Noise Canceller Using a New Modified Adaptive Step Size LMS Algorithm

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Abstract: - In this paper, the performance of adaptive noise canceller (ANC) in stationary environment is improved by using a new proposed variable step size LMS algorithm. The algorithm is called (Absolute Average Error- Based Adjusted step size LMS algorithm (AAE-ASSLMS)). The adjusted step size is based on the absolute average value of the current and the previous sample errors. The algorithm has low level steady-state misadjustment compared with the standard LMS and another Variable Step Size LMS (VSSLMS) algorithm for ANC. The proposed algorithm achieved (16, 13) dB difference of attenuation factor in a steady state compared with the LMS and VSSLMS algorithms respectively. Moreover, the proposed algorithm is insensitive to the both power variation of reference input and different signal to noise ratios at the primary input of ANC.

Key-Words: - Adaptive noise canceller, LMS algorithm, Variable step size.

1 Introduction

Adaptive noise canceller (ANC) was widely used in digital communication systems in order to reduce or eliminate an unwanted noise from a received signal. The simple structure for adaptive noise canceller was the Finite Impulse Response filter (FIR) which can be trained using Least Mean Square adaptive algorithm (LMS) which was first proposed by Widrow and Hoff at Stanford University, Stanford, CA in 1960 [1]. This algorithm is often used for the adaptation of the ANC system because it is easy to implement and require a small number of calculations but this algorithm suffers from slow convergence [2]. The step size influences two important parameters, namely the low level of misadjustment or Minimum Mean Square Error (MMSE) (steady-state state behavior) and the convergence speed (transient behavior). The step size is directly proportional to the convergence speed, and it is inversely proportional to the MMSE which makes the compromise process difficult [3].Therefore, to overcome this problem, one can start with large step size, to enhance the convergence speed, and gradually reduce it to attain its minimum value, to achieve desirable MMSE (i.e. low level of misadjustment) [3].

Several methods of varying the step size are proposed with different criteria: squared instantaneous error [3], the square of the time-averaged estimate of the autocorrelation of current and previous error [4] , the accumulated instantaneous error concept [5], negative exponential form of the error variation in between and to consecutive iterations [6] , or they used the squared norm of the smoothed gradient vector [7] . Moreover, the authors in [8] used the error differences between two adjacent iteration periods to qualitatively estimate the state of the algorithm, established a new nonlinear relationship between the step size and the error [9], or a nonlinear function between the step factor $\mu(n)$ and the error signal e(n) [10].

. In this paper time varying step size is chosen due to its powerful effect on the performance of the system. Moreover, the structure of the adaptive noise canceller will not be changed, and this technique requires fewer overheads in computations, which are an important factor for hardware implementation. The proposed algorithm in this paper is called Absolute Average Error Adjusted Step Size LMS algorithm (AAE-ASSLMS) algorithm. The value of the time-varying step size in this proposed algorithm is adjusted according to the absolute average value of the current and the previous estimation errors. Although LMS algorithm was simple, ease of computation and it does not requiring averaging or differentiating in the adaptation process, but it has some drawbacks. These drawbacks (disadvantages) are:-

a) Slow convergence adaption process.
b) High sensitivity to the eigenvalues spread of input correlation matrix (R).
c) Has trade off between low level of miss-adjustment and fast convergence.

Therefore , the main goals (advantages) of our proposed algorithm are:-

a. Fast convergence time with low level steady-state misadjustment.
b. Large attenuation factor in a steady state.
c. Insensitive to the both power variation of reference input and different signal to noise ratios at the primary input of ANC.

This new proposed algorithm shows through computer simulation and using real speech signal, good performance compared with traditional LMS and other variable step size LMS algorithm (VSSLMS) which was proposed in 2012 [10].

