Diversity Detection in Non-Gaussian Noise over Fading Channels by Generalized Detector

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Abstract: - In this paper, we consider the problem of *M*-ary signal detection based on the generalized approach to signal processing (GASP) in noise over a single-input multiple-output (SIMO) channel affected by the frequency-dispersive Rayleigh distributed fading and corrupted by the additive non-Gaussian noise modelled as spherically invariant random process. We derive both the optimum generalized detector (GD) structure based on GASP and a suboptimal reduced-complexity GD applying the low energy coherence approach jointly with the GASP in noise. Both GD structures are independent of the actual noise statistics. We also carry out a performance analysis of both GDs and compare with the conventional receivers. The performance analysis is carried out with reference to the case that the channel is affected by a frequency-selective fading and for a binary frequency-shift keying (BFSK) signalling format. The results obtained through both a Chernoff-bounding technique and Monte Carlo simulations reveal that the adoption of diversity also represents a suitable means to restore performance in the presence of dispersive fading and impulsive non-Gaussian noise. It is also shown that the suboptimal GD incurs a limited loss with respect to the optimum GD and this loss is less in comparison with the conventional receiver.

Key-Words: - Generalized detector (GD), additive non-Gaussian noise, array processing, diversity, detection performance, generalized approach to signal processing (GASP), spherically invariant random processes.

1 Introduction

In the design of wireless communication systems, two main disturbance factors are to be properly accounted for, i.e. the fading and additive noise. As to the former, it is usually taken into account by modelling the propagation channel as a linear-time-varying filter with random impulse response [1], [2]. Indeed, such a model is general enough to encompass the most relevant instances of fading usually encountered in practice, i.e. frequency- and/or time-selective fading, and flat-flat fading. As to the additive noise, such a disturbance has been classically modelled as a possibly correlated Gaussian random process. However, the number of studies in the past few decades has shown, through both theoretical considerations and experimental results, that Gaussian random processes, even though they represent a faithful model for the thermal noise, are largely inadequate to model the effect of real-life noise processes, such as atmospheric and man-made noise [3]–[6] arising, for example, in outdoor mobile communication systems.

It has also been shown that non-Gaussian disturbances are commonly encountered in indoor environments, for example, offices, hospitals, and factories [7], [8], as well as in underwater communications applications [9]. These disturbances have an impulsive nature, i.e. they are characterized by a significant probability of observing large interference levels Since conventional receivers exhibit dramatic performance degradations in the presence of non-Gaussian impulsive noise, a great attention has been directed toward the development of non-Gaussian noise models and the design of optimized detection structures that are able to operate in such hostile environments.

Among the most popular non-Gaussian noise models considered thus far, we cite the alpha-stable model [10], the Middleton Class-A and Class-B noise [11], the Gaussian-mixture model [12] which, in turn, is a truncated version, at the first order, of the Middleton Class-A noise, and the compound Gaussian model [13], [14]. In particular, in the recent past, the latter model, subsuming, as special cases, many marginal probability density functions (pdfs) that have been found appropriate for modelling the impulsive noise, like, for instance, the Middleton Class-A noise, the Gaussian-mixture noise [14], and the symmetric alpha-stable noise [15]. They can be deemed as the product of a Gaussian, possibly complex random process times a real non-negative one [16].

Physically, the former component, which is usually referred to as speckle, accounts for the conditional validity of the central limit theorem, whereas the latter, the so-called texture process, rules the gross characteristics of the noise source. Very interesting property of compound-Gaussian processes is that, when observed on time intervals whose duration is significantly shorter than the average decorrelation time of the texture component, they reduce to spherically invariant random processes (SIRPs) [17] and [18], which have been widely adopted to model the impulsive noise in wireless communications [13], [14], and [19], multiple access interference in direct-sequence spread spectrum cellular networks [20], and clutter echoes in radar applications [21], [22].

In the present paper, we consider the problem of detecting one of M signals transmitted upon a zero-mean fading dispersive channel and embedded in SIRP noise by the generalized detector (GD) based on the generalized approach to signal processing (GASP) in noise [23]–[28]. The similar problem has been previously addressed. In [14], the optimum receiver for flat-flat Rayleigh fading channels has been derived, whereas in [29], the case of Rayleigh-distributed, dispersive fading has been considered. It has been shown therein that the receiver structure consists of an estimator of the short-term conditionnal, i.e. given the texture component, noise power and of a bank of M estimators/correlators keyed to the estimated value of the noise power.

Theoretically, the GD can be applied to detect any signal, i.e. the signal with known or unknown, deterministic or random parameters. The GD implementation in wireless communication and radar is discussed in [30]–[34] and [28], respectively. The signal detection performance improvement using GD in radar sensor system is investigated in [35]–[38]. The first attempt to investigate the GD employment in cognitive radio networks has been discussed in [39].

Since such a structure is not realizable, a suboptimum detection structure has been introduced and analyzed in [40]. In this paper, we design the GD extending conditions discussed in [29], [40] to the case that a diversity technique is employed. It is well known that the adoption of diversity techniques is effective in mitigating the negative effects of the fading, and since conventional diversity techniques can incur heavy performance loss in the presence of impulsive disturbance [41], it is of interest to envisage the GD for optimized diversity reception in non-Gaussian noise.

We show that the optimum GD is independent of the joint pdf of the texture components on each diversity branch. We also derive a suboptimum GD, which is amenable to a practice. We focus on the relevant binary frequency-shift-keying BFSK) signalling case and provide the probability of error as well as the optimum GD and the suboptimum GD. We assess the channel diversity order impact and noise spikiness on the performance.

The remainder of the present paper is organized as follows. The problem statement is declared in Section 2. The brief description of GD structure and the main functioning principles are delivered in Section 3. Additionally the design of optimal and suboptimal GD structures is discussed in Section 3. Special cases, namely the channels with flat-flat Rayleigh fading and the channels with slow frequency-selective Rayleigh fading are discussed in Section 4. Evaluation of the probability of error for designed GD structures in the presence of spherically invariant disturbance is carried out in Section 5. Simulation results allowing us to define the Chernoff bounds for the given probability of error are presented in Section 6. Some conclusions are made in Section 7.

2 Problem Statement

The problem is to derive the GD aimed at detecting one out of M signals propagating through single-input multiple-output (SIMO) channel affected by dispersive fading and introducing additive non-Gaussian noise. In other words, we have to deal with the following M-ary hypothesis test:

$$\mathscr{H}_i \Rightarrow \begin{cases} x_1(t) = s_{1,i}(t) + n_1(t) \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ x_P(t) = s_{P,i}(t) + n_P(t) \end{cases}, \qquad (1)$$
$$i = 1,\dots,M \quad t \in [0,T]$$

where P is the channel diversity order and $[0,T]$ is the observation interval; the waveforms $\{x_p(t)\}_{p=1}^{P}$ are the complex envelopes of the P distinct channel outputs; $\{s_{p,i}(t)\}_{p=1}^{P}, i = 1,\dots,M$ represent the baseband equivalents of the useful signal received on the P diversity branches under the ith hypothesis.

Since the channel is affected by dispersive fading, we may assume [2] that these waveforms are related to the corresponding transmitted signals $u_i(t)$

$$s_{p,i}(t) = \int_{-\infty}^{\infty} h_p(t,\tau) u_i(t-\tau) d\tau \ , \quad t \in [0,T] \qquad (2)$$

where $h_p(t,\tau), p=1,\ldots,P$ is the random impulse response of the channel p-th diversity branch and is modelled as a Gaussian random process with respect to the variable t. In keeping with the uncorrelated-scattering model, we assume that the random processes $h_p(t,\tau), p=1,\ldots,P$ are all statistically independent.

