Joint Estimation of Angle, Doppler and Polarization Parameters for Narrowband MIMO Multipath Channels Using Polarization Sensitive Antenna Arrays

RAMONI ADEOGUN
Victoria University of Wellington
School of Engineering and Computer Science
Kelburn, Wellington
NEW ZEALAND
ramoni.adeogun@ecs.vuw.ac.nz

Abstract: A novel subspace based joint angle of arrival (AOA), angles of departure (AOD), Doppler shifts and polarization states parameter estimation scheme for polarized two-dimensional (2D) double directional MIMO multipath channels is proposed in this paper. A narrowband system with non-polarized uniform linear array at the transmitter and cross-polarized antenna array at the receiver is considered. The proposed algorithm perform a simple transformation on the estimated channel state information (CSI) matrix in such a manner that multidimensional ESPRIT can be utilized to exploit the translational invariance in the angle, Doppler and polarization dimensions. Simulation results show that the proposed algorithm can accurately estimate the AOA, AOD, Doppler shifts and polarization angles of the multipath channel even for closely spaced scattering sources.

Key–Words: Polarization, MIMO, Multidimensional ESPRIT, Multipath Parameter Estimation

1 Introduction

Mobile MIMO wireless communication systems using polarized antenna arrays at the transmitter and/or receiver have received considerable research attention recently. Since polarization diversity reduces the spatial correlation between antennas [1], more antennas can be deployed within the limited space. Thus, polarized MIMO systems offer several potential benefits (e.g. increased capacity, space and cost efficiency and improved parameter estimation accuracy) when compared with classical uni-polarized MIMO systems.

In order to characterize the performance of MIMO systems with polarization sensitive antenna arrays, several channel models have been proposed which account for polarization in both 2D (e.g. 3GPP SCM [2], WINNER phase II [3]) and three-dimension (see e.g. [4], [5], [6]). The receiver and/or scatterers movement in the propagation environment make the multipath channel time-variant. However, the parameters of the channel such AOA, AOD, delay, Doppler shifts, and polarization angles exhibit very slow variation when compared with the actual channel. These parameters can therefore be considered quasistatic over a sufficiently long time interval.

Estimation of multipath parameters such as AOA, AOD, delay, Doppler shifts and polarization parameters is a key problem for several applications including sonar, radar, wireless communications, localization and wireless intelligent networks [7]. Although several algorithms have been proposed for the estimation of AOAs, DOAs and Doppler shifts from received signal at the mobile station antenna array, there exists very few results on the estimation of angle of departure (AOD) and polarization parameters. In [8], a method for joint angle and delay estimation (JADE) was proposed to jointly estimate the AOA and DOA of narrowband multipath sources. A similar method exploiting the rotational invariance structure in a collection of space-time estimates was also proposed in [9]. The authors in [10] proposed a Multiple Signal Classification (MUSIC) [11] based approach for the joint estimation of AOA, AOD and DOA in narrowband MIMO channels. The methods in [8] and [10] exploit the eigen-structure of the channel covariance matrix. The parameters are however estimated via a 2D search of the peaks of the MUSIC spectrum making the algorithms computationally intensive. In [12], a method based on the classical beamforming...
techniques was proposed to jointly estimate the AOA, AOD and DOA of multipath sources in narrowband MIMO systems. A 2D unitary ESPRIT based method for joint AOA and AOD estimation was proposed in [13].

Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) [14] is a computationally efficient and high resolution parameter estimation algorithm. It eliminates the spectral search requirement in other subspace methods and does not require explicit knowledge of the array geometry. Using the idea of multiple rotational invariance [15], [16], we develop a novel algorithm for the joint estimation of AOA, AOD, Doppler shifts and polarization angles of propagation paths in narrowband MIMO propagation environments.

The rest of the paper is organised as follows. The system model is presented in Section 2. The double directional polarized multipath MIMO model is discussed in Section 3. In Section 4, we develop the proposed joint parameter estimation algorithm. The simulation results and performance analysis are presented in Section 5. Finally conclusions are drawn in Section 6.