The LMS algorithm requires (2L+1) multiplications and (2L+1) additions operations for each time sample, while the new proposed algorithm required (2L+2) multiplications and (3L+1) additions. These extra arithmetic operations of the new proposed algorithm can be overcome using modern fast DSP processors or FPGA. The hardware design of new proposed variable step size LMS algorithm based FIR adaptive noise canceller (ANC) is implemented using schematic design entry , but the details of hardware design will be present in another paper which will be submit to this or another journal. The system structure of FPGA-based AAE-ASSLMS algorithm is divided into the delay unit, average unit, comparator unit, absolute unit, accumulator, arithmetic shift register, and tap unit. The performance of FPGA-based system structure of AAE-ASSLMS algorithm is synthesized and simulated on Xilinx Spartan-3E platform using ISE 10.1i Xilinx tool simulator.

2 Basic Concept of LMS Algorithm

The Least Mean Square (LMS) algorithm was first developed by Widrow and Hoff in 1960 through their studies of pattern recognition [2]. From there it has become one of the most widely used algorithms in adaptive filtering. The filter weights of the adaptive filter (L) (Figure 1) are updated in each iteration according to the following formula [1].

$$\boldsymbol{W}(n+1) = \boldsymbol{W}(n) + 2\mu e(n)\boldsymbol{X}(n) \tag{1}$$

Here $X(n)$ is the input vector of reference noise, such that

$$\boldsymbol{X}(n)=[x(n)x(n-1)\ldots x(n-L+1)] ,$$

where n is time index. The vector

$$\boldsymbol{W}(n) = [\, w_0(n)\; w_1(n)\; w_2(n) \ldots w_{L-1}(n)]^T$$

represents the coefficients (weights) of the adaptive FIR filter vector.

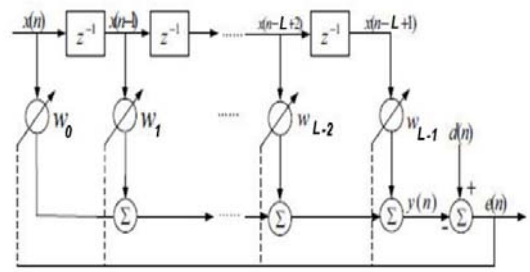

Figure1. Structure of adaptive FIR filter

The system output signal is called an error signal (e) which is used to update the adaptive filter's impulse response (weight coefficients) using a suitable adaptive algorithm such that the error signal is progressively minimized. The input noise $\boldsymbol{X}(n)$ is filtered to produce an output $y(n)$. The parameter μ is known as the step size parameter which assigned a small positive value. In order to ensure stability (or convergence) of the LMS algorithm; the step size parameter is bounded by the following equation [2]:

$$0 < \mu < \frac{2}{tap\ weight\ power} \tag{2}$$

3 New Proposed Algorithm

AAE-ASSLMS regards as modified version of the standard LMS algorithm. AAE-ASSLMS algorithm used variable step size that will be adjusted according to absolute average value of the current and the previous estimator errors as follows:

$$\mu(n+1) = \mu(n) - \left|\sum_{c=0}^{L} e(n-c) * \beta\right| \quad (3)$$

Where $0 < \beta < 1$

And

$$\mu(n+1) = \begin{cases} \mu_{min} & if \quad \mu(n+1) < \mu_{min} \\ \mu(n+1) & otherwise \end{cases} \quad (4)$$

The way in which $\mu(n)$ is changing in (3) depends on previous value of step size and also on the absolute average estimation error. As shown in (3) one can start with initial large step size (which is equal to the μ_{Max}) , to enhance the convergence speed, and gradually reduce it to attain its minimum value, to achieve a low level of misadjustment. To achieve best performance the step size should decrease to the next, smaller step in smoothing transient manner. Therefore, in this new proposed algorithm the subtraction process is used to make the next step size $\mu(n+1)$ always smaller than the current step size $\mu(n)$.Therefore this algorithm is suitable for stationary environment only. Furthermore, the absolute value is used to get a decreasing step size in order to arrive at the minimum step size.