As a consequence, the waveforms $\{s_{p,i}(t)\}_{p=1}^{P}$ are themselves the independent complex Gaussian random processes that we assume to be zero-mean and with the covariance function given by

$$Cov(t,\tau)=E[s_{p,i}(t)s^*_{p,i}(\tau)]\ , \quad i=1,\ldots,M \quad t,\tau\in[0,T] \tag{3}$$

and independent of p (the channel correlation properties are identical of each branch) and upper bounded by a finite positive constant. This last assumption poses the constraint on the average receive energy in the i-th hypothesis $\mathscr{E}_i=\int_0^T Cov_i(t,t)dt<\infty$.

We also assume in keeping with the model [42] that $E[s_{p,i}(t)s_{p,i}(\tau)]=0$. This is not a true limitation in most practical instances, and it is necessarily satisfied if the channel is wide sense stationary. Finally, as to the additive non-Gaussian disturbances $\{n_p(t)\}_{p=1}^{P}$, we resort to the widely adopted compound model, i.e. we deem the waveform $n_p(t)$ as the product of two independent processes:

$$n_p(t)=\nu_p(t)g_p(t)\ ,\quad p=1,\ldots,P \tag{4}$$

where $\nu_p(t)$ is a real non-negative random process with marginal pdf $f_{\nu_p}(\cdot)$ and $g_p(t)$ is a zero-mean complex Gaussian process. If the average decorrelation time of $\nu_p(t)$ is much larger than the observation interval $[0,T]$, then the disturbance process degenerates into SIPR [17], i.e.

$$n_p(t)=\nu_p g_p(t)\ ,\quad p=1,\ldots,P\ . \tag{5}$$

From now on, we assume that such a condition is fulfilled, and we refer to [13] and [14] for further details on the noise model, as well as for a list of all of the marginal pdfs that are compatible with (5). Additionally, we assume $E[\nu_p^2]=1$ and that the correlation function of the random process $g_p(t)$ is either known or has been perfectly estimated based on (5). While previous papers had assumed that the noise realization $n_1(t),\ldots,n_P(t)$ were statistically independent, in this paper, this hypothesis is relaxed.

To be more definite, we assume that the Gaussian components $g_1(t),\ldots,g_P(t)$ are uncorrelated (independent), whereas the random variables $\nu_1,\ldots,\nu_P$ are arbitrary correlated. We thus denote by $f_{\nu_1,\ldots,\nu_P}(\nu_1,\ldots,\nu_P)$ their joint pdf. It is worth pointing out that the above model subsumes the special case that the random variables $\nu_1,\ldots,\nu_P$ are either statistically independent or fully correlated, $\nu_1=\cdots=\nu_P$.

Additionally, it permits modelling a much wider class of situations that may occur in practice. For instance, if one assumes that the P diversity observations are due to a temporal diversity, it is apparent that if the temporal distance between consecutive observations is comparable with the average décorrelation time of the process $\nu(t)$, then the random variables $\nu_1,\ldots,\nu_P$ can be assumed to be neither independent nor fully correlated. Such a model also turns out to be useful in clutter modelling in that if the diversity observations are due to the returns from neighbouring cells, the corresponding texture components may be correlated [43].

For a sake of simplicity, consider the white noise case, i.e. $n_p(t)$ possesses an impulsive covariance $\forall p$

$$Cov_n(t,\tau)=2\mathscr{N}_0 E[\nu_p^2]\delta(t-\tau)=2\mathscr{N}_0\delta(t-\tau)\ , \tag{6}$$

where $2\mathscr{N}_0$ is the power spectral density (PSD) of the Gaussian component of the noise processes $g_1(t),\ldots,g_P(t)$. Notice that this last assumption does not imply any loss of generality should the noise possess a non-impulsive correlation.

Then, due to the closure of SIRP with respect to linear transformations, the classification problem could be reduced to the above form by simply preprocessing the observables through a linear whitening filter. In such a situation, the $s_{p,i}(t)$ represent the useful signals at the output of the channel cascade and of the whitening filter. Due to the linearity of such systems, they are still Gaussian processes with known covariance functions.

Finally, we highlight here that the assumption that the useful signals and noise covariance functions (3) and (6) are independent of the index p has been made to simplify notation.

3 GD Structure

3.1 Main functioning principles

As we mentioned before, the GD is constructed in accordance with the generalized approach to signal processing (GASP) in noise [23]–[28]. The GASP introduces an additional noise source that does not carry any information about the signal with the purpose to improve the qualitative signal detection performance. This additional noise can be considered as the reference noise without any information about the signal to be detected.

The jointly sufficient statistics of the mean and variance of the likelihood function is obtained in the case of GASP employment, while the classical and modern signal processing theories can deliver only a sufficient statistics of the mean or variance of the likelihood function (no the jointly sufficient statistics of the mean and variance of the likelihood function). Thus, GASP implementation allows us to obtain more information about the input process or received signal.

Owing to this fact, the detectors constructed based on GASP basis are able to improve the signal detection performance in comparison with other conventional detectors. The GD consists of three channels (see Fig. 1): the correlation channel (the preliminary filter PF, the multipliers 1 and 2, the model signal generator MSG); the autocorrelation channel (PF, the additional filter AF, multipliers 3 and 4, summator 1); and the compensation channel (summators 2 and 3, accumulator 1).

As we can see from Fig. 1, under the hypothesis $\mathcal{H}_1$ (a "yes" signal), the GD correlation channel generates the signal component $s_i^{\text{mod}}[k]s_i[k]$ caused by interaction between the model signal and the incoming signal and the noise component $s_i^{\text{mod}}[k]\xi_i[k]$ caused by interaction between the model signal $s_i^{\text{mod}}[k]$ (the MSG output) and the noise $\xi_i[k]$ (the PF output) Under the hypothesis $\mathcal{H}_1$, the GD autocorrelation channel generates the signal energy $s_i^2[k]$ and the random component $s_i[k]\xi_i[k]$ caused by interaction between the $s_i[k]$ signal and the noise $\zeta_i[k]$ (the PF output). The main purpose of the GD compensation

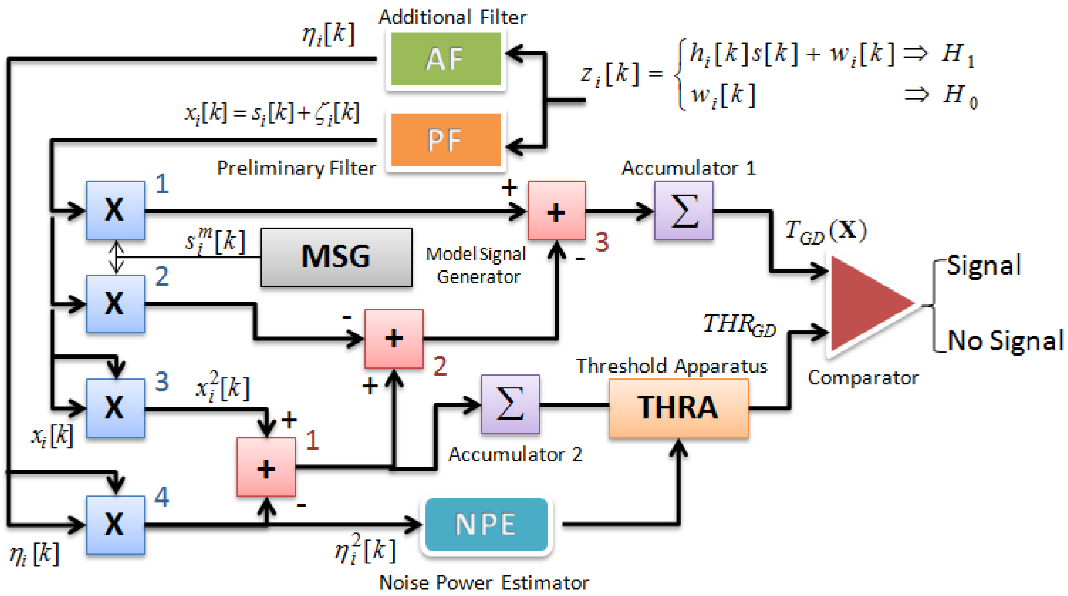

Figure 1. Principal flowchart of GD.

channel is to cancel the GD correlation channel noise component $s_i^{\text{mod}}[k]\xi_i[k]$ and the GD autocorrelation channel random component $s_i[k]\xi_i[k]$ based on the same nature of the noise $\xi_i[k]$. For description of the GD flowchart we consider the discrete-time processes without loss of any generality.