2 System Model

Consider a narrowband MIMO system with M non-polarized uniform linear array (ULA) antenna elements at the transmitter and N cross-polarized antenna elements at the mobile station (MS). The received baseband signal at the MS can be expressed as

\[ r(t) = H(t)s(t) + v(t) \]  

(1)

where \( r(t) = [r_1(t), r_2(t), \ldots, r_{2N}(t)]^T \in \mathbb{C}^{2N \times 1} \) is the received signal vector, \( s(t) = [s_1(t), s_2(t), \ldots, s_M(t)]^T \in \mathbb{C}^{M \times 1} \) is the transmitted signal vector on the \( M \) transmit antennas, \( v(t) = [v_1(t), v_2(t), \ldots, v_{2N}(t)]^T \in \mathbb{C}^{2N \times 1} \) is the zero mean complex Gaussian receive noise vector and \( H(t) \in \mathbb{C}^{2N \times M} \) is the polarized MIMO impulse response matrix. Assuming \( L \) pilot training symbols \([p_1(\ell), p_2(\ell), \ldots, p_L(\ell)]\) each of length \( K \) is transmitted on each of the \( M \) transmit antenna during the \( L \) symbol durations. Assuming that the channel is invariant over each symbol duration, the received pilot symbol is given by

\[ Y(\ell) = [y_1(\ell), \ldots, y_K(\ell)]; \quad \forall \ell = 1, 2, \ldots, L \]

\[ = H(\ell)S_p(\ell) + V(\ell) \]  

(2)

where \( S_p(\ell) = [p_1(\ell), \ldots, p_K(\ell)] \in \mathbb{C}^{2N \times K} \) is the pilot signal matrix for the \( \ell \)th symbol duration, \( V(\ell) = [v_1(\ell), \ldots, v_K(\ell)] \in \mathbb{C}^{2N \times K} \) is the noise matrix and the channel matrix \( H(\ell) \) is defined as

\[
H(\ell) = \begin{bmatrix}
h_{1,1} & h_{1,2} & \cdots & h_{1,M} \\
h_{2,1} & h_{2,2} & \cdots & h_{2,M} \\
\vdots & \vdots & \ddots & \vdots \\
h_{2N,1} & h_{2N,1} & \cdots & h_{2N,M} 
\end{bmatrix} \in \mathbb{C}^{2N \times M}
\]

(3)

The Least square (LS) channel estimation method can be applied on (2) to find the pilot channel estimates \( \hat{H} \) which minimizes the objective function

\[
G(\hat{H}(\ell)) = ||Y(\ell) - \hat{H}(\ell)S_p(\ell)||^2 \quad \forall \ell = 1, 2, \ldots, L
\]

\[
= (Y(\ell) - \hat{H}(\ell)S_p(\ell))^H(Y(\ell) - \hat{H}(\ell)S_p(\ell))
\]

\[
= Y^H(\ell)Y(\ell) - Y^H(\ell)\hat{H}(\ell)S_p(\ell)
\]

\[
- S_p^H\hat{H}^H(\ell)Y(\ell)
\]

\[
+ S_p^H(\ell)\hat{H}^H(\ell)\hat{H}(\ell)S_p(\ell)
\]

(4)

where \([.]^H\) denotes the Hermitian conjugate transpose. Differentiating (4) and equating to zero gives the pilot channel estimates as

\[
\hat{H}(\ell) = (S_p^H(\ell)S_p(\ell))^{-1}S_p^H(\ell)Y(\ell) \quad \ell = 1, 2, L
\]

\[
= (S_p(\ell))^\dagger Y(\ell)
\]

(5)

where \((.)^\dagger\) denotes the Moore-penrose psudoinverse.

3 Double Directional Polarized MIMO Model

The \( p \)th propagation path can be characterized by delay \( \tau_p \), angle of arrival (AOA) \( \theta_p \), angle of departure (AOD) \( \phi_p \), Doppler frequency \( \nu_p \), polarization angles \( (\varphi_p, \varepsilon_p) \) and complex amplitude \( \alpha_p \). Assuming that the multipath parameters are constant over the region of interest, the double direction MIMO impulse response can be modelled as

\[
H(\ell) = \sum_{p=1}^{P} \alpha_p a_r(\theta_p, \varphi_p, \varepsilon_p) a_t^T(\phi_p) \exp(j2\pi \ell \Delta t \nu_p)
\]