To ensure stability, the variable step size $\mu(n)$ is constrained to the pre-determined minimum step size value μ_{min} , such that $\mu(n+1)$ is set to μ_{min} when it falls below of the lower bound. The parameter (β) is a relax factor that controls the convergence time as well as the level of misadjustment of the algorithm at a steady state. Then the update equation (1) for the weight vector for AAE-ASSLMS algorithm will be:-

$$\boldsymbol{W}(n+1) = \boldsymbol{W}(n) + 2\mu(n)e(n)\boldsymbol{X}(n) \quad (5)$$

Table 1 shows the sequence steps for the proposed algorithm.

In this paper in addition to LMS, a comparison between the performance of the AAE-ASSLMS and another Variable Step Size (VSSLMS) [10] algorithm is introduced. This algorithm constructed a nonlinear function between the step factor $\mu(n)$ and the error signal $e(n)$ as follow [10]:-

$$\mu_n = \rho(1 - \text{sech}[\alpha\, e(n)^{\gamma}]) \quad (6)$$

Where ‘ sech ‘ is Hyperbolic secant function , ρ is constant that affected the convergence rate and both α and γ affected the error when the algorithm was almost steady [10].

Table 1. Proposed algorithm

AAE-ASSLMS ALGORITHM
MATRIX AND VECTOR DEFINITIONS
$\boldsymbol{X}(n) = [x(n)x(n-1) \dots\ x(n-L+1)]$
$\boldsymbol{W}(n) = [\ w_0(n)\ w_1(n)\ w_2(n)\ \dots\ w_{L-1}(n)]^T$
INITIALIZATION
$\boldsymbol{W}(n) = 0 \qquad \mu_0 = \mu_{max} \qquad 0 < \beta < 1$
FOR EACH TIME SAMPLE
$y(n) = \boldsymbol{W}(n)\boldsymbol{X}(\boldsymbol{n})^{\boldsymbol{T}}$
$e(n) = d(n) - y(n)$
$\boldsymbol{W}(n+1) = \boldsymbol{W}(n) + 2\mu(n)e(n)\boldsymbol{X}(n)$
STEP SIZE UPDATE
$\mu(n+1) = \mu(n) - \left\lvert\sum_{c=0}^{L} e(n-c) * \beta\right\rvert$
$if\ \ \mu(n+1) < \mu_{min}$ then $\mu(n+1) = \mu_{min}$
Otherwise $\mu(n+1) = \mu(n+1)$
end

4 Performance Analyses for the Proposed Algorithm

An approximate performance analysis for the proposed variable step size algorithm is presented in this section. The proposed algorithm computes a set of weights $\boldsymbol{W}(n)$ that seeks to minimize $|e(n)|^2$, where $|e(n)|^2$ is calculated as:-

$$|e(n)|^2 = |d(n) - \boldsymbol{W}^T\boldsymbol{X}(n)|^2 \quad (7)$$

Taking the expected value $E\ [.]$ of (5) yields:-

$$E[\boldsymbol{W}(n+1)] = \quad E[\boldsymbol{W}(n)] + E[\mu(n)e(n)\boldsymbol{X}(n)] \quad (8)$$

To make the analysis tractable, simple assumption was used [3]:-

$$E[\mu(n)e(n)\boldsymbol{X}(n)] = E[\mu(n)]E[e(n)\boldsymbol{X}(n)] \quad (9)$$

However, this assumption was approximately true if (β) was small, and then μ(n)) will vary slowly around its mean value. Equation (8) can be written as:-

$$E[\boldsymbol{W}(n+1)] = E[\boldsymbol{W}(n)] + E[\,\mu(n\,)]E[\,e(n)\boldsymbol{X}(n)] \quad (10)$$

Let $\boldsymbol{W}_o(n)$ denote the time varying optimal coefficient vector for estimating the desired response signal. Then ,

$$E[\boldsymbol{W}(n+1)] = E[\boldsymbol{W}(n)] - E[\,\mu(n\,)]\,R\,E[\boldsymbol{W}(n) - \boldsymbol{W}_o(n)\,] \quad (11)$$