Evidently, this cancelation is possible only satisfying the condition of equality between the signal model $s_i^{\text{mod}}[k]$ and incoming signal $s_i[k]$ over the whole range of parameters. The condition $s_i^{\text{mod}}[k] = s_i[k]$ is the main functioning condition of the GD. Satisfying this condition, we are able to define the incoming signal parameters.

Naturally, in practice, the signal parameters are random. How we can satisfy the GD main functioning condition and define the signal parameters in practice if there is no a priori information about the signal and there is an uncertainty in signal parameters, i.e. signal parameters are random, is discussed in detail in [23]–[25].

Under the hypothesis $\mathcal{H}_0$ – a "no" incoming signal – satisfying the GD main functioning condition, i.e. $s_i^{\text{mod}}[k] = s_i[k]$, we obtain the background noise $\eta_i^2[k] - \xi_i^2[k]$ only at the GD output. Additionally, the practical implementation of the GD decision statistics requires an estimation of the noise variance σ_n^2 using the reference noise $\eta_i[k]$ at the AF output. AF is the reference noise source and the PF bandwidth is matched with the band width of the incoming signal to be detected. The threshold apparatus (THRA) device defines the GD threshold.

The linear systems (the PF and AF) can be considered as the band-pass filters with the impulse responses $h_{PF}[m]$ and $h_{AF}[m]$, respectively. For simplicity of analysis, we assume that these filters have the same amplitude-frequency characteristics or impulse responses by shape. Moreover, the AF central frequency is detuned relative to the PF central frequency on such a value that the incoming signal can not pass through the AF. The PF bandwidth is matched with the incoming signal bandwidth. Thus, the incoming signal and noise can be appeared at the PF output and the only noise is appeared at the AF output.

If a value of detuning between the AF and PF central frequencies is more than $4 \div 5\Delta f_s$, where Δf_s is the signal bandwidth, the processes at the AF and PF outputs can be considered as the uncorrelated and independent processes and, in practice, under this condition, the coefficient of correlation between PF and AF output processes is not more than 0.05 that was confirmed by experiment in [44] and [45].

The processes at the AF and PF outputs present the input stochastic samples from two independent frequency-time regions. If the noise $w[k]$ at the PF and AF inputs is Gaussian, the noise at their outputs is Gaussian, too, owing to the fact that PF and AF are the linear systems and we believe that these linear systems do not change the statistical parameters of the input process. Thus, the AF can be consiered as a generator of reference noise with a priori information a "no" signal (the reference noise sample). A detailed discussion of the AF and PF can be found in [25, Chapter 3] and [26, Chapter 5].

The noise at the PF and AF outputs can be presented in the following form:

$$\begin{cases} w_{PF}[k] = \zeta[k] = \sum\limits_{m=-\infty}^{\infty} h_{PF}[m]w[k-m] \ , \\ w_{AF}[k] = \eta[k] = \sum\limits_{m=-\infty}^{\infty} h_{AF}[m]w[k-m] \ . \end{cases} \tag{7}$$

Under the hypothesis $\mathcal{H}_1$, the signal at the PF output (see Fig. 1) can be defined as $x_i[k] = s_i[k] + \xi_i[k]$, where $\xi_i[k]$ is the observed noise at the PF output and $s_i[k] = h_i[k]s[k]$; $h_i[k]$ are the channel coefficients indicated here only in general case. Under the hypothesis $\mathcal{H}_0$ and for all i and k, the process $x_i[k] = \xi_i[k]$ at the PF output is subjected to the complex Gaussian distribution and can be considered as the independent and identically distributed (i.i.d) process. The process at the AF output is the reference noise $\eta_i[k]$ with the same statistical parameters as the noise $\xi_i[k]$.

The decision statistics at the GD output presented in [23] and [25, Chapter 3] is extended to the case of antenna array employment when an adoption of multiple antennas and antenna arrays is effective to mitigate the negative attenuation and fading effects. The GD decision statistics can be presented in the following form:

$$T_{GD}(\mathbf{X}) = \sum_{k=0}^{N-1}\sum_{i=1}^{M} 2x_i[k]s_i^{\text{mod}}[k] - \sum_{k=0}^{N-1}\sum_{i=1}^{M} x_i^2[k] + \sum_{k=0}^{N-1}\sum_{i=1}^{M} \eta_i^2[k] \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} THR_{\text{GD}} \ , \tag{8}$$

where $\mathbf{X} = [\mathbf{x}(0),...,\mathbf{x}(N-1)]$ is the vector of the random process forming at the PF output and THR_{GD} is the GD detection threshold. Under the hypotheses $\mathcal{H}_1$ and $\mathcal{H}_0$, and when the amplitude of the signal is equal to the amplitude of the model signal, i.e. $s_i^{\text{mod}}[k] = s_i[k]$, the GD decision statistics $T_{GD}(\mathbf{X})$ takes the following form, respectively:

$$\begin{cases} \mathcal{H}_1 : T_{GD}(\mathbf{X}) = \sum\limits_{k=0}^{N-1}\sum\limits_{i=1}^{M} s_i^2[k] + \sum\limits_{k=0}^{N-1}\sum\limits_{i=1}^{M} \eta_i^2[k] \\ \qquad - \sum\limits_{k=0}^{N-1}\sum\limits_{i=1}^{M} \zeta_i^2[k] \ , \\ \mathcal{H}_0 : T_{GD}(\mathbf{X}) = \sum\limits_{k=0}^{N-1}\sum\limits_{i=1}^{M} \eta_i^2[k] - \sum\limits_{k=0}^{N-1}\sum\limits_{i=1}^{M} \zeta_i^2[k] \ . \end{cases} \tag{9}$$

In (9) the term $\sum_{k=0}^{N-1}\sum_{i=1}^{M} s_i^2[k] = E_s$ corresponds to the average signal energy, and the term $\sum_{k=0}^{N-1}\sum_{i=1}^{M}\eta_i^2[k] - \sum_{k=0}^{N-1}\sum_{i=1}^{M}\zeta_i^2[k]$ presents the background noise at the GD output. The GD background noise is a difference between the noise power forming at the PF and AF outputs. Practical implementation of the GD decision statistics requires an estimation of the noise variance σ_n^2 using the reference noise at the AF output.

3.2 Optimum GD structure design

Given the M-ary hypothesis test (1), the synthesis of the optimum GD structure in the sense of attaining the minimum probability of error P_{er} requires evaluating the likelihood functionals under any hypothesis and adopting a maximum likelihood decision-making rule. Formally, we have

$$\hat{\mathscr{H}} = \mathscr{H}_i \Rightarrow \Lambda[\mathbf{x}(t);\mathscr{H}_i] > \max_{k\neq i}\Lambda[\mathbf{x}(t);\mathscr{H}_k] \qquad (10)$$

with $\mathbf{x}(t)=[x_1(t),\ldots,x_P(t)]^T$. The above functionals are usually evaluated through a limiting procedure (see Fig. 2).

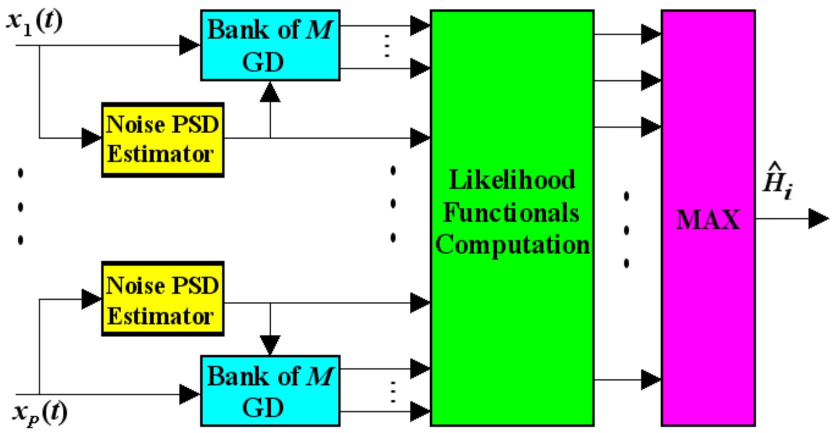

Figure 2. Flowchart of optimum GD in compound Gaussian noise.