(6)

where \( \ell \) denotes the sample index, \( \Delta t \) is the time domain sampling interval, \( \nu_p = \frac{K}{2} \cos(\theta_p - \theta_v) \), \( \theta_v \) is the mobile station direction, \( a_t(\phi_p) \) is the \( N \times 1 \) transmit array steering vector in the direction of a plane
wave departing with \( \phi_p \) and \( a_r(\theta_p, \varphi_p, \varepsilon_p) \) denotes the \( 2M \times 1 \) polarization dependent receive array steering vector. For a ULA of omnidirectional antennas spaced \( \delta r \) apart, the receive steering vector is defined as

\[
a_r(\theta_p, \varphi_p, \varepsilon_p) = \begin{bmatrix} 1 \\ z_p^2 \\ \vdots \\ z_p^{M-1} \end{bmatrix} \otimes \begin{bmatrix} -\cos \vartheta_p \\ \sin \vartheta_p \cos \theta_p \exp(j\varphi_p) \end{bmatrix}
\]

where \( z_p = \exp(-j2\pi \delta r \sin \theta_p) \) and the angles \( \vartheta_p \) and \( \varphi_p \) are the re-parametrization of the polarization on the Poincare sphere. Details of the relation between the ellipticity angle \( \varphi_p \), tilt angle \( \varphi_p \), and \( \varphi_p, \varepsilon_p \) can be found in [17]. The transmit array steering vector is similarly defined for a ULA as

\[
a_t(\phi_p) = \begin{bmatrix} \frac{1}{2} \\ \exp(-j2\pi \delta t \sin \phi_p) \\ \vdots \\ \exp(-j2(M-1)\pi \delta t \sin \phi_p) \end{bmatrix}
\]

where \( \delta t \) is the transmit antenna spacing in wavelengths.

4 Joint Parameter Estimation

Applying a vectorization operation to the \( L \) pilot channel estimates in (5), we obtain the \( 2NM \times L \) matrix

\[
\mathcal{H} = \begin{bmatrix} \text{vec}(H(1)) \\ \text{vec}(H(2)) \\ \vdots \\ \text{vec}(H(L)) \end{bmatrix}
\]

where \( \text{vec}(A) \) denotes the vectorization operation which stacks the columns of \( A \). By sliding a \( 2NM \times R \) window through the matrix in (9) and performing the vectorizing the resulting matrix, we obtain

\[
h(s) = \text{vec}(\mathcal{H}(\cdot, s : R + s)) \; s = 1, 2, \ldots, S
\]

where \( S = L - R + 1 \). Using (7) and the vectorization in (10), it can be shown that (10) can be expressed as

\[
h(s) = Z_s(\theta, \phi, \varphi, \varepsilon, \nu) \alpha
\]

where \( Z_s(\theta, \phi, \varphi, \varepsilon, \nu) = [a_r(\theta_1, \varphi_1, \varepsilon_1) \otimes a_t(\varphi_1) \otimes a_r(\nu_1), \ldots, a_r(\theta_p, \varphi_p, \varepsilon_p) \otimes a_t(\varphi_p) \otimes a_r(\nu_p)] \in \mathbb{C}^{2NM \times P} \) is the space-time-polarization manifold matrix, \( \alpha = [\alpha_1, \alpha_2, \ldots, \alpha_P]^T \in \mathbb{C}^P \times 1 \) is a vector containing the complex amplitudes. The time domain manifold vector \( a_d^s(\nu_p) \) is defined as

\[
a_d^s(\nu_p) = [u_p^{(s-1)}, u_p^{(s)}, \ldots, u_p^{(s+R-1)}]^T
\]

where \( u_p = \exp(j2\pi \Delta \nu_p) \). In the presence of estimation noise, the covariance matrix estimate is defined as

\[
\hat{\mathbf{C}} = \frac{1}{S} \sum_{s=1}^{S} \hat{h}(s)\hat{h}(s)^H = Z_s(\theta, \phi, \varphi, \varepsilon, \nu)R_{\alpha\alpha}Z_s^H(\theta, \phi, \varphi, \varepsilon, \nu) + \sigma^2 \mathbf{I}
\]