Taking $E\,[.]$ of the $\boldsymbol{W}_o(n+1) = a\,\boldsymbol{W}_o(n) + A_z(n)$ (where a is less than but close to one and A_z is an independent zero mean sequence) yields:-

$$E[\boldsymbol{W}_o(n+1)] = a\,E[\boldsymbol{W}_o(n)] \quad (12)$$

Then the error of the weight vector $\widetilde{\boldsymbol{W}}(n) = \boldsymbol{W}(n) - \boldsymbol{W}_o(n)$ satisfies the equation:-

$$E[\widetilde{\boldsymbol{W}}(n+1)] = [\,I - E[\mu(n\,)]R\,E\big[\widetilde{\boldsymbol{W}}(n)\big] + (1-a)E\,[\boldsymbol{W}_o(n)] \quad (13)$$

This equation can be obtained by adding and subtracting the term $E[\widetilde{\boldsymbol{W}}(n+1)]$ in the L.H.S and the term $E\,[\boldsymbol{W}_o(n)]$ in the R.H.S. of (11). Equation (13) was stable if and only if [3]:-

$$\prod_{n=0}^{L}(\,I - E[\mu(n\,)]R\,) = \rightarrow 0 \quad as \quad L \rightarrow \infty \quad (14)$$

A sufficient condition for (14) to hold was [3]:-

$$E[\mu(n\,)] < \frac{2}{\lambda_{MAX}\,R} \quad (15)$$

Where $\lambda_{MAX}\,R$ is the maximum eigenvalue of the matrix. Furthermore, for any $|a| < 1$, $E\,[\boldsymbol{W}_o(n)] \rightarrow 0\; as\, n\, \rightarrow \infty$. Hence under (14):- $E\big[\widetilde{\boldsymbol{W}}(n)\big] \rightarrow 0\;\; as\, n\, \rightarrow \infty$.

For the stationary case where $\boldsymbol{W}_o(n) = \boldsymbol{W}(n)$ was even simpler in that (13) becomes homogenous difference equation [3]. Then (14) was necessary and sufficient condition for $E[\boldsymbol{W}(n)] \rightarrow \boldsymbol{W}_o(n)\;as\, n\, \rightarrow \infty$.

The performance of proposed, VSSLMS and traditional LMS algorithms are validated by simulations of an adaptive noise as shown in Figure 2. As shown in this figure, the ANC has two inputs called primary (d) and reference(x) inputs where (d) is desired signal that contains the clean signal (s) plus an uncorrelated noise (n_o). H (z) is the a noise path channel that produced noise signal (n_o)which is uncorrelated with the input signal (s) but correlated with the (x) .The reference input is filtered and subtracted from a primary input (d) which is containing both signal and noise. As a result, the primary noise (n_o) is attenuated or eliminated by cancellation. This filtering and subtraction must be controlled by an appropriate process in order to obtain a noise reduction with little risk of distorting the signal or increasing the noise level [2].

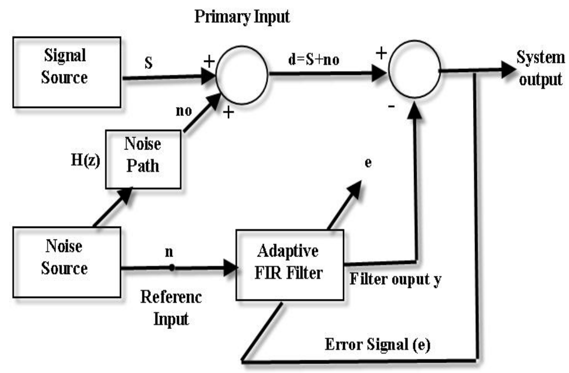

Figure 2. ANC Model

5 Simulation Results

In this simulation, we evaluate the performance of the standard LMS, VSSLMS and the AAEASSLMS in terms of the weight coefficients estimation, the error function, and the attenuation factor. The ANC parameters are chosen as following (given that all the values of these parameters for LMS, VSSLMS and AAE-ASSLMS algorithms were chosen to achieve better performance in terms of fast convergence time and low level misadjustment in order to make fairly a comparison between these algorithms) :-

a) The order of FIR adaptive filter (L) for all simulation was eight taps.

b) The source noise used for all simulations was white Gaussian noise with zero mean and different variance σ^2 .

c) The optimum step size for the standard LMS was chosen to be 0.05 according to equation (2).

d) The optimum value for parameters used for the VSSLMS was chosen according to the ref [10] to be α = 3.0 , $\rho =$ 0.008 , and γ =1 .

e) The optimum value of 捡$_{max}$ and $\mu_{min}$for the AAE-ASSLMS was chosen to be 0.05 and 0.0005 respectively. The constant β is chosen to be 0.00018 according to the both performance analysis in section 4 and the simulation tests shown later.