We evaluate the likelihood $f_{\mathbf{x}_Q|\mathscr{H}_i}(\mathbf{x}_Q)$ of the Q-dimensional random vector $\mathbf{x}=[x_1,\ldots,x_Q]^T$ whose entries are the projections of the received signal along the first Q elements of suitable basis $\mathscr{B}_i$. Therefore, the likelihood functional corresponding to $\mathscr{H}_i$ is

$$\Lambda[\mathbf{x}(t);\mathscr{H}_i] = \lim_{Q\to\infty}\frac{f_{\mathbf{x}_Q|\mathscr{H}_i}(\mathbf{x}_Q)}{f_{\mathbf{n}_{AF_Q}}(\mathbf{n}_{AF_Q})} \quad , \qquad (11)$$

where $f_{\mathbf{n}_{AF_Q}}(\mathbf{n}_{AF_Q})$ is the likelihood corresponding to the reference sample with *a priori* information a "no" signal is obtained in the additional reference noise forming at the AF output, i.e. no useful signal is observed at the P channel outputs.

In order to evaluate the limit (11), we resort to a different basis for each hypothesis. We choose for the i-th hypothesis the Karhunen-Loeve basis $\mathscr{B}_i$ determined by the covariance function of the useful received signal under the hypothesis $\mathscr{H}_i$. Projecting the waveform received on the p-th diversity branch along the first N axes of the i-th basis yields the following N-dimensional vector:

$$\mathbf{x}_{N,p}^i = \mathbf{s}_{N,p}^i + \nu_p\mathbf{g}_{N,p}^i \quad , \quad p=1,\ldots,P \qquad (12)$$

where $\mathbf{s}_{N,p}^i$ and $\mathbf{g}_{N,p}^i$ are the corresponding projections of the waveforms $s_{p,i}(t)$ and $g_p(t)$.

Since $\mathscr{B}_i$ is the Karhunen-Loeve basis for the random processes $s_{1,i}(t),\ldots,s_{P,i}(t)$, the entries of $\mathbf{s}_{N,p}^i$ are a sequence of uncorrelated complex Gaussian random variables with variances $(\sigma_{s_{1,i}}^2,\ldots,\sigma_{s_{N,i}}^2)$, which are the first N eigenvalues of the covariance function $Cov_i(t,u)$, whereas the entries of $\mathbf{g}_{N,p}$ are a sequence of uncorrelated Gaussian variables with variance $2\mathscr{N}_0$. Here we adopt the common approach of assuming that any complete orthonormal system is an orthonormal basis for white processes [14] and [46].

Upon defining the following NP-dimensional vector

$$\mathbf{x}_N^i = [\mathbf{x}_{N,1}^{iT},\mathbf{x}_{N,2}^{iT},\ldots,\mathbf{x}_{N,P}^{iT}]^T \qquad (13)$$

the likelihood functional taking into consideration subsection 3.1 and [23] and [25] can be written in the following form

$$\Lambda[\mathbf{x}_N^i;\mathscr{H}_i] = \frac{f_{\mathbf{x}_N^i|\mathscr{H}_i}(\mathbf{x}_N^i)}{f_{\mathbf{n}_{AF_N}^i|\mathscr{H}_0}(\mathbf{n}_{AF_N}^i)}$$

$$= \frac{\int \prod_{p=1}^{P} \prod_{j=1}^{N} \frac{1}{\sigma^2_{s_{j,i}} + 4\sigma_n^4 y_p^2} \exp\left[-\frac{|x^i_{j,p}|^2}{\sigma^2_{s_{j,i}} + 4\sigma_n^4 y_p^2} \right] f_{\mathbf{v}}(\mathbf{y}) d\mathbf{y}}{\int \prod_{p=1}^{P} \frac{1}{(4\sigma_n^4 y_p^2)^N} \exp\left[-\frac{|\mathbf{n}^i_{AF_{j,p}}|^2}{4\sigma_n^4 y_p^2} \right] f_{\mathbf{v}}(\mathbf{y}) d\mathbf{y}} \tag{14}$$

where $x^i_{j,p}$ is the j-th entry of the vector $\mathbf{x}^i_{N,p}$, the integrals in (14) are over the set $[0,\infty)^P$

$$\begin{cases} \mathbf{v} = [\nu_1, \ldots, \nu_P], \\ \mathbf{y} = [y_1, \ldots, y_P] \end{cases} \tag{15}$$

and

$$d\mathbf{y} = \prod_{i=1}^{P} dy_i \ . \tag{16}$$

The convergence in measure of (14) for increasing N to the likelihood functional $\Lambda[\mathbf{x}(t); \mathcal{H}_i]$ is ensured by the Grenander theorem [46]. In order to evaluate the above functional, we introduce the following substitution

$$y_p = \frac{\| \mathbf{x}^i_{N,p} \|}{\sqrt{4\sigma_n^4 z_p}} \ , \ \ p = 1, 2, \ldots, P \tag{17}$$

where $\| \cdot \|$ denotes the Euclidean norm.

Applying the same limiting procedure as in [29], we come up with the following asymptotical expression:

$$\Lambda[\mathbf{x}(t); \mathcal{H}_i] = \lim_{N\to\infty} \prod_{p=1}^{P} \Lambda^p_{gN} \left[\mathbf{x}^i_{N,p}, \frac{\| \mathbf{x}^i_{N,p} \|^2}{4\sigma_n^4 N}; \mathcal{H}_i \right] , \tag{18}$$

where

$$\Lambda^p_{gN}(\mathbf{x}^i_{N,p}, y_p^2; \mathcal{H}_i)$$

$$= \exp\left\{ \sum_{j=1}^{N} \left[\frac{\sigma^2_{s_{j,i}} |x^i_{j,p}|^2}{4\sigma_n^4 y_p^2 (\sigma^2_{s_{j,i}} + 4\sigma_n^4 y_p^2)} - \ln\left(1 + \frac{\sigma^2_{s_{j,i}}}{4\sigma_n^4 y_p^2} \right) \right] \right\} \tag{19}$$

represents the ratio between the conditional likelihoods for $\mathcal{H}_i$ and $\mathcal{H}_0$ based on the observation of the signal received on the p-th channel output only.

Equation (18) also requires evaluating

$$Z_p = \lim_{N\to\infty} \frac{\| \mathbf{x}^i_{N,p} \|^2}{N} \ , \tag{20}$$

that, following in [29], can be shown to converge in the mean square sense to the random variable $4\sigma_n^4 \nu_p^2$ for any of the Karhunen-Loeve basis $\mathcal{B}_i, i = 1, \ldots, M$. Due to the fact that the considered noise is white, this result also holds for the large signal-to-noise ratios even though, in this case, a large number of summands is to be considered in order to achieve a given target estimation accuracy.

Notice also that $4\sigma_n^4 \nu_p^2$ can be interpreted as a short-term noise power spectral density (PSD), namely, the PSD that would be measured on sufficiently short time intervals on the p-th channel output.

