Low-Complexity Image Denoising via Analytical Form of Generalized Gaussian Random Vectors in AWGN

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Abstract: The application of the wavelet transform in image denoising has shown remarkable success over the last decade. In this paper, we present new Bayesian estimators for spherically-contoured generalized Gaussian (GG) random vectors in additive white Gaussian noise (AWGN). The derivations are an extension of existing results for Pearson type VII random vectors. In fact, Pearson type VII distribution have higher-order moment in statistical parameter for fitted the data such as mean, variance, and kurtosis. Indeed, where high-order statistics were used, better performance can be obtained but with much higher computational complexity. In Specific case, GG random vectors is similar to Pearson Type VII random vectors. However, the specific case of GG random vectors have only first few statistical moments such as variance. So, the proposed method can be calculated very fast, with out any contours. In our experiments, our proposed method gives promising denoising results with moderate complexity.

Key–Words: Bayesian Estimation, Spherically-Contoured Generalized Gaussian (GG) Random Vectors, Wavelet Denoising

1 Introduction

The wavelet transform have been developed and successfully used in several image processing applications such as compression and denoising [1]-[6]. Indeed, wavelet-based image denoising methods are formulated as a Bayesian estimation problem. So, the prior knowledge about the probability distribution of wavelet coefficients is required. Wavelet coefficients of natural images usually possess peaked, symmetric, zero-mean distributions, with heavier than Gaussian tails [7, 8]. Moreover, it is well-known fact that wavelet coefficients possess strong dependencies among parents' coefficients (adjacent scale locations) [9]- [11]. So, multivariate modeling offers advantages over univariate modeling.

Pearson type VII random vectors is one of the distribution that successfully use for image denoising [12]. However, it has higher-order moment in statistical parameter for fitted wavelet coefficients, a computationally complex parameter estimation step was required. So, the goal of this paper is we propose the simply distribution is similar to Pearson type VII distribution. The proposed distribution should have only first few moments such as mean and variance.

In fact, generalized Gaussian (GG) distribution have been developed and successfully used in image processing such as estimation, compression, and denoising problems [13]-[17]. In [6, 18], the researchers develop maximum a posterior (MAP) estimation in combination with univariate GG distribution for wavelet coefficients. In [14], the researchers proposed Bayesian estimator for GG random vectors in additive white Gaussian noise (AWGN). However, there is no closed-form solution for the proposed method, computationally complex iterative numerical solution was required. In this paper, we derive MAP estimators for GG random vectors in AWGN. We obtain a closed-form expression for our proposed. So, our method can be calculated very fast, without any contours.

We recall that zero-mean of marginal GG distribution has the density [18]

$$f_x(x) = \frac{\beta A(\beta)}{2\Gamma(1/\beta)\sigma} \exp\left(-\left(\frac{A(\beta)\,|x|}{\sigma}\right)^{\beta}\right), x \in R \quad (1)$$

where $\Gamma(z) = \int_0^\infty e^{-t}t^{z-1}dt$ is standard Gamma function, $\sigma > 0$ is standard deviation. $\beta > 0$ is shape parameter, $A(\beta) = \sqrt{\Gamma(3/\beta)/\Gamma(1/\beta)}$.

The GG model is chosen because of the two following assumptions: (1) The GG distribution has been introduced due to its more peaky and heavy-tailed shape compared to the Gaussian distribution [19]. (2)

In our experience, if we fix the shape parameter $\beta = 3/2$, coarsely the range of values commonly encountered in image processing [18], then GG model is similar to Pearson type VII distribution. Consequently, only variance are used in the modeling process.

The paper is structured as follows. Section 2, after a brief review on the basic idea of Bayesian denoising, we obtain a closed-form expression for MAP estimation of the GG random vectors in AWGN. Section 3, the proposed method is applied for wavelet-based denoising of several images corrupted with additive Gaussian noise at various noise levels. Simulation results demonstrate the effectiveness of our proposed algorithm compared with state-of-the-arts methods. The experimental results show that our algorithm achieves better performance visually and in terms of peak signal to noise ratio (PSNR). Finally conclusion and discussion are given in Section 4