(f) The impulse response of the noise path H (z) was randomly chosen as:

([0.2, -0.15, 1.0, 0.21, 0.03, 0]).

(g) Original speech $s(n)$ with number of samples is shown in Figure 3.

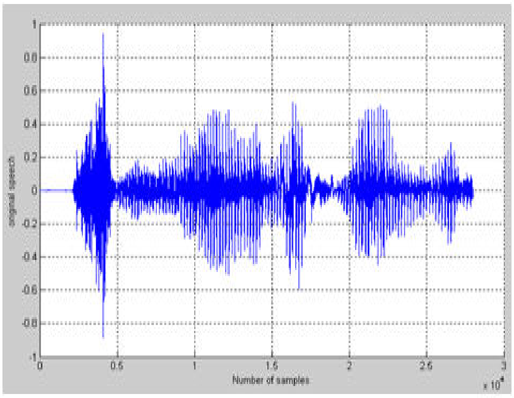

Figure 3. Original speech Signal.

5.1 Weight coefficients estimation

The ability of algorithms for tracking the noise path coefficients (i.e. transfer function of noise path $H(z)$) is investigated in this section. Figure 4 shows the weight coefficients of the noise path (([0.2, -0.15, 1.0, 0.21, 0.03, 0]). Figures (5 ,6, and 7) shows the weight coefficients estimation performance for LMS, VSSLMS and AAE-ASSLMS when signal to noise ratio at primary input equals (0 dB) respectively.

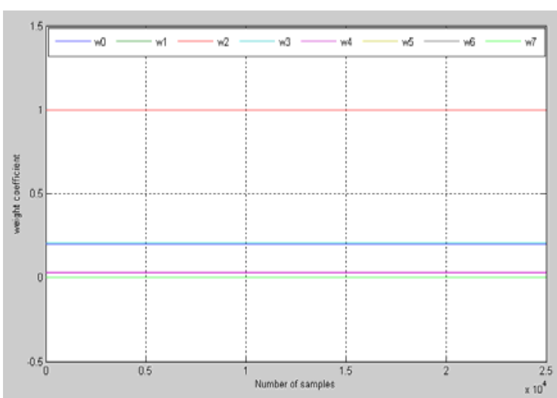

Figure (4) Weight coefficients of the noise path H(z).

5.2 Error Function

This function represents the difference between the original signal *s(n)* and the error signal *e(n)* . Three cases will be considered as follows:-

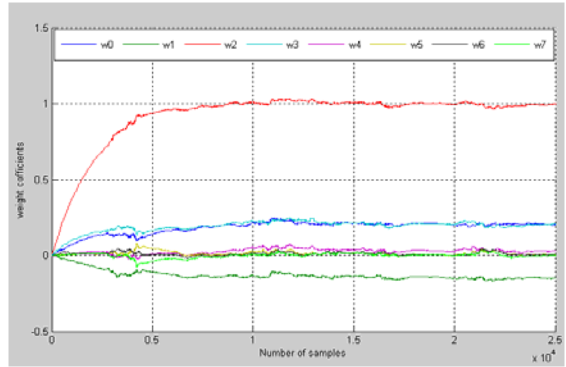

Figure 5. Weight coefficients estimation for LMS algorithm at SNR (0 dB).