Thus, the classification problem under study admits the sufficient statistics

$$\ln \Lambda[\mathbf{x}(t); \mathcal{H}_i] = \sum_{p=1}^{P} \sum_{j=1}^{\infty} \left[\frac{\sigma^2_{s_{j,i}} |x^i_{j,p}|^2}{Z_p(\sigma^2_{s_{j,i}} + Z_p)} - \ln\left(1 + \frac{\sigma^2_{s_{j,i}}}{Z_p} \right) \right] \tag{21}$$

The above equations demonstrate that the optimum GD structure for the problem given in (1) is completely canonical in that for any $f_{\nu_1,\ldots,\nu_P}(\nu_1, \ldots, \nu_P)$ and, for any noise model in the class of compound-Gaussian processes and for any correlation of the random variables $\nu_1, \ldots, \nu_P$, the likelihood functional is one and the same. Equation (21) can be interpreted as a bank of P estimator-GDs [42] plus a bias term depending on the eigenvalues of the signal correlation under the hypothesis $\mathcal{H}_i$.

The optimum test based on GASP can be written in the following form:

$$\hat{\mathcal{H}} = \mathcal{H}_i \Rightarrow$$

$$\sum_{p=1}^{P} \frac{1}{Z_p} \left\{ \int_0^T [2x_p(t)\hat{s}^*_{p,i} - x_p(t)x_p(t-\tau)dt + \int_0^T \eta_p^2(t)dt \right\} - b_{p,i}$$

$$> \sum_{p=1}^{P} \frac{1}{Z_p} \left\{ \int_0^T [2x_p(t)\hat{s}^*_{p,k} - x_p(t)x_p(t-\tau)dt + \int_0^T \eta_p^2(t)dt \right\} - b_{p,k} \ , \quad \forall k \neq i \tag{22}$$

where $\hat{s}_{p,i}(t)$ is the linear minimum mean square estimator of $s_{p,i}(t)$ embedded in white noise with PSD Z_p, namely,

$$\hat{s}_{p,i}(t) = \int_0^T h_{p,i}(t,u) x_p(u) du \ , \tag{23}$$

where $h_{p,i}(t,u)$ is the solution to the Wiener-Hopf equation

$$\int_0^T Cov_i(t,z)h_{p,i}(z,\tau)dz + Z_p h_{p,i}(t,\tau) = Cov_i(t,\tau) \ . \tag{24}$$

As to the bias terms $b_{p,i}$, they are given by

$$b_{p,i} = \sum_{j=1}^{\infty} \ln\left[1+\frac{\sigma^2_{s_{j,i}}}{Z_p}\right], \ i=1,\ldots,M \text{ and } p=1,\ldots,P \ . \tag{25}$$

The block diagram of the corresponding GD is shown in Fig. 2. The received signals $x_1(t),\ldots,x_P(t)$ are fed to P estimators of the noise short-term PSD, which are subsequently used for synthesizing the bank of MP minimum mean square error filters $h_{p,i}(\cdot,\cdot), \forall p=1,\ldots,P, \forall i=1,\ldots,M$ to implement the test (22). The newly proposed GD structure is a generalization, to the case of multiple observations, of that proposed in [30], to which it reduces to $P=1$.

3.3 Suboptimum GD structure design

Practical implementation of the decision rule (22) requires an estimation of the short-term noise PSDs on each diversity branch and evaluation of the test statistic. This problem requires a real-time design of *MP* estimator-GDs that are keyed to the estimated values of the short-term PSDs. This would require a formidable computational effort, which seems to prevent any practical implementation of the new receiving structure. Accordingly, we develop an alternative suboptimal GD structure with lower complexity.

Assume that the signals $\{s_{p,i}(t): \forall p=1,\ldots,P, \forall i=1,\ldots,M\}$ possess a low degree of coherence, namely, that their energy content is spread over a large number of orthogonal directions. Since

$$\bar{\mathscr{E}}_i = \sum_{j=1}^{+\infty} \sigma^2_{s_{j,i}} \ , \tag{26}$$

the low degree of coherence assumption implies that the covariance functions $Cov_i(t,\tau)$ have a large number of nonzero eigenvalues and do not have any dominant eigenvalue.

Under these circumstances, it is plausible to assume that the following low energy coherence condition is met:

$$\sigma^2_{s_{j,i}} << 2\mathscr{N}_0 \ , \quad i=1,\ldots,M \ , \ j=1,2,\ldots \ . \tag{27}$$

If this is the case, we can approximate the log-likelihood functional (21) with its first-order McLaurin series expansion with starting point $\sigma^2_{s_{j,i}} / Z_p = 0$.

Following the same steps as in [40], we obtain the following suboptimal within the limits GASP decision-making rule:

$$\begin{aligned}\hat{\mathscr{H}} = \mathscr{H}_i \Rightarrow \sum_{p=1}^{P} \frac{1}{Z_p^2}&\left\{\int_0^T\int_0^T x_p(t)x_p^*(\tau)Cov_i(t,\tau)dtd\tau + \int_0^T n^2_{AF_p}(t)dt\right\} - \frac{\bar{\mathscr{E}}_i}{Z_p} \\ > \sum_{p=1}^{P} \frac{1}{Z_p^2}&\left\{\int_0^T\int_0^T x_p(t)x_p^*(\tau)Cov_k(t,\tau)dtd\tau + \int_0^T n^2_{AF_p}(t)dt\right\} - \frac{\bar{\mathscr{E}}_k}{Z_p} \ , \quad \forall k \neq i \ .\end{aligned} \tag{28}$$

The new GD again requires estimating the short-term noise PSDs $Z_1,\ldots,Z_P$. Unlike the optimum GD (22), in the suboptimum GD (28), the *MP* mini-mum mean square error filters $h_{p,i}(\cdot,\cdot), \forall p=1,\ldots,P, \forall i=1,\ldots,M$ whose impulse responses depend on $Z_1,\ldots,Z_P$ through (24) are now replaced with M filters whose impulse response $Cov_i(t,\tau)$ is independent of the short-term noise PSDs realizations, which now affect the decision-making rule as mere proportionality factors. The only difficulty for practical implementation of such a GD scheme is the short-term noise PSD estimation through (20). However, as already mentioned, such a drawback can be easily circumvented by retaining only a limited number of summands.

4 Special Cases

4.1 Channels with flat-flat Rayleigh fading

Let us consider the situation where the fading is slow and non-selective so that the signal observed on the p-th channel output under the hypothesis $\mathscr{H}_i$ takes the form

$$s_{p,i}(t) = A_p \exp\{j\theta_p\}u_i(t) \ , \tag{29}$$

where $A_p \exp\{j\theta_p\}$ is a complex zero-mean Gaussian random variable. The signal covariance function takes a form:

$$Cov_i(t,\tau) = \bar{\mathscr{E}}_i u_i(t)u_i^*(\tau) \ , \tag{30}$$

where the assumption has been made that $u_i(t)$ possesses unity norm. Notice that this equation represents the Mercer expansion of the covariance in a basis whose first unit vector is parallel to $u_i(t)$.

It should be noted that since the Mercer expansion of the useful signal covariance functions contains just one term, the low energy coherence condition is, in this case, equivalent to a low *SNR* condition. It thus follows that the low energy coherence GD can be now interpreted as a locally optimum GD, thus implying that for large *SNR*s, its performance is expectedly much poorer than that of the optimum GD. The corresponding eigenvalues are

$$\sigma^2_{s_{1,i}} = \bar{\mathcal{E}}_i \ , \ \sigma^2_{s_{k,i}} = 0 \ , \ \forall k \neq 1 \ . \tag{31}$$