2 Bayesian Denoising

In this Section, the idea of MAP estimator will be explained. The denoising of an image corrupted by AWGN with variance σ_n^2 will be considered. For a wavelet coefficient X_1, let $X_{2,}\ldots,X_d$ represent its parent, i.e., $X_{2,}X_3,\ldots,X_d$ is the wavelet coefficient at the same position as the wavelet coefficient X_1, but at the next coarser scale. We suppose that the coefficients are contaminated by additive noise, that is $Y_1 = X_1 + N_1,\ Y_2 = X_2 + N_2,\ldots, Y_\mathrm{d} = X_\mathrm{d} + N_\mathrm{d}$ where $Y_1, Y_2, \ldots, Y_d$ are noisy observations of $X_1, X_2, \ldots, X_d$, and $N_1, N_2, \ldots, N_d$ are noise samples respectively. To take into account the statistical dependencies between a coefficient and its parent, we combine them into vector form as follow: $\mathbf{Y} = \mathbf{X} + \mathbf{N}$. Let us continue on developing the MAP estimator which is equivalent to $\hat{\mathbf{x}} = \arg\max_{\mathbf{x}} \left[f_{\mathbf{Y}|\mathbf{X}}(\mathbf{y}|\mathbf{x})\, f_{\mathbf{X}}(\mathbf{x}) \right]$. After some manipulations, this equation can be written as [11]

$$\hat{\mathbf{x}} = \arg\max_{\mathbf{x}} \left[\ln\left(f_{\mathbf{N}}(\mathbf{y}-\mathbf{x})\right) + \ln\left(f_{\mathbf{X}}(\mathbf{x})\right) \right]. \quad (2)$$

The proposed model for noise-free wavelet coefficients distribution, $f_{\mathbf{X}}(\mathbf{x})$, is important in Eq. (2). Generally, it is hard to find a model for this random vectors.

Indeed, there does not appear to exist a generally agreed upon multivariate extension of the univariate GG distribution. However, we proposed the following zero-mean d-dimensions spherically-contoured GG random vectors for wavelet coefficients and its parent, the proposed model is related GG random vectors form in [20],

$$f_{\mathbf{x}}(\mathbf{x}) = \frac{K(d,\beta)}{\sigma^{d/2}} \exp\left(-\left(\frac{A(\beta)\,\|\mathbf{x}\|}{\sigma} \right)^{\beta} \right), \quad (3)$$

where $K(d,\beta)$ is normalized function. $\beta, \sigma > 0$ denote the shape parameter and standard deviation respectively, $A(\beta) = \sqrt{\Gamma(3/\beta)/\Gamma(1/\beta)}$. We assume that the noise is i.i.d white Gaussian [21], the noise PDF is given by

$$f_{\mathbf{N}}(\mathbf{n}) = \frac{1}{(2\pi\sigma_n^2)^{\frac{d}{2}}} \exp\left(\frac{-\|\mathbf{n}\|^2}{2\sigma_n^2} \right). \quad (4)$$

Solving Eq. (2) using Eq. (3) and Eq. (4) gives

$$\ln(f_{\mathbf{N}}(\mathbf{y}-\mathbf{x})) + \ln(f_{\mathbf{x}}(\mathbf{x})) = \ln\left(\frac{1}{(2\pi\sigma_n^2)^{d/2}} \right)$$

$$-\frac{\|\mathbf{y}-\mathbf{x}\|^2}{2\sigma_n^2} + \ln\left(\frac{K(d,\beta)}{\sigma^{d/2}} \right) - \left(\frac{A(\beta)\,\|\mathbf{x}\|}{\sigma} \right)^{\beta},$$

$$\frac{\partial\left[\ln(f_{\mathbf{N}}(\mathbf{y}-\mathbf{x})) + \ln(f_{\mathbf{x}}(\mathbf{x}))\right]}{\partial x_i} = 0.$$

Then,

$$\frac{y_i - x_i}{\sigma_n^2} - \beta \left(\frac{A(\beta)}{\sigma} \right)^{\beta} \|\mathbf{x}\|^{\beta-2} x_i = 0.$$

Let $r = \|\mathbf{x}\|$ we have

$$x_i \left[1 + \beta\sigma_n^2 \left(\frac{A(\beta)}{\sigma} \right)^{\beta} r^{\beta-2} \right] = y_i. \quad (5)$$

Taking the square root of the sum of the square over $1 \le i \le d$ gives

$$1 + \beta\sigma_n^2 \left(\frac{A(\beta)}{\sigma} \right)^{\beta} r^{\beta-2} = \frac{\|\mathbf{y}\|}{r}, \quad (6)$$

$$r + \beta\sigma_n^2 \left(\frac{A(\beta)}{\sigma} \right)^{\beta} r^{\beta-1} - \|\mathbf{y}\| = 0. \quad (7)$$

Let the shape parameter $\beta = 3/2$, GG random vectors is similar to Pearson type VII random vectors, we have

$$r + \frac{3}{2}\sigma_n^2 \left(\frac{A(3/2)}{\sigma} \right)^{3/2} r^{1/2} - \|\mathbf{y}\| = 0.$$

