar{\gamma}}\right)^{N+1}  \tag{22}\\
\xi_{3}^{\infty}(x) \simeq \prod_{m=1}^{N}\left(x \mu \beta_{S D_{m}}^{Q}\right) \sum_{n=1}^{N} \operatorname{Pr}\left(n^{*}=n\right) \sum_{k=1}^{K} \\
\times \operatorname{Pr}\left(k^{*}=k\right)\left(1-e^{-\mu \beta_{R P}}\right) e^{-\mu \beta_{S P}} \\
\quad \times\left[x\left(\mu \beta_{S R_{k}}^{Q}+\beta_{R_{k} D_{m}}^{P}\right)\right] \propto\left(\frac{1}{\bar{\gamma}}\right)^{N+1} \tag{23}
\end{gather*}
$$



Fig. 2: OP and asymptotic behavior versus system SNR $\bar{\gamma}$ for different numbers of secondary destinations with $K=2, P_{t}=\mathcal{Q}=0.5$.

$$
\begin{align*}
& \xi_{4}^{\infty}(x) \simeq \prod_{m=1}^{N}\left(x \mu \beta_{S D_{m}}^{Q}\right) \sum_{n=1}^{N} \operatorname{Pr}\left(n^{*}=n\right) \sum_{k=1}^{K} \\
& \quad \times \operatorname{Pr}\left(k^{*}=k\right) e^{-\mu\left(\beta_{S P}+\beta_{R P}\right)} \\
& \quad \times\left[x \mu\left(\beta_{S R_{k}}^{Q}+\beta_{R_{k} D_{m}}^{Q}\right)\right] \propto\left(\frac{1}{\bar{\gamma}}\right)^{N+1} . \tag{24}
\end{align*}
$$

Thus, substituting these results into (14) and (20), the corresponding asymptotic approximation for upper and lower bounds of OP can be obtained. As observed from the above expressions, the diversity gain equals to $N+1$ indicating it only depends on the number of secondary destinations, which means the diversity order is independent of the number of secondary relays.

## 4 Numerical and Computer Simulation Results

In this section, Monte-Carlo simulation is provided to validate our analytical expressions. Specifically, we consider a 2-D plane network, where the SU source is located at $(0,0)$, the SU relays and destinations are clustered together and located at $(1 / 2,0)$ and $(1,0)$, respectively, the PU is located at $(0,1)$, and the eavesdropper EVE is located at $(1 / 2,1)$. Without loss of generality, we assume that the average channel gains is determined by the distance among the nodes, and we set the threshold $\gamma_{\text {th }}$ to 5 dB for Figs. 2 and 4 and the path loss coefficient $\rho$ to 4. The "Anal." and "Asy."


Fig. 3: OP and asymptotic behavior versus system SNR $\bar{\gamma}$ for different numbers of secondary relays with $N=2, P_{t}=\mathcal{Q}=0.5$.
curves in Figs. 2 and 3 represent the lower bounds of OP and the asymptotic approximation for lower bounds of OP, respectively, the upper bound of the OP is not shown to avoid entanglement. Observed from Figs. 2-4, the derived lower and upper bounds of OP are both very tight with their corresponding simulation results, thus validating the correctness of our analysis results.

Fig. 2 depicts the impact of the number of secondary destinations $N$ on reliability OP of the CRN with $K=2$ and $P_{t}=\mathcal{Q}=0.5$. As observed from this figure, reliability performance improves as the number of secondary destinations $N$ increases. In addition, it can be seen that the diversity gain increases as $N$ increases. In addition, the diversity order of reliability OP is independent of the number of relays i.e., $K$, just as our preceding analysis.

Fig. 3 shows the outage probability versus system SNR $\bar{\gamma}$ for different numbers of secondary relays when threshold $\gamma_{\text {th }}=\{5,10\} \mathrm{dB}$, assuming $N=2, P_{t}=\mathcal{Q}=0.5$. It should be noted that the outage performance improves as $\gamma_{\text {th }}$ decreases. As shown, when $\gamma_{\text {th }}$ is a certain value, the OP curves overlap completely when the number of secondary relays $K=2$, 4 , which means diversity and coding gains of the considered network are independent of the number of secondary relays.

Fig. 4 displays the impact of maximum transmit power $P_{t}$ on reliability OP when $K=N=2$ with interference temperature $\mathcal{Q}=[3,9,15] \mathrm{dB}$. It is observed that when $\mathcal{Q}$ is a certain value, the OP tends


Fig. 4: Impact of $P_{t}$ on outage probability with $K=$ $N=2$.
to be stable with the increase in $P_{t}$. This is because, when $P_{t}$ is large enough, $\mathcal{Q}$ will limit the transmit power of the secondary nodes thus determining the outage performance. Similar impact of the maximum tolerable interference power $\mathcal{Q}$ on OP with fixed $P_{t}$ can be observed which is omitted here due to space limitation.

## 5 Conclusion

The outage performance of multi-user and multi-relay AF CRNs for MinW scheme has been investigated over i.n.i.d. Rayleigh fading channels in this paper. Specifically, closed-form lower and upper bounds as well as asymptotic expressions of reliability OP have been derived. Our analysis reveals that the diversity gain of reliability OP for MinW scheme equals to $N+1$, where $N$ is the number of secondary destinations, independent of the number of secondary relay nodes. Finally, simulation results are provided to validate the analysis.

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