Accordingly, the minimum mean square error filters to be substituted in (26) have the following impulse responses:

$$h_{p,i}(t,\tau) = \frac{\bar{\mathcal{E}}_i}{\bar{\mathcal{E}}_i + Z_p} u_i(t) u_i^*(\tau) \ , \tag{32}$$

where the bias term is simply $b_{p,i} = \ln\{1 + \frac{\bar{\mathcal{E}}_i}{Z_p}\}$.We explicitly notice here that such a bias term turns out to depend on the estimated PSD Z_p. Substituting into (22), we find the optimum test

$$\hat{\mathcal{H}} = \mathcal{H}_i \Rightarrow \sum_{p=1}^{P} \frac{\bar{\mathcal{E}}_i}{Z_p(\bar{\mathcal{E}}_i + Z_p)}$$
$$\times \left| \int_0^T [2x_p(t)u_i^*(t) - x_p(t)x_p(t-\tau)]dt + \int_0^T n^2_{AF_p}(t)dt \right|^2 - b_{p,i}$$
$$> \max_{k \neq i} \sum_{p=1}^{P} \frac{\bar{\mathcal{E}}_k}{Z_p(\bar{\mathcal{E}}_k + Z_p)}$$
$$\times \left| \int_0^T [2x_p(t)u_k^*(t) - x_p(t)x_p(t-\tau)]dt + \int_0^T n^2_{AF_p}(t)dt \right|^2 - b_{p,k} \tag{33}$$

whereas its low energy coherence suboptimal approximation can be written in the following form:

$$\hat{\mathcal{H}} = \mathcal{H}_i \Rightarrow \sum_{p=1}^{P} \frac{1}{Z_p^2}$$
$$\times \left| \int_0^T [2x_p(t)u_i^*(t) - x_p(t)x_p(t-\tau)]dt + \int_0^T n^2_{AF_p}(t)dt \right|^2 - \frac{\bar{\mathcal{E}}_i}{Z_p}$$
$$> \max_{k \neq i} \sum_{p=1}^{P} \frac{1}{Z_p^2}$$
$$\times \left| \int_0^T [2x_p(t)u_k^*(t) - x_p(t)x_p(t-\tau)]dt + \int_0^T n^2_{AF_p}(t)dt \right|^2 - \frac{\bar{\mathcal{E}}_k}{Z_p} \tag{34}$$

It is worth pointing out that both GDs are akin to the "square-law combiner" GD [25] that is well known to be the optimum GD in GASP [23]–[27] viewpoint for array signal detection in Rayleigh flat -flat fading channels and Gaussian noise. The relevant difference is due to the presence of short-term noise PSDs $Z_1, \ldots, Z_P$, which weigh the contribution from each diversity branch. In the special case of equienergy signals, the bias terms in the above decision-making rules end up irrelevant, and the optimum GD test (33) reduces to a generalization of the usual incoherent GD, with the exception that the decision statistic depends on the short-term noise PSD realizations.

4.2 Channels with slow frequency-selective Rayleigh fading

Now, assume that the channel random impulse response can be written in the following form:

$$\chi_p(t,\tau) = \chi_p(\tau) = \sum_{k=0}^{L-1} A_{p,k} \exp\{j\theta_{p,k}\}\delta(\tau - kW^{-1}) , \tag{35}$$

where $A_{p,k}\exp\{j\theta_{p,k}\}$ is a set of zero-mean, independent complex Gaussian random variables, and L is the number of paths. Equation (35) represents the well known taped delay line channel model, which is widely encountered in wireless mobile communications. It is readily shown that in such a case, the received useful signal, upon transmission of $u_i(t)$, has the following covariance function:

$$Cov_i(t,\tau) = \sum_{k=0}^{L-1} \overline{A_k^2} u_i(t - kW^{-1}) u_i^*(\tau - kW^{-1}) \ , \quad i = 1, \ldots, M \tag{36}$$

where $\overline{A_k^2}$ is the statistical expectation (assumed independent of p) of the random variables $A^2_{p,k}$. These correlations admit L nonzero eigenvalues, and a procedure for evaluating their eigenvalues and eigenfunctions can be found in [47]. In the special case that the L paths are resolvable, i.e. $T \le W^{-1}$, the optimum GD (18) assumes the following simplified form:

$$\hat{\mathcal{H}} = \mathcal{H}_i \Rightarrow$$

$$\sum_{p=1}^{P}\sum_{j=0}^{L-1}\left\{\frac{\overline{A_j^2}\left|\int_0^T x_p(t)u_i^*(t-jW^{-1})dt+\int_0^T n_{AF_p}^2(t)dt\right|^2}{Z_p(Z_p+\mathcal{E}_i\overline{A_j^2})}\right.$$

$$\left.-\ln\left\{1+\frac{\mathcal{E}_i\overline{A_j^2}}{Z_p}\right\}\right\}$$

$$>\max_{k\neq i}\sum_{p=1}^{P}\sum_{j=0}^{L-1}\left\{\frac{\overline{A_j^2}\left|\int_0^T x_p(t)u_k^*(t-jW^{-1})dt+\int_0^T n_{AF_p}^2(t)dt\right|^2}{Z_p(Z_p+\mathcal{E}_k\overline{A_j^2})}\right.$$

$$\left.-\ln\left\{1+\frac{\mathcal{E}_k\overline{A_j^2}}{Z_p}\right\}\right\}, \qquad (37)$$

where $\mathcal{E}_i$ is the energy of the signal $u_i(t)$. The low energy coherence suboptimal GD (24) is instead written as

$$\hat{\mathcal{H}}=\mathcal{H}_i \Rightarrow$$

$$\sum_{p=1}^{P}\sum_{j=0}^{L-1}\left\{\frac{\overline{A_j^2}}{Z_p^2}\left|\int_0^T x_p(t)u_i^*(t-jW^{-1})dt+\int_0^T n_{AF_p}^2(t)dt\right|^2\right.$$

$$\left.-\frac{\mathcal{E}_i\overline{A_j^2}}{Z_p}\right\}$$

$$>\max_{k\neq i}\sum_{p=1}^{P}\sum_{j=0}^{L-1}\left\{\frac{\overline{A_j^2}}{Z_p^2}\left|\int_0^T x_p(t)u_k^*(t-jW^{-1})dt+\int_0^T n_{AF_p}^2(t)dt\right|^2\right.$$

$$\left.-\frac{\mathcal{E}_k\overline{A_j^2}}{Z_p}\right\} \quad . \qquad (38)$$

Optimality of (37) obviously holds for one-short detection, namely, neglecting the intersymbol interference induced by the channel band limitedness.

5 Performance Assessment

In this section, we focus on the performance of the proposed GD structures. A general formula to evaluate the probability of error P_{er} of any receiver in the presence of spherically invariant disturbance ta-kes the following form:

$$P_{er}=\int P_{er}(e\,|\,\mathbf{v})f_{\mathbf{v}}(\mathbf{v})d\mathbf{v} \quad , \qquad (39)$$

where $P_{er}(e\,|\,\mathbf{v})$ is the receiver probability of error in the presence of Gaussian noise with PSD on the p-th diversity branch $2\mathcal{N}_0\nu_p^2$.

The problem to evaluate P_{er} reduces to that of first analyzing the Gaussian case and then carrying out the integration (39). In order to give an insight into the GD performance, we consider a BFSK signaling scheme, i.e. the base-band equivalents of the two transmitted waveforms are related as

$$u_2(t)=u_1(t)\exp\{j2\pi\Delta ft\} \quad , \qquad (40)$$

where $\Delta f=T^{-1}$ denotes the frequency shift. Even for this simple case study, working out an analytical expression for the probability of error of both the optimum GD and of its low energy coherence approximation is usually unwieldy even for the case of Gaussian noise.

With regard to the optimum GD structure, upper and lower bounds for the performance may be established via Chernoff-bounding techniques. Generalizing to the case of multiple observations, the procedure in [42], the conditional probability of error given $\nu_1,\ldots,\nu_P$ can be bounded as

$$\frac{\exp\{2\mu(0.5\,|\,\mathbf{v})\}}{2\left(1+\sqrt{0.25\pi\ddot{\mu}(0.5\,|\,\mathbf{v})}\right)}\le P_{er}(e\,|\,\mathbf{v})$$

$$\le\frac{\exp\{2\mu(0.5\,|\,\mathbf{v})\}}{2\left(1+\sqrt{0.25\pi\ddot{\mu}(0.5\,|\,\mathbf{v})}\right)} \quad , \qquad (41)$$

where $\mu(\cdot\,|\,\mathbf{v})$ is the following conditional semiinvariant moment generating function

$$\mu(x\,|\,\mathbf{v})$$

$$=\lim_{N\to\infty}\ln E\left\{\exp\left[x\sum_{p=1}^{P}\ln\Lambda_{gN}^{p}(\mathbf{x}_{N,p}^{i};\nu_p^2:\mathcal{H}_1)\right]\middle|\,\mathcal{H}_0,\mathbf{v}\right\}$$

$$=\sum_{j=1}^{\infty}\sum_{p=1}^{P}\left\{(1-x)\ln\left[1+\frac{\sigma_{s_j}^2}{4\sigma_n^4\nu_p^2}\right]-\ln\left[1+\frac{\sigma_{s_j}^2(1-x)}{4\sigma_n^4\nu_p^2}\right]\right\} \qquad (42)$$

with $\{\sigma_{s_j}^2\}_{j=1}^{\infty}$ being the set of common eigenvalues. Substituting this relationship into (41) and averaging with respect to $\nu_1,\ldots,\nu_P$ yields the unconditional bounds on the probability of error P_{er} for the optimum GD (22).

6 Simulation Results

To proceed further in the GD performance there is a need to assign both the marginal pdf, as well as the channel spectral characteristics. We assume hereafter the generalized Laplace noise, i.e. the marginal pdf of the p-th noise texture component takes the following form:

$$f_{\nu_p}(x) = \frac{2\nu^{\nu}}{\Gamma(\nu)} x^{2\nu-1} \exp\{-\nu x^2\} \ , \ x > 0 \qquad (43)$$

where ν is a shape parameter, ruling the distribution behaviour. In particular, the limiting case $\nu \to \infty$ implies $f_{\nu_p}(x) = \delta(x-1)$ and, eventually, the Gaussian noise, where increasingly lower values of ν account for increasingly spikier noise distribution.

Regarding the channel, we consider the case of the frequency-selective, slowly fading channel, i.e. the channel random impulse response is expressed by (35), implying that the useful signal correlation is that given in (36). For simplicity, we also assume that the paths are resolvable. In the following plots, the P_{er} is evaluated by the following way: a) through a semianalytic procedure, i.e. by numerically averaging the Chernoff bound (41) with respect to the realizations of the $\nu_1, \ldots, \nu_P$, and b) by resorting to a Monte Carlo counting procedure. In this later case, the noise samples have been generated by multiplying standard, i.e. with zero-mean and unit-variance complex Gaussian random variates time the rand-om realizations of $\nu_1, \ldots, \nu_P$.

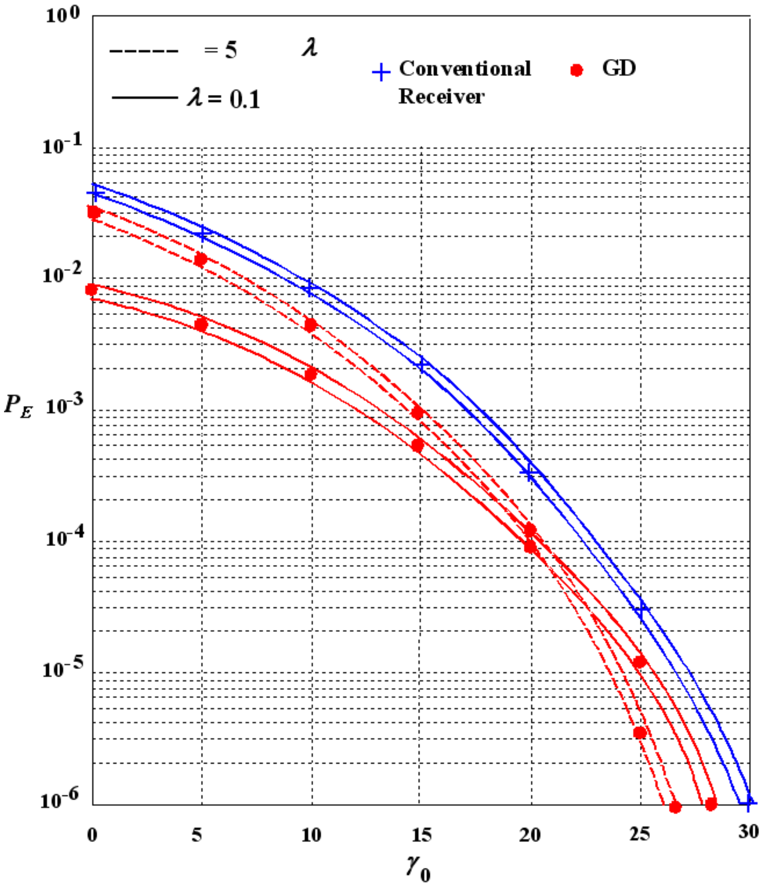

Figure 3. Chernoff bounds for the probability of error P_{er} of the optimum GD.

The Chernoff bound for the optimum GD versus the averaged received radio-frequency energy contrast that is defined as $\gamma_0 = P\sum_{j=1}^{L}\sigma_{s_j}^2(4\sigma_n^4)^{-0.5}$ at $P = 2$ and for two values of the noise shape parameter ν is shown in Fig.3. The noise texture components have been assumed to be independent.

Inspecting the curves, we see that the Chernoff bound provides a very reliable estimate of the actual probability of error P_{er}, as the upper and lower bounds follow each other very tightly. As expected, the results demonstrate that in the low P_{er} region, the spikier the noise, i.e. the lower ν, the worse the GD performance. Conversely, the opposite behaviour is observed for small values of γ_0. This fact might appear, at a first look, surprising. It may be analytically justified in light of the local validity of Jensen's inequality [42] and is basically the same phenomenon that makes digital modulation schemes operating in Gaussian noise to achieve, for low values of γ_0, superior performance in Rayleigh flat-flat fading channels than in no-fading channels. Notice, this phenomenon is in accordance with that observed in [14].

In order to validate the Chernoff bound, we also show, on the same plots, some points obtained by Monte Carlo simulations. These points obviously lie between the corresponding upper and lower probability of error P_{er} bounds. Additionally, we compare the GD Chernoff bound with that for the conventional optimum receiver [48]. Superiority of GD structure is evident.

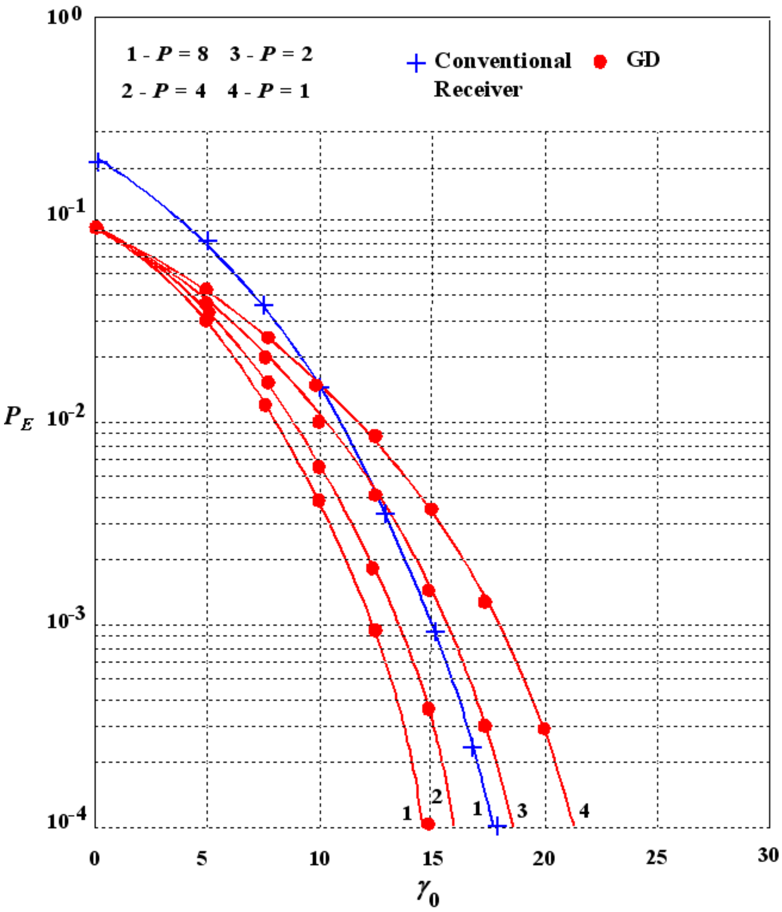

Figure 4. The probability of error P_{er} at several values of P.