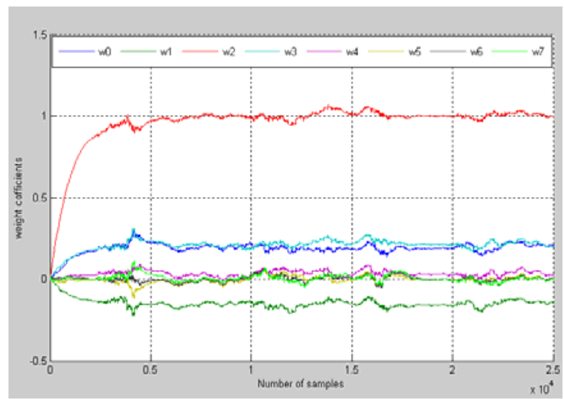

Figure 6. Weight coefficients estimation for VSSLMS algorithm at SNR (0 dB).

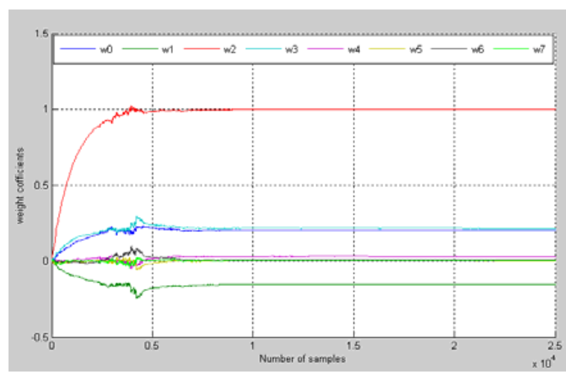

Figure 7. Weight coefficients estimation for AAE-ASSLMS algorithm at SNR (0 dB)

5.2.1 Case 1 fixed SNR at primary input

In this case fixed signal to noise ratio (0 dB) at primary input is used with the original speech signal shown previously in Figure 3 .

Figure 8 shows error function performance for the proposed algorithm, LMS and VSSLMS algorithms. As shown in this figure the proposed algorithm has a faster convergence time and a lower misadjustment compared with LMS and VSSLMS algorithms.

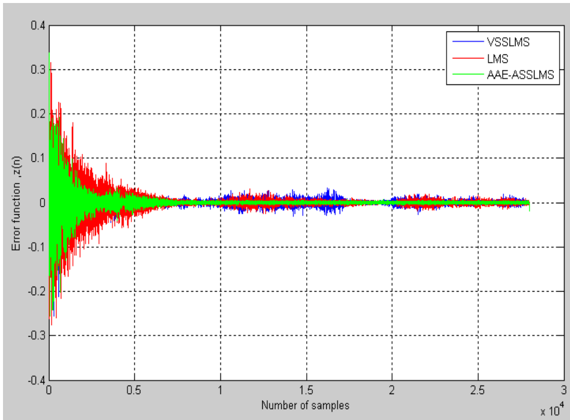

Figure 8. Error function performance for fixed SNR (0 dB) at primary input

5.2.2 Case 2 Power variation at reference input

In this case one of the characteristics of reference input (power variation) will vary as a function of time. Therefore, the primary speech signal will be affected by power variation of the reference input as shown in Figure 9. Figure 10 shows the Error function performance for the different algorithms for this case. It is obvious that the proposed algorithm gives a lower level of the misadjustment compared with the other algorithms for the same convergence time approximately. Moreover, the proposed algorithm is insensitive to the noise power variation in the primary input.

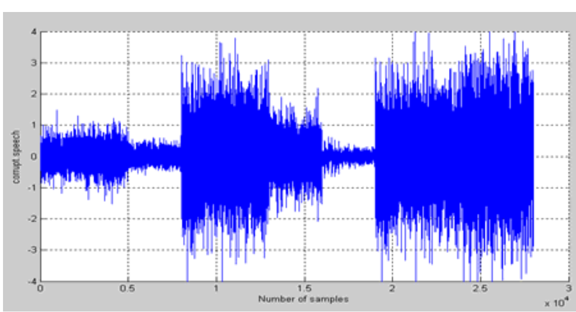

Figure 9. Corrupted speech signal at the primary input due to power variation of reference input.

5.2.3 Case 3 different SNR's at primary input

Another case will be considered here when different signal to noise ratios (SNR's) at primary input are applied.