So, $r =$

$$\left(\frac{ -\frac{3}{2}\sigma_n^2 \left(\frac{A(3/2)}{\sigma} \right)^{3/2} + \sqrt{ \frac{9}{4} (\sigma_n^2)^2 \left(\frac{A(3/2)}{\sigma} \right)^{3} + 4\,\|\mathbf{y}\| } }{2} \right)^{2} \quad (8)$$

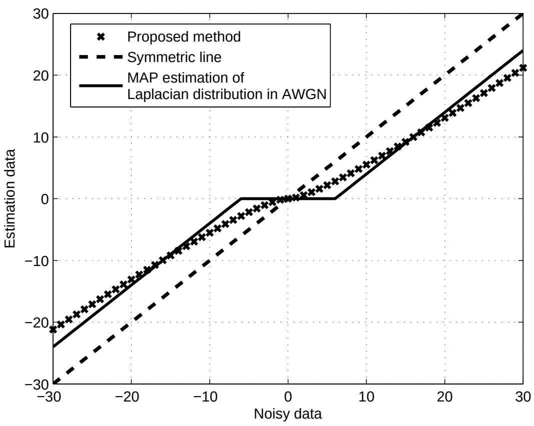

Figure 1: Star line: proposed shrinkage function, univariate case, with $\sigma^2 = 20, \sigma_n^2 = 19$. Solid line: MAP estimation of Laplacian distribution in AWGN.

,where $A(3/2) = 1/\Gamma(2/3)$. Substituting Eq. (6) in Eq. (5) gives

$$\hat{x}_i = \frac{(r)_+}{\|\mathbf{y}\|} y_i, \tag{9}$$

where $(g)_+ = \max(0, g)$. In univariate case ($d = 1$), the proposed method in Eq. (9) given the shrinkage function in following form $\hat{x}_i = (r)_+ sign(y_1)$. Fig. 1 show the proposed shrinkage function produced from Eq. (9), univariate case, and MAP estimation of Laplacian distribution in AWGN [21]. As we can see in Fig. 1, the proposed shrinkage function is nonlinear.

In fact, in order for the processor in Eq. (9) to be of any practical use, one should be able estimate the standard deviation σ for the noise-free signal and noise variance σ_n^2 from observed data. As proposed in [22], a robust estimate of noise variance is obtained using the median absolute deviation of coefficients at the first level of an wavelet decomposition. For standard deviation estimation from noisy observation have been previously proposed in [23].

3 Experimental Results

This section presents image denoising examples using our proposed method, bivariate case ($d = 2$), and compare it with MAP estimators for radial exponential [24], Laplace [21], Pearson type VII [12], and two-sided Gamma random vectors [25] in AWGN, every distributions are more peaked and the tails are heavier. Three 512×512 grayscale images, namely Lena, Barbara, and Boat are used to assess the algorithm's performance. Those test images can be obtained from the same sources as mentioned in [3]. In discrete wavelet transform, the Daubechies length-10 filter and 7×7 window size for estimation of local variances are used. Here, we have not considered different window's size. These algorithms are evaluated with different additive Gaussian noise levels, $\sigma_n = 5$, 10, 20, and 30. We use PSNR as an objective criterion for image performance evaluation. Results can be seen in Table 1, highest PSNR value is star. Each PSNR value in Table 1 is averaged over five runs. We compare the visual qualities of different denoising results of Barbara image, $\sigma_n = 20$, in Fig. 2 show cropped part with the face. The experimental results show that the proposed method yield good denoising results.

4 Conclusion and Discussion

We derive MAP estimators for GG random vectors in AWGN. Here, we obtain a closed-form expression for our proposed method. The specific case of GG random vectors have only first few statistical moments such as variance. Indeed, where high-order statistics were used, better performance can be obtained but with much higher computational complexity. So, the proposed method can be calculated very fast, with out any contours. The experimental results show that the proposed method yields good denoising results.