In Fig. 4, the effect of the channel diversity order is investigated. Indeed, the optimum GD performance versus γ_0 is represented for several values of P and with $\nu = 1$. The $Z_1,\ldots,Z_P$ have been assumed exponentially correlated with the coefficient of correlation equal to $\rho = 0.2$. A procedure for generating these exponentially correlated random variables for integer and semi-integer values of ν is reported in [49]. As expected, as P increases, the GD performance ameliorates, thus confirming that diversity represents a suitable means to restore performance in severely hostile scenarios.

Also, we compare the GD performance with the conventional optimum receiver one [48] and we see that the GD keeps superiority in this case, too. The optimum GD performance versus γ_0 for the generalized Laplace noise at $\nu = 1, P = 4$, and for several values of the correlation coefficient ρ is demonstrated in Fig. 5. It is seen that the probability of error P_{er} improves at vanishingly small values of ρ. For small values of ρ, the GD takes much advantage in the diversity observations. For high values of ρ, the realizations $Z_1,\ldots,Z_P$ are very similar and much less advantage can be gained through the adoption of a diversity strategy. Such GD performance improvement is akin to that observed in signal diversity detection in the presence of flat-flat fading and Gaussian noise. We see that the GD outperforms the conventional optimum receiver [48] by the probability of error P_{er}.

In Fig. 6, we compare the optimum GD performance versus that of the low energy coherence GD.

We assumed $\rho = 0.2$ and $P = 4$. It is seen that the performance loss incurred by the low energy coherence GD with respect to the optimum GD is kept within a fraction of 1 dB at the probability of error equal to $P_{er} = 10^{-4}$. Simulation results that are not presented in the paper show that the crucial factor ruling the GD performance is the noise shape parameter, whereas the particular noise distribution has a rather limited effect on the probability of error P_{er}.

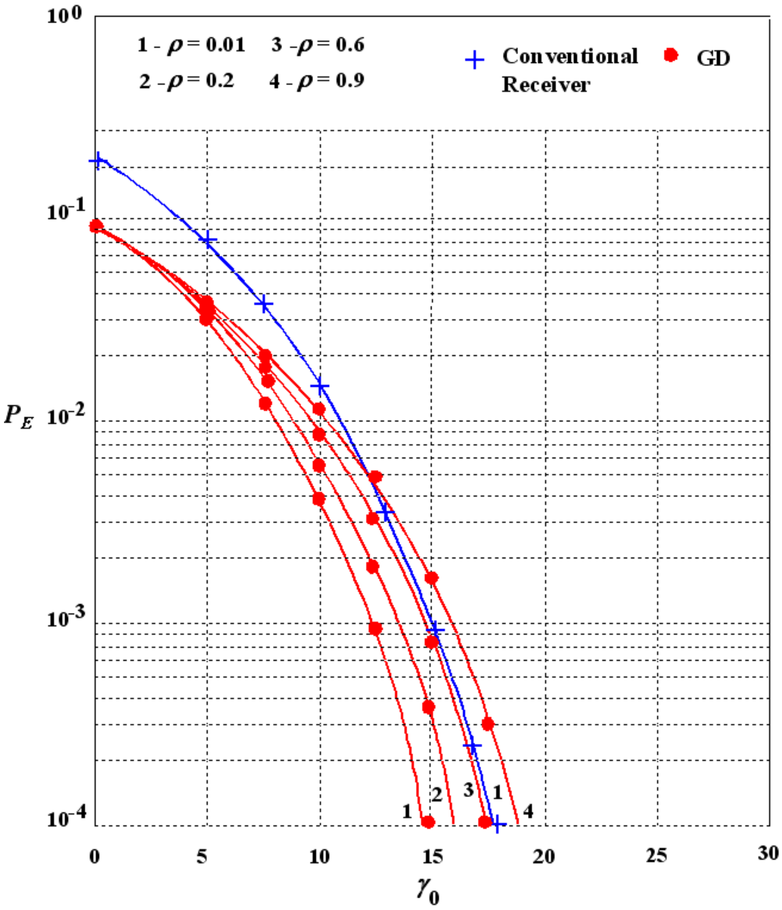

Figure 5. The probability of error P_{er} at several values of the coefficient of correlation.

7 Conclusions

In this paper, we have considered the problem of diversity detection of one out of M signals transmitted over a fading dispersive channel in the presence of non-Gaussian noise. We have modelled the additive noise on each channel diversity branch through a spherically invariant random process, and the optimum GD has been shown to be independent of the actual joint pdf of the noise texture components present on the channel diversity outputs. The optimum GD is similar to the optimum GD for Gaussian noise, where the only difference is that the noise PSD $2\mathcal{N}_0$ is substituted with a perfect estimate of the short-term PSD realizations of the impulsive additive noise.

We also derived a suboptimum GD matched with GASP based on the low energy coherence hypothesis. At the performance analysis stage, we focused on frequency-selective slowly fading channels and on a BFSK signalling scheme and evaluated the GD performance through both a semi-analytic bounding technique and computer simulations. Numerical results have shown that the GD performance is affected by the average received energy contrast, by the channel diversity order, and by the noise shape parameter, whereas it is only marginally affected by the actual noise distribution. Additionally, it is seen that in impulsive environments, diversity represents a suitable strategy to improve GD performance.

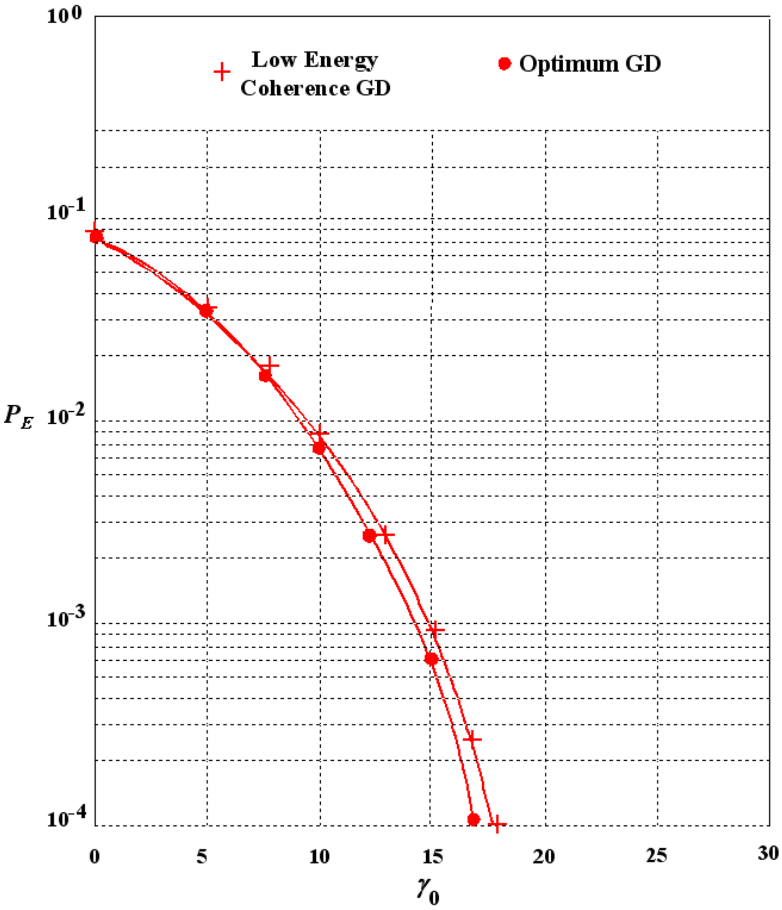

Figure 6. The probability of error P_{er} for the optimal and low energy coherence GDs.

Acknowledgment

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