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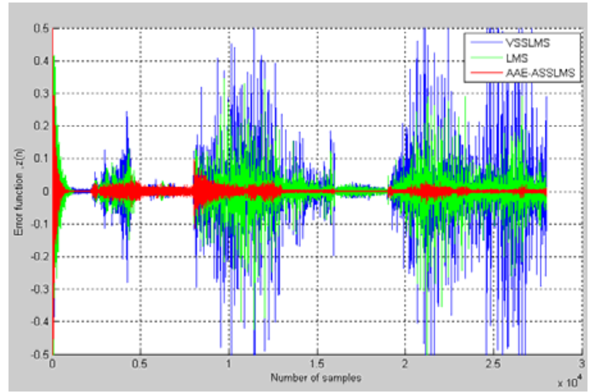


Figure 10. Error function performance for different algorithms when the reference input is power variation.

Figure 11 shows the error function performance for LMS, VSSLMS and the proposed algorithm respectively for different signal to noise ratios. These figures consist of two parts ,the part on the left hand represents the number of sample from the (0-2500) and the part on the right hand represents the number of sample from the (2501-30000). These two parts are required to explain the tradeoff between the convergence time and the misadjustment at steady state for different signal to noise ratio. From these figures, it is obvious that the proposed algorithm gives faster convergence time and lower misadjustment at steady state for different signal to noise ratios at the primary input.

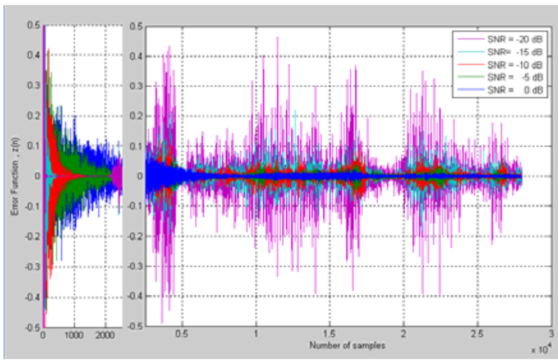
The attenuation factor was measured in (dB), and it represents the ratio between the original signal $s(n)$ to the difference between original signal $s(n)$ and the error signal $e(n)$. More negative value of attenuation means the best performance. The average attenuation factor can be calculated as follows:

$$Attenuation(n) = -20log10\,(U/Q) \qquad (16)$$

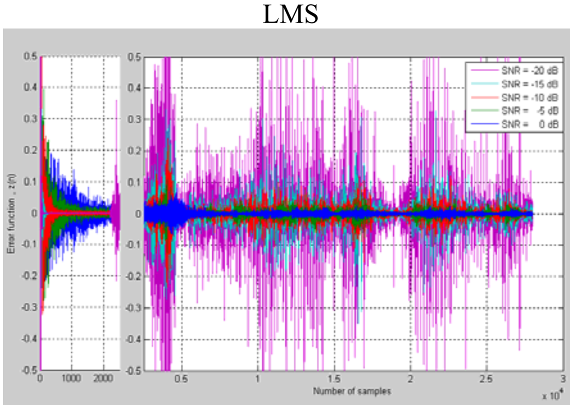
$$where \qquad U = \sum_{i=n}^{WL} s(i)$$

$$and \qquad Q = \sum_{i=n}^{WL} s(i) - e(i)$$

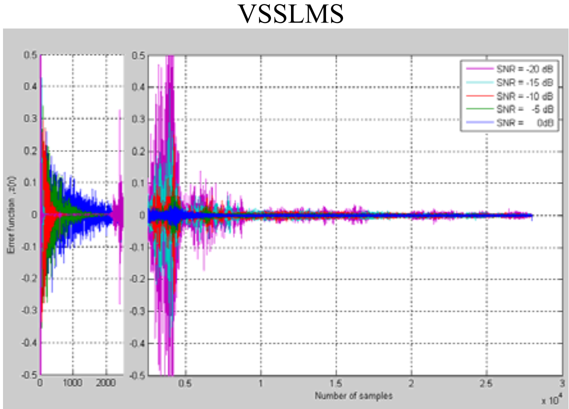
Where WL represents the window length, long window length takes more running time in computer but makes the attenuation factor performance smoother. In this paper, the window length is chosen to be (4000). Figure 12 shows the performance of the attenuation factors for the LMS, VSSLMS and proposed algorithms. As shown in these figures the proposed algorithm has a higher attenuation performance and lower misadjustment at the steady state compared with the LMS and VSSLMS algorithms. Average attenuation can be calculated using equation (9) (for example, from Figure12. The average attenuation factor for LMS algorithm is (-11dB), VSSLMS (-8 dB) and proposed algorithm (-24 dB)). As shown in this figure the proposed algorithm gives a lower (more negative) level of attenuation than the other algorithms. The proposed algorithm achieved (16, 13) dB difference compared with the other algorithms.



LMS



VSSLMS



AAE-ASSLMS

Figure 11. Error function performance of different algorithms for different (SNR's) at primary input.

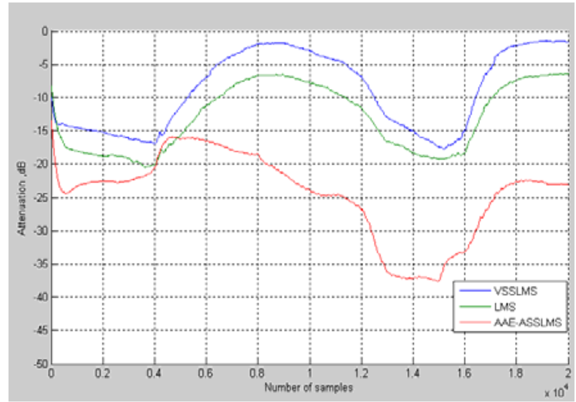


Figure 12. Attenuation performance for different algorithms at steady state.

6 The effect of Constant (β) on the Algorithm Performance:

In this section the effect of the constant (β) on the step size updating profile for the proposed algorithm is introduced. As explained previously the constant (β) is a very important factor for determining the convergence time only of the proposed algorithm (because the misadjustment of the proposed algorithm is treated by the (μ_{min})). Figure 13 shows the error function performance of the proposed algorithm for different constant (β) at signal to noise ratio equals (0 dB).

As shown in this figure the optimum value of constant (β) is 0.00018 that makes the convergence time fast and the misadjustment level is low.

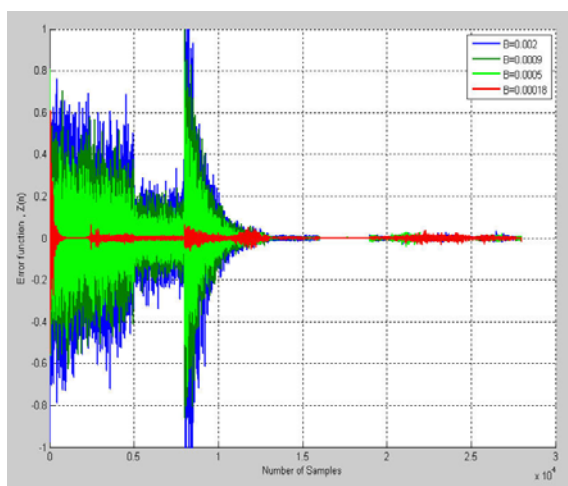


Figure13. Error function performance of the proposed algorithm for different constant (β).

7. Conclusions

This paper focused on enhancement performance of the standard LMS using new proposed algorithm (AAEASSLMS). The performance of the proposed algorithm was illustrated by simulations of ANC. Through simulation results using real speech signal, the proposed algorithm shows low level of miss-adjustment compared with LMS and VSSLMS algorithms respectively. The attenuation factor of ANC was enhanced using proposed algorithm compared with other algorithms. Moreover, the proposed algorithm is insensitive to the both power variation of reference input and different signal to noise ratios at the primary input of ANC. Moreover, the simulation results show that the optimum value of constant (β) was 0.00018.

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