where \( \cdot^H \) denotes the Hermitian transpose, \( R_{\alpha\alpha} = \frac{1}{S} \sum_{s=1}^{S} \alpha \alpha^H \) and \( \sigma^2 \) is the noise variance. Let \( U_s \) be the signal subspace eigenvectors corresponding to the \( P \) largest eigenvalues of \( \hat{\mathbf{C}} \). Since \( \hat{U}_s \) and \( Z_s \) span the same signal subspace, the invariance structure [14] in \( Z_s \) can be exploited to estimate the five parameters by applying an ESPRIT-like procedure on \( U_s \). This requires a set of selection matrices defined as

\[
\begin{align*}
J_{1r} &= [I_{(N-1)} \ 0_{(N-1)}] \\
J_{2r} &= [0_{(N-1)} \ I_{(N-1)}] \\
J_{1t} &= [I_{(M-1)} \ 0_{(M-1)}] \\
J_{2t} &= [0_{(M-1)} \ I_{(M-1)}] \\
J_{1d} &= [I_{(R-1)} \ 0_{(R-1)}] \\
J_{2d} &= [0_{(R-1)} \ I_{(R-1)}] \\
J_{1p} &= [I_{(2-1)} \ 0_{(2-1)}] \\
J_{2p} &= [0_{(2-1)} \ I_{(2-1)}]
\end{align*}
\]

where the subscript indices \( r, t, d, p \) denotes receive, transmit, Doppler and polarization respectively, \( I_b \) is an \( b \times b \) identity matrix and \( 0_b \in \mathbb{R}^b \) is a vector of zeros. The invariance equations in each of the four dimensions can therefore be defined as

\[
\begin{align*}
J_{r2}U_s &= J_{r1}U_s \Theta \\
J_{t2}U_s &= J_{t1}U_s \Phi \\
J_{d2}U_s &= J_{d1}U_s N \\
J_{p2}U_s &= J_{p1}U_s T
\end{align*}
\]

Since \( U_s \) and \( Z_s \) are rotated forms of each other lying in the same signal subspace, it can be easily shown that eigendecomposition of the matrices \( \Theta, \Phi, N \) and
The diagonal eigenvalue matrices are then obtained using (16). Denoting the common eigenvector matrices in (16), we utilized a scheme similar to the mean eigenvalue decomposition (MEVD) pairing scheme [18]. Denoting \[ \Theta = \sum \Lambda \Sigma^{-1} \] (17)

where \( \Sigma \) denote the common eigenvector matrices in (16). The diagonal eigenvalue matrices are then obtained using

\[
\begin{align*}
\Xi_r & = \Sigma^{-1} \Theta \Sigma & (18a) \\
\Xi_t & = \Sigma^{-1} \Phi \Sigma & (18b) \\
\Xi_d & = \Sigma^{-1} N \Sigma & (18c) \\
\Xi_p & = \Sigma^{-1} T \Sigma & (18d)
\end{align*}
\]

where \( \Xi_r = \text{eig}(\Theta) \), \( \Xi_t = \text{eig}(\Phi) \), \( \Xi_d = \text{eig}(N) \) and \( \Xi_p = \text{eig}(T) \). The AOA, AOD and Doppler shifts can respectively be estimated from (18a), (18b) and (18c) as

\[
\begin{align*}
\theta & = \sin^{-1} \left( \frac{-\text{arg}(\text{diag}(\Xi_\theta))}{2 \pi \delta r} \right) \\
\phi & = \sin^{-1} \left( \frac{-\text{arg}(\text{diag}(\Xi_\phi))}{2 \pi \delta t} \right) \\
\nu & = \left( \frac{\text{arg}(\text{diag}(\Xi_\nu))}{\Delta t} \right)
\end{align*}
\]

(19)

In order to estimate the polarization angles, we define the ratio of the entries of the polarimetric vector in (7) for each propagation path as

\[
e_p = \frac{-\cos \varphi}{\sin \theta_p \cos \theta_p \exp(j \nu_p)}
\]

(20)

Denoting \( e = [e_1, e_2, \ldots, e_p] \), it can be shown that (18d) gives an estimate of \( e \) as

\[
\hat{e} = \text{diag}(\Xi_p)
\]

(21)

Using (21), (20) and estimates of the AOA, the polarization angles for each path are obtained as

\[
\varphi_p = \tan^{-1}\left( \frac{1}{e_p \cos \theta_p} \right)
\]

\[
\nu_p = \arg \left( \frac{1}{e_p \cos \theta_p} \right)
\]

(22)

where \(|.|\) denote the absolute value of the associated scalar.