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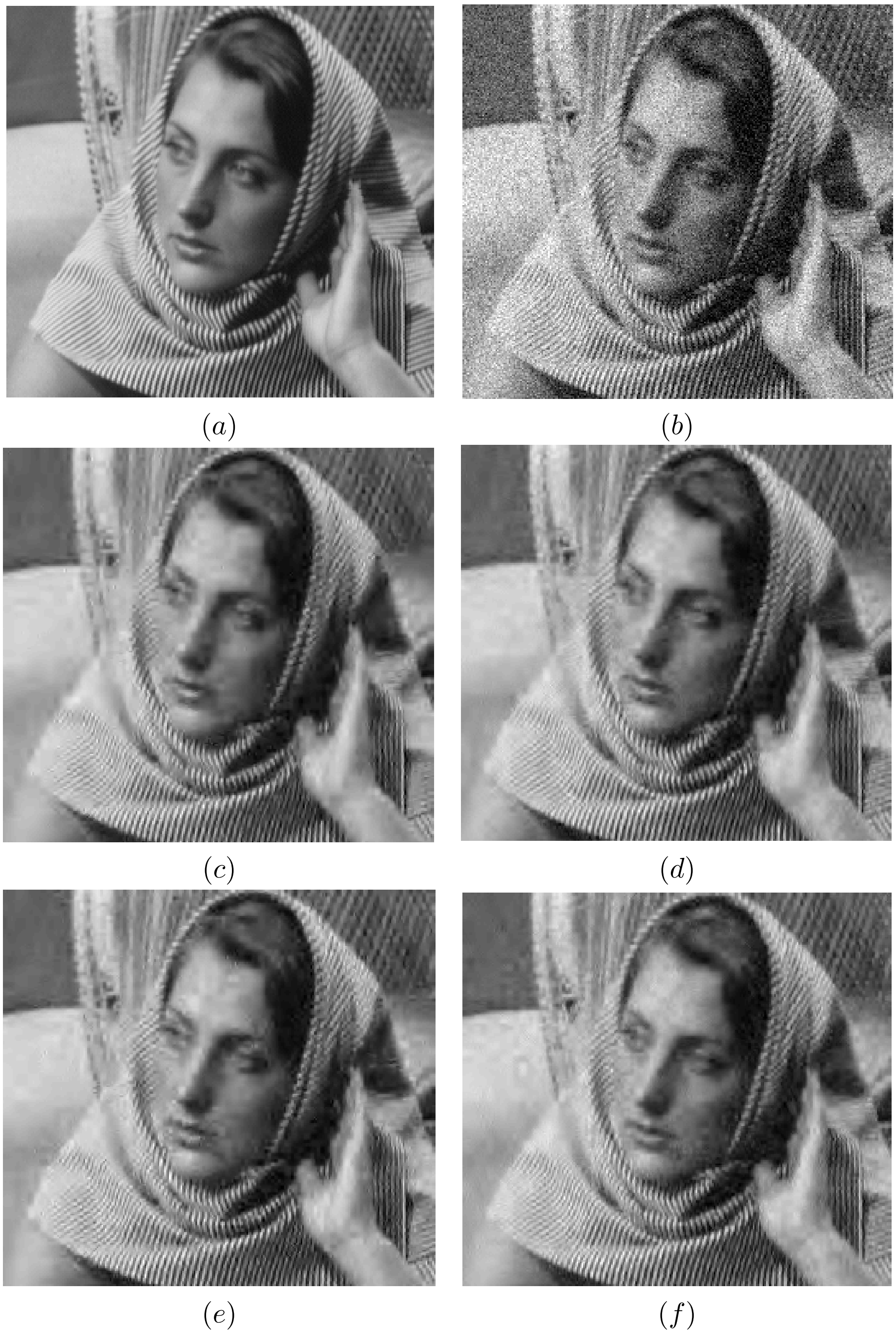

Figure 2: Comparison of denoising results of Barbara image (cropped) with $\sigma_n = 20$: (a) Noise-free. (b) Noisy image. (c) MAP estimation, Laplace random vector [21] (PSNR = 28.04). (d) MAP estimation, Pearson type VII random vectors [12] (PSNR = 28.64). (e) MAP estimation, two-sided Gamma random vectors [25] (PSNR = 28.27). (f) Proposed method (PSNR = 28.66).

Table 1: Average PSNR values of denoising images over five runs for Lena, Barbara, and Boat.

Standard Deviation Noise (σ_n)	5	10	20	30
Lena				
radial exponential random vectors [24]	37.43	34.39	31.24	29.36
Laplace random vectors [21]	37.38	34.29	31.13	29.32
Pearson type VII random vectors [12]	37.74	34.35	31.21*	29.35*
two-sided Gamma random vectors [25]	37.57	34.30	31.12	29.23
Proposed method	37.77*	34.45*	31.21*	29.23
Barbara				
radial exponential random vectors [24]	36.23	32.19	28.27	26.18
Laplace random vectors [21]	35.98	31.94	28.04	25.94
Pearson type VII random vectors [12]	36.70*	32.46	28.64	26.60*
two-sided Gamma random vectors [25]	36.43	32.23	28.27	26.19
Proposed method	36.70*	32.54*	28.66*	26.58
Boat				
radial exponential random vectors [24]	35.33	32.42	29.19	27.34
Laplace random vectors [21]	35.25	32.38	29.11	27.27
Pearson type VII random vectors [12]	35.73	32.51	29.29	27.44*
two-sided Gamma random vectors [25]	35.72	32.47	29.17	27.31
Proposed method	35.83*	32.57*	29.30*	27.40

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