## 5 Simulation Results

In this section, we analyse the performance of the ESPRIT based joint parameter estimation method. The estimation error of the algorithm is evaluated in terms of root mean square error (RMSE) criterion defined as

\[
\text{RMSE}(\sigma) = \sqrt{\frac{1}{J} \sum_{j=1}^{J} |\hat{\sigma} - \sigma|^2}
\]

(23)

where \( J \) denotes the number of Monte Carlo trials. We considered a MIMO system with 2 antenna elements and 2 pairs of polarization sensitive crossed dipoles at the transmitter and receive respectively. The mobile velocity \( V = 50 \text{ Kmph} \) and 500 samples of the channel with sampling interval \( \Delta t = 2 \text{ ms} \) is generated for each realization.

**Case I**: We consider a four-path channel model with parameters as specified in Table 1. The complex amplitudes of the paths is normally distributed (i.e. \( \alpha_p \sim N(0,1) \)). Figures 1-5 show the root mean square error as a function of SNR for each of the parameters. As shown in the figures, the performance of proposed algorithm improves with increasing SNR. However, the RMSE in the low SNR region is within acceptable limits.

**Case II**: A more realistic channel consisting of ten propagation paths is considered in this stage. The RMSE error for each of the parameters is averaged over the paths. As shown in Figure 6, the algorithm is able to effectively resolve all the ten paths. This shows that the identifiability limit of the method is far beyond the number of antennas at both the transmit and receive end.

**Case III**: A more difficult scenario where the paths are closely spaced and share a common AOA and AOD is simulated. The AOA for all the paths are spaced \( \frac{\pi}{109} \) apart and are equal to the AOD. The true and estimated values of the parameters are presented
Table 1: Parameters of the four-path channel

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Path 1</th>
<th>Path 2</th>
<th>Path 3</th>
<th>Path 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>θ (rad)</td>
<td>0.9198</td>
<td>-1.4366</td>
<td>0.7540</td>
<td>-0.1295</td>
</tr>
<tr>
<td>φ (rad)</td>
<td>-0.6080</td>
<td>-0.1726</td>
<td>0.1714</td>
<td>-1.0426</td>
</tr>
<tr>
<td>ϑ (rad)</td>
<td>1.4437</td>
<td>0.4593</td>
<td>0.3788</td>
<td>0.1402</td>
</tr>
<tr>
<td>ς (rad)</td>
<td>0.9384</td>
<td>2.6469</td>
<td>0.5443</td>
<td>0.2322</td>
</tr>
<tr>
<td>ν (rad/s)</td>
<td>370.453</td>
<td>81.807</td>
<td>445.655</td>
<td>606.234</td>
</tr>
</tbody>
</table>

As shown in the table, the parameters are accurately estimated. This shows that the scheme can achieve high resolution of parameter estimates even for closely spaced paths using the polarization diversity.

6 Conclusion

We propose a novel ESPRIT based algorithm for the joint estimation of AOA, AOD, Doppler shifts and polarization parameters for MIMO wireless systems with polarization sensitive antenna array at the mobile station. The computationally efficient subspace based ESPRIT algorithm allows us to perform efficient multiple estimation of the five parameters using four dimensional algorithm. Simulation results show that the proposed algorithm can achieve super resolution estimation of channel parameters even for closely spaced scattering sources. Future work will analyse the performance of the proposed algorithm using measured channel data and more realistic correlated sensor noise assumption.
Figure 3: RMSE of polarization parameter $\varsigma$ with the Proposed Scheme for a Four-Path Channel

Figure 4: RMSE of AOD Estimation with the Proposed Scheme for a Four-Path Channel

Figure 5: RMSE of Doppler shifts Estimation with the Proposed Scheme for a Four-Path Channel

Figure 6: Averaged RMSE of Parameter Estimates with the Proposed Scheme for Ten-Path Channel
References:


