An improved redundant observability model for optimal placement of PMUs with different channel capacities

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Abstract

This paper proposes a new optimization model for optimal placement of PMUs having different channel capacities with improved measurement redundancy. The proposed model provides complete system observability during normal operating conditions as well as contingencies like single line outage and PMU outages. It has been tested on different IEEE test systems and a practical 246 bus Indian system using LINDOGLOBAL solver in GAMS software package. The effect of usage of PMUs with varying channels and fixed channels has been studied using channel utilization factor and total installation cost. It has been found that placing PMUs with varying channel capacity requires less number of channels for complete system observability of the system leading to reduction in installation cost. Further, the proposed redundant observability formulation has been compared with the cost minimization model of PMU placement to demonstrate the improved redundancy obtained with equal number of PMUs.

Keywords:
Phasor measurement unit, integer programming, channel limits, measurement redundancy, zero injection bus

1. Introduction

Due to rapidly growing electricity demand and deregulation of electricity supply industry, power systems are operated closer to their stability boundaries causing reduction in their marginal security. In such circumstances, for keeping the system secure, an accurate monitoring of system states is essential. This was conventionally done using SCADA systems. The measurements provided by this system were not synchronized which led to errors in state estimation. To overcome this limitation, wide area measurement systems based on Phasor Measurement Units (PMU) are employed [1, 2].

PMUs are devices which provide synchronous measurements of voltage and current phasors in the power system. Synchronicity is achieved using a clock pulse generated from Global Positioning System (GPS). This property of the PMU along with its high slew rate improves the accuracy of its measurements and makes it an ideal measurement device [3].

Due to the high installation cost of PMU and its related equipment, it is impractical to place them at all the buses. Moreover, PMU placed at a bus observes itself and all its interconnected buses provided it has sufficient channels. Therefore, the number of PMUs required to make the power system observable is always less than the total number of buses in that power system. Thus, identification of optimal locations for installation of PMU is one of the main problems to be focused upon in this area [4, 5].

Optimal PMU placement problem was first attempted in [6] using a combination of bisecting search algorithm and simulated annealing. The former was used to find the optimal number of PMUs for complete system observability and the latter was used to find the optimal locations for placing the same. Since then, many approaches have been introduced for PMU placement. They can be broadly classified as meta-heuristic [7–13] and deterministic approaches. Meta-heuristic approach uses intelligent search techniques to find the optimal locations for PMU placement. Deterministic approaches use optimization algorithms like Integer Linear Programming (ILP) [14, 15], semi-definite programming [16] and Integer Quadratic Programming (IQP) [17, 18].

Optimal PMU placement with improved redundancy is performed in [19–24]. In [19, 20], an integer programming approach is used for finding the optimal PMU locations with improved redundancy under normal operation and contingencies. The dual objectives of minimizing the number of PMUs and improving redundancy are combined into a single objective function in these models. Binary search algorithm has been used in [21] to optimize the number of PMUs for complete system observability and the solution having better measurement redundancy was chosen as an optimal solution in case of multiple solutions. But, this method is computationally very intensive which limits its application to smaller systems. A PMU placement model for improving the measurement redundancy of critical buses, which are selected based on topology, dynamic and transient stability of system, was presented in [22].

In the aforementioned literature, it was assumed that a PMU placed on a bus can observe all its interconnected buses. However, the channel capacity of PMU is limited due to which some of the interconnected buses may not be observed. For maintaining complete system observability under such situations, the effect of limited channel capacity was incorporated into the optimization model in [25] and solved using a Binary Integer Linear Programming (BILP) approach. Similar models using ILP was formulated in [26–29]. Reference [30] proposed an integer programming model for studying the effect of limited channel capacity during normal operations and contingencies like single line outage and PMU outages.

Apart from ILP, meta-heuristic algorithm like GA have also been used to solve the limited channel capacity problem. In [31], a cellular genetic algorithm based model is used for solving the optimal PMU placement problem with limited channel capacity during normal operation and power system contingencies. The model in [32] uses a combination of ILP and genetic algorithm to optimize the PMU placement considering the number of analog channels. A four stepped algorithm based on GA for simultaneously minimizing the number of PMUs along with the number of channels is proposed in [33].

It is noticed that the usage of PMUs of different channel capacities is not considered in most of the literatures [25, 26, 28–33] . However, placement of PMUs having different channel capacities for optimal placement reduces the number of measurement channels needed for complete system observability thereby reducing the installation cost. In view of the above facts, an attempt has been made in this paper to improve the measurement redundancy while optimally placing the PMUs of different channel capacity.

This paper proposes a new redundant observability model to determine the optimal placement of PMUs with varying channel capacity. In this model, the channel capacity of PMU at a particular bus is selected on the basis of its maximum observability. This reduces the number of channels required for complete system observability during normal operations and contingencies like single line outage and PMU outages reducing the installation cost. The effectiveness of the usage of PMUs with varying channels in the proposed model is examined by comparing its channel utilization factors and total installation cost with fixed channels for different IEEE test systems and a practical system. Further, the improvement in measurement redundancy is verified by comparing the System Observability Redundancy Index (SORI)[3] of the proposed model with that of the cost minimization model. The SORI of the test system for a particular channel capacity is the sum of the total number of direct and indirect observations made using the given set of PMUs. The indirect observations are made using zero injection effect.

The contributions of the paper are summarized below.

(a) An optimization model capable of determining the optimal locations of PMUs with varying channel capacities has been developed. While optimally placing the PMUs with varying channel capacities, it is important to use the PMUs with best suitable channel capacity at a bus such that the number of PMUs is minimized and the use of channel capacity is maximized. Thus, the placement of a higher channel capacity PMU at a bus with fewer interconnections should be avoided. Keeping this in view, a new constraint has been included in the proposed model to place the PMUs with varying channel capacities at the buses such that the utilization of channels is maximized.

(b) A new objective function is defined in order to obtain the PMU locations such that the measurement redundancy is maximized and the number of PMUs is minimized even when PMUs with varying channel capacity are used. A simple method has been identified to calculate the value of weightage factor given to the measurement redundancy.

(c) A new constraint is added to take care of the fact that a PMU placed at a bus measures the voltage phasor of that bus irrespective of the channel limits. In the absence of this constraint, the voltage phasor of the PMU placed bus may be measured by some other PMU in the system which is an undesirable phenomenon.

The remaining sections of the paper are organized as follows. Section 2 presents the basic formulation for optimal PMU placement. The impact of limited channel capacity on optimal PMU placement with improved measurement redundancy is detailed in Section 3. The formulations detailed in the earlier sections are tested on IEEE test systems and the results are tabulated in Section 4. Section 5 concludes the paper.

2. Basic PMU Placement Problem

Since the installation of PMU and its associated equipment is very expensive, it is necessary to minimize the cost of installation of PMUs. Thus, the main objective of PMU placement problem is to determine the minimum number of PMUs required for complete system observability. This can be mathematically represented as

$$Min \quad \sum_{i=1}^{N} c_i x_i \tag{1}$$

where, N is the number of buses in the power system and c_i denotes the cost of PMU placed at i^{th} bus and x_i is a binary variable, which indicates whether the PMU is placed at i^{th} bus. If the value of x_i is one, then the PMU is placed at the i^{th} bus, otherwise not [30]. N indicates the number of buses in the power system. This objective function is optimized according to certain observability constraints. These constraints are derived from the observability rules mentioned below [3].

(1) PMU placed on a bus can measure the voltage phasor of that bus and the current phasors emanating from it making the host bus directly observable and all the connected buses observable using Kirchhoff's law.

(2) If voltage phasors of the two interconnected buses are known then current phasor of the connected branch can be calculated through Ohm's law.

For complete system observability, each bus should be observed at least once, which can be expressed mathematically as

$$\sum_{j=1}^{N} a_{ij} x_j \geq 1, \qquad \forall i \in I \tag{2}$$

where, a_{ij} is the binary connectivity parameter of buses i and j. It attains a value of one when the buses i and j are connected. I denote the set of buses in the power system.

There are some buses in the power system which are neither connected to any generators nor loads. These buses are used only for transferring the power from one point to another and are called zero injection buses. If zero injection buses are modelled in the observability constraints, then the total number of PMUs required for complete power system observability can be further decreased. They are modelled into the observability constraints subject to certain rules given below [27].

(1)When the buses incident to an observable zero-injection bus, are all observable except one, then the unobservable bus is also identified as observable by applying KCL at zero-injection bus.

(2)When all the buses incident to an unobservable zero-injection bus are observable, then the zero-injection bus is also identified as observable by applying KCL at the incident node.

These rules can be mathematically modelled as [27]

$$\sum_{j=1}^{N} a_{ij}x_j + \sum_{j\in ZIB} a_{ij}z_j y_{ij} \geq 1, \qquad \forall i \in I \tag{3}$$

$$\sum_{i=1}^{N} a_{ij}y_{ij} = z_j, \qquad \forall j \in ZIB \tag{4}$$

where, z_j and y_{ij} are binary variables used to include the zero injection effect into the observability constraints. If z_j is equal to one then the j^{th} bus is a zero injection bus, otherwise not. The value of y_{ij} indicates whether i^{th} bus is observed through the zero injection effect of bus j. If the value of y_{ij} is one, then it can be inferred that i^{th} bus is observed through the zero injection effect of bus j. ZIB denotes the set of zero injection buses in the system.

3. Proposed Formulation

With the same number of PMUs being used, the PMU placement problem has more than one solution for complete system observability. The best solution among them will be the one with maximum measurement redundancy. For selecting the best solution, a new criterion of maximizing measurement redundancy is added to the cost minimization making it a bi-objective problem. Maximization of measurement redundancy will help in better utilization of the available channels of PMU. The proposed objective function after combining the cost minimization and measurement redundancy is

$$\sum_{i=1}^{N} c_i x_i + \beta \sum_{i=1}^{N}(-f_i) \tag{5}$$

where,

$$f_i = \sum_{j=1}^{N} a_{ij}x_j, \qquad \forall i \in I \tag{6}$$

Here, f_i is the observability constraint of the i^{th} bus whose value indicates the number of times that bus is observed using the given set of PMUs in the system. Minimizing the negative value of f_i will maximize the observability of i^{th} bus thereby improving the measurement redundancy. The parameter β is the normalization factor for the redundancy maximization function. If the value of β is high, then more PMUs will be needed for complete system observability. Therefore, the value of β should be selected such that the cost minimization function is not affected. In [19], β is defined as the inverse of total times all the buses that can be ideally observed in a power system. But, for large practical systems, calculation of β using this equation will be tedious. Therefore, a new definition is proposed for β as shown in (7).

$$\beta = \frac{1}{N * C} \tag{7}$$

Here, C is the maximum number of connections of a bus in that system. Ideally, the value of β is always less than unity. The value of β derived through equation (7) is lesser than that of [19] which in turn helps in increasing the measurement redundancy of the system.

In the previous section, it is assumed that the PMU has enough number of channels to measure the voltage phasors of its neighboring buses. However, the maximum number of measurement channels of a PMU is usually not more than eight due to technical limitations and cost constraints [34]. Due to this limitation, PMU placed at a bus cannot fully observe all the interconnected buses if the number of interconnections is higher than the number of current channels of the PMU. For instance, when a PMU having n current channels is placed on a bus which is connected to m other buses such that $m > n$, then $m - n$ buses will remain unobservable even though a PMU is placed at its adjacent bus. To overcome this problem, the observability constraints need to be modified to include the effects of channels limits. This is done by adding another binary variable to the observability constraint equation in (6) as shown below [27].

$$g_i = \sum_{j=1}^{N} a_{ij}w_{ij}x_j, \qquad \forall i \in I \tag{8}$$

The parameter w_{ij} denotes whether the bus i is observed using a PMU placed at bus j. The value of $w_{ij} = 1$ indicates that the PMU placed on j^{th} bus measures the current phasor between i^{th} and j^{th} buses using one of its current channels. The value of w_{ii} or $w_{jj} = 1$ indicates that the PMU placed on i^{th} bus measures the voltage phasors of the i^{th} bus using its voltage channel.

For improving the measurement redundancy when the channel capacity of PMU is limited, the modified objective function will be

$$\sum_{i=1}^{N} c_i x_i + \beta \sum_{i=1}^{N}(-g_i) \tag{9}$$

Since the effect of zero injection buses is independent of channel capacity of PMUs, it can be modelled into the observability constraints as in (3). The modified observability constraints with limited channels can be written as

$$\sum_{j=1}^{N} a_{ij}w_{ij}x_j + \sum_{j\in ZIB} a_{ij}z_jy_{ij} \geq 1, \qquad \forall i \in I \qquad (10)$$

$$\sum_{i=1}^{N} a_{ij}y_{ij} = z_j, \qquad \forall j \in ZIB \qquad (11)$$

The total number of measurements made by a PMU should be less than or equal to its total channel capacity. This constraint is expressed as

$$\sum_{i=1}^{N} a_{ij}w_{ij} \leq w_j^{max}, \qquad \forall j \in I \qquad (12)$$

where, w_j^{max} is the channel capacity of the PMU placed at j^{th} bus [27].

In [27], the channel limit of the PMU is assumed to be constant. However, placing a PMU with higher channel capacity at a bus with less number of connections is uneconomical. Therefore, in this work, PMUs with different channel capacities have been considered. The channel capacity of the PMU at a particular bus is determined by the number of connections of the bus. To accommodate this, a new variable u_j is defined, which represents the maximum observability of that bus. It is determined by summing the j^{th} row of the binary connectivity matrix.

$$u_j = \sum_{i=1}^{N} a_{ij} \qquad (13)$$

The value of u_j is compared with the channel capacities of the PMU available and the most appropriate channel capacity is selected for that particular bus. Thus the constraint (12) is modified as given below.

$$\sum_{i=1}^{N} a_{ij}w_{ij} \leq \left\{ \begin{array}{ll} k1, & if \ \ u_j \leq k1 \\ k2, & if \ \ k1 < u_j \leq k2 \\ k3, & k2 < u_j \end{array} \right\}, \qquad \forall j \in I \qquad (14)$$

here, k1, k2 and k3 represents the channel capacities of PMU used in the system.

The buses adjacent to the PMU placed bus may or may not be observed depending on its channel limits. The following constraint is formulated based on this logic [27].

$$w_{ij} \leq x_j, \qquad for \ i \neq j, \quad \forall i, j \in I \qquad (15)$$

Since the total number of channels are limited in PMU, it becomes necessary to keep one channel for measuring voltage phasor of the host bus in the optimization model. This is realised through the following equation

$$w_{ij} = x_j, \qquad for \ i = j, \quad \forall i, j \in I \qquad (16)$$

This constraint is not implemented in [27] which is one of the main limitations of that model. If it is not enforced, then the host bus is made observable by the current channel of some other PMU in the system. The voltage channel of the PMU is remaining idle in such cases which is an undesirable phenomenon. This is illustrated below with the help of an example.

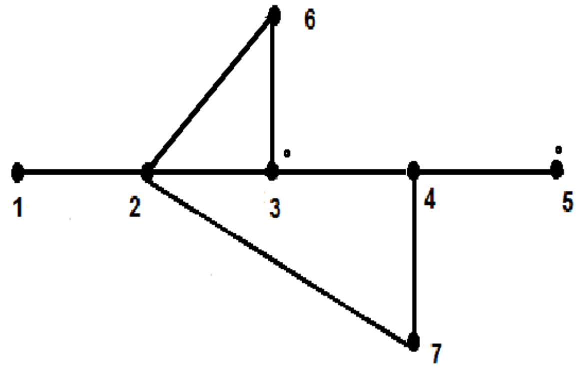

Figure 1: 7 bus system [20]

Let us consider a 7 bus system having two zero injection buses 3 and 5 as shown in Fig. 1. The objective function can be written as follows

$$Z = \sum_{i=1}^{7} x_i + \beta \sum_{i=1}^{7} (-g_i) \qquad (17)$$

$$g_i = \sum_{j=1}^{7} a_{ij}w_{ij}x_j + \sum_{j\in 3,5} a_{ij}z_jy_{ij} \qquad (18)$$

Here, g_i is the observability constraint of the i^{th} bus. The value of β is found to be 0.0357 using (7). The cost of the PMU is considered as 1 pu for simplicity. The observability constraints of the 7-bus system are given below

$$\begin{array}{l} w_{11} * x_1 + w_{12} * x_2 \geq 1; \\ w_{21} * x_1 + w_{22} * x_2 + w_{23} * x_3 + w_{26} * x_6 + w_{27} * x_7 + y_{23} \geq 1; \\ w_{33} * x_3 + w_{32} * x_2 + w_{34} * x_4 + w_{36} * x_6 + y_{33} \geq 1; \\ w_{44} * x_4 + w_{43} * x_3 + w_{45} * x_5 + w_{47} * x_7 + y_{43} + y_{45} \geq 1; \\ w_{55} * x_5 + w_{54} * x_4 + y_{55} \geq 1; \\ w_{66} * x_6 + w_{62} * x_2 + w_{63} * x_3 + y_{63} \geq 1; \\ w_{77} * x_7 + w_{74} * x_4 + w_{72} * x_2 \geq 1; \\ y_{23} + y_{33} + y_{43} + y_{63} = 1; \\ y_{45} + y_{55} = 1; \end{array} \qquad (19)$$

The channel capacity of PMU placed at a particular bus is determined by (13) and (14). In this example, it is assumed that the channel capacities of PMU are 2 and 4 respectively. So $k1$ and $k2$ are set as 2 and 4 respectively. It is noticed from (19) that maximum observability of all the buses except buses 1 and 5 is greater than 2. So these buses are suitable for the placement of PMU having $k2$ channels whereas PMU having $k1$ channels can be placed at buses 2 and 5. This mathematically represented in (20)

$$
\begin{aligned}
&w_{11} + w_{21} \leq 2;\\
&w_{12} + w_{22} + w_{32} + w_{62} + w_{72} \leq 4;\\
&w_{33} + w_{23} + w_{43} + w_{63} \leq 4;\\
&w_{44} + w_{34} + w_{54} + w_{74} \leq 4;\\
&w_{55} + w_{45} \leq 2;\\
&w_{66} + w_{26} + w_{36} \leq 4;\\
&w_{77} + w_{47} + w_{27} \leq 4;
\end{aligned}
\tag{20}
$$

In some cases, the PMU installed at a bus may not measure the voltage phasor of its own although it observes the other interconnected buses. For instance at bus 2, there may be a case when the value of w_{22} is 0 and all the other channels $(w_{12}, w_{32}, w_{62}, w_{72})$ are 1. This means that bus 2 is not self observed which is contradictory to the principles of PMU placement. Moreover, the voltage channel of the PMU is remaining idle. The following set of equations are introduced to remove this limitation.

$$
\begin{aligned}
&w_{11} = x_1; w_{22} = x_2;\\
&w_{33} = x_3; w_{44} = x_4;\\
&w_{55} = x_5; w_{66} = x_6;\\
&w_{77} = x_7;
\end{aligned}
\tag{21}
$$

The buses which are connected to the PMU placed bus will be observed depending on the channel capacity of the PMU. This is expressed in (22)

$$
\begin{aligned}
&w_{21} \leq x_1; w_{12} \leq x_2;\\
&w_{32} \leq x_2; w_{62} \leq x_2; w_{72} \leq x_2;\\
&w_{23} \leq x_3; w_{43} \leq x_3; w_{63} \leq x_3;\\
&w_{34} \leq x_4; w_{54} \leq x_4; w_{74} \leq x_4;\\
&w_{45} \leq x_5;\\
&w_{26} \leq x_6; w_{36} \leq x_6;\\
&w_{47} \leq x_7; w_{27} \leq x_7;
\end{aligned}
\tag{22}
$$

The set of equations (19)-(22) are solved using LINDOGLOBAL solver in GAMS software package. The optimal PMU locations of the considered 7- bus system is found to be buses 2 and 3 when PMUs having channel capacities of 2 and 4 are used.

3.1. PMU Outage Scenario

In a fully observable system, when a PMU becomes faulty, one or more buses will become unobservable. This problem can be solved by increasing the observability of all the buses in the power system from one to two. Therefore, the observability constraint in (17) is reformulated as [27]

$$g_i + \sum_{j=1}^{N} a_{ij} y_{ij} \geq 2, \qquad \forall i \in I \tag{23}$$

where g_i is the observability constraint of the i^{th} bus and the value of $a_{ij}y_{ij}$ denotes whether the i^{th} bus is observed through zero injection effect. If $\sum_{j=1}^{N} a_{ij}y_{ij} = 1$ then the i^{th} bus is observed through one of the zero injection buses connected to it. If i^{th} bus is not connected to any zero injection bus then $\sum_{j=1}^{N} a_{ij}y_{ij} = 0$. The number of PMUs required for effectively covering the PMU outage scenario will be much higher than that of the cost minimization or redundant observability models. Therefore, this constraint will be applied only for important buses in large practical systems.

3.2. Line Outage Scenario

Line outage in the power system causes changes in the connectivity matrix. When a line connecting buses i and j is taken out, then the value of a_{ij} and a_{ji} in the connectivity matrix becomes zero making one or more buses unobservable. For making the system observable in this situation, new observability constraints which includes the effect of line outage across line i-j should be formulated. They are formulated based on the following equations [27]

$$f_i^k \geq 1, \qquad \forall i \in I, \quad \forall k \in K \tag{24}$$

where

$$f_i^k = \sum_{j=1}^{N} a_{ij}^k w_{ij}^k x_j + \sum_{j \in ZIB}^{N} a_{ij}^k z_j y_{ij}^k, \quad \forall i \in I, \ \forall k \in K \tag{25}$$

$$\sum_{i=1}^{N} a_{ij}^k y_{ij}^k = z_j, \qquad \forall j \in ZIB, \ \forall k \in K \tag{26}$$

In the above equations, f_i^k is the post contingency observability constraint of the i^{th} bus and K denotes the set of lines in a power system. Parameters a_{ij}^k and y_{ij}^k are the post contingency values of a_{ij} and y_{ij}. If the line connecting buses i and j is taken out, then the values of both a_{ij}^k and y_{ij}^k will be 0.

4. Simulation Results and Discussions

Table 1: Test Systems

Test	Zero injection buses	Line outage
IEEE 14 bus	7	1-2
IEEE 118 bus	5,9,30,37,38,63,64,68,71,81	101-102
NRPG 246 bus	54,56,59,61,62,63,69,70,71,72,73,74,75,80, 81,86,102,103,104,107,122,126,129,131, 147,154,,154,155,167,175,179,180,183, 209,210,211,212,213,214,215,216,217, 221,222, 226,229,230,231,232,233,234, 236,237,238,239,240,241,243,244	49-50

The performance of the proposed model has been examined on various IEEE test systems viz. IEEE 14-bus, 118-bus systems and a Northern Regional Power Grid - India (NRPG) 246-bus system[35, 36]. The details of these test systems are presented in Table I. The proposed model has been validated considering two different sets of PMU for obtaining optimal locations under normal operating conditions as well as

Table 2: Fixed and variable charges of PMU [34]

Component	Cost ($)
PMU (Fixed Cost)	20k
Voltage channel	3k
Current channel	3k

Table 3: Comparison of TICs and CUFs obtained with the proposed model considering PMUs of fixed and varying channel capacity during normal operation

Test System	No of PMUs for optimal placement with					No of channels needed		TIC of PMUs with ($)		CUF of PMUs with	
	FC	VC				FC	VC	FC	VC	FC	VC
		Two	Four	Six	Total						
Set A											
IEEE 14 bus	3	0	1	2	3	18	16	114k	108k	0.833	0.9375
IEEE 118 bus	28	0	11	17	28	168	146	1064k	998k	0.8154	0 .9383
NRPG 246 bus	53	1	12	40	53	318	290	2014k	1930k	0.8551	0.9379
Set B											
		Three	Five	Seven	Total						
IEEE 14 bus	3	0	3	0	3	21	15	123k	105k	0.7142	1
IEEE 118 bus	28	6	10	12	28	196	152	1148k	1016k	0.7193	0.9276
NRPG 246 bus	53	6	22	25	53	371	303	2173k	1969k	0.7574	0.9339

* FC: Fixed Channels
VC: Varying Channels

Table 4: Comparison of TICs and CUFs obtained with the proposed model considering PMUs of fixed and varying channel capacity during single line outage and PMU outages

Test System	No of PMUs for optimal placement with					No of channels needed		TIC of PMUs with ($)		CUF of PMUs with	
	FC	VC				FC	VC	FC	VC	FC	VC
		Two	Four	Six	Total						
Single Line Outage - Set A											
IEEE 14 bus	4	0	0	4	4	24	24	152k	152k	.875	.875
IEEE 118 bus	29	0	8	21	29	174	158	1102k	1054k	.8505	.9367
NRPG 246 bus	54	1	12	41	54	324	296	2052k	1968k	.8611	.9425
Single Line Outage - Set B											
		Three	Five	Seven	Total						
IEEE 14 bus	4	0	3	1	4	28	22	164k	146k	.75	.9545
IEEE 118 bus	29	5	9	15	29	203	165	1189k	1075k	.8177	.9454
NRPG 246 bus	54	4	23	27	54	378	316	2214k	2028k	.7698	.9208
PMU Outage - Set A											
IEEE 14 bus	7	0	3	5	8	42	42	266k	286k	.7857	.8809
IEEE 118 bus	63	4	29	31	64	378	310	2394k	2210k	.7026	.896
NRPG 246 bus	125	20	49	57	126	750	578	4750k	4254k	.732	.9117
PMU Outage - Set B											
		Three	Five	Seven	Total						
IEEE 14 bus	7	1	6	1	8	49	40	287k	280k	.6734	.9250
IEEE 118 bus	63	24	21	19	64	441	310	2583k	2210k	.6485	.9137
NRPG 246 bus	125	46	49	31	126	875	600	5125k	4320k	.6171	.8983

during line outage and PMU outage contingencies. The details of these sets are given below.

Set A - considers PMUs with channel capacities of two, four and six.

Set B - considers PMUs with channel capacities of three, five and seven.

Due to the non-linear observability constraints, the proposed model has been solved using Mixed Integer Quadratic Constrained Programming (MIQCP) method in LINDOGLOBAL solver.

Table 5: Comparison of SORI of the proposed model against the cost minimization model

Test System	Set A				Set B			
	Cost Minimization Model		Proposed Model		Cost Minimization Model		Proposed Model	
	No. of PMUs	SORI	No. of PMUs	SORI	No. of PMUs	SORI	No. of PMUs	SORI
Normal operation								
IEEE 14 bus	3	16	3	16	3	16	3	16
IEEE 118 bus	28	137	28	147	28	143	28	151
NRPG 246 bus	53	299	53	330	53	311	53	341
Line outage								
IEEE 14 bus	4	19	4	22	4	19	4	22
IEEE 118 bus	29	150	29	158	29	153	29	165
NRPG 246 bus	54	298	54	337	54	315	54	349
PMU outage								
IEEE 14 bus	8	38	8	38	8	38	8	38
IEEE 118 bus	64	270	64	288	64	281	64	296
NRPG 246 bus	126	558	126	585	126	595	126	597

Table 6: Comparison of the proposed model with other existing models

System		IEEE 14 bus		IEEE 30 bus		IEEE 57 bus		IEEE 118 bus	
		No. of PMUs	TIC ($)	No. of PMUs	TIC ($)	No. of PMU	TIC ($)	No. of PMUs	TIC ($)
PM	Set A	3	108k	7	242k	11	382k	28	992k
	Set B	3	105k	7	239k	11	379k	28	1004k
[25]		3	114k	7	266k	12	456k	28	1064k
[27]		3	114k	7	266k	11	418k	28	1064k
[31]		3	114k	7	266k	11	418k	29	1102k
[37]		3	114k	7	266k	12	456k	28	1064k
[38]		3	114k	7	266k	12	456k	28	1064k

* PM: Proposed Model

The absolute stopping tolerance and relative stopping tolerance of the LINDOGLOBAL solver is set to zero to obtain a global optimal solution. The maximum iteration limit of the solver is set as 40000 and tolerance for gradient of nonlinear functions is set as $1e^{-7}$. Moreover, the maximum simulation time is set to 1000 seconds. The effect of using PMUs with varying channel capacities (set A or set B) is studied by comparing the results when PMUs of fixed channel capacity are used. Results obtained using PMUs of set A and set B are compared against PMUs with limited channel capacity of six and seven respectively. For better comparison of results, two new criteria, Channel Utilization Factor (CUF) and Total Installation Cost (TIC) are defined. CUF is the ratio of total direct observations (TDO) made by the set of PMUs to the total number of PMU channels (TPC) present in the system. As the value of TPC decreases, the CUF increases, which indicates that fewer channels are remaining idle.

$$CUF = \frac{TDO}{TPC} \tag{27}$$

TIC is the sum of installation cost of all the PMUs in the system. Installation cost (IC) of a PMU is the sum of fixed costs (FC) which includes the cost associated with the PMU panel, power-supply provision, global positioning system (GPS) installation etc and the cost of voltage and current channels[34].

$$IC_i = FC_i + (n_1 * CV_i + n_2 * CC_i) \tag{28}$$

$$TIC_i = \sum_{i \in PMUL} IC_i \tag{29}$$

Here, PMUL denotes the set of PMU locations in the system. CV and CC denotes the cost of voltage channels and current channels respectively. n_1 and n_2 gives the number of voltage and current channels in the given PMU. The details about fixed cost, cost of voltage and current channels of a PMU are obtained from [34] and is given in Table 2.

Table 3 compares the CUFs and TICs obtained with the proposed model during normal operation when PMUs with fixed and varying channel capacities are placed in the system. It is observed that both the cases require same number of PMUs for full system observability. However, the total number of channels required is less when PMUs with varying channel capacities are used. For instance, to monitor the IEEE 118 bus system, using the PMUs of fixed channel capacity of six requires 168 channels. However, the same system can be fully observed using just 146 channels when PMUs belonging to set A are used. This reduction in the number of channels reduces the TIC by about 6.20% and improves the CUF of the system by almost 12%. Hence, it can be concluded that PMUs of varying channel capacity are more economical than PMUs with fixed channel capacity.

It is also noticed that, for optimal PMU placement of a particular test system using the proposed model, PMUs belonging to set B requires more number of channels when compared to set A, which results in higher TIC. This difference in the total number of channels increases as the size of the test system increases. For full system observability of larger test systems like NRPG 246 system, PMUs belonging to set B requires thirteen more channels than set A. On the other hand, usage of PMUs belonging to set B for optimal placement provide increased measurement redundancy than set A due to the presence of increased number of channels.

Table 4 compares the CUFs and TIC obtained with the proposed model during single line outage and PMU outages, when PMUs with fixed and varying channel capacity are placed in the system. In both the cases, PMUs of varying channel capacities gives a better CUF and reduced TIC than PMUs of fixed channel capacities, irrespective of the size of the test system. It can be noticed that for a particular test system, in most of the cases, the value of CUF during single line outage is lesser than that of normal operating conditions. This is due to the fact that the number of PMUs required for full system observability of a system during single line outage is usually higher than that of normal operating condition. If the number of PMUs required during normal operation and during single line outage is same, then their CUFs are also equal.

It can also be noticed that, during PMU outage, the proposed model with PMUs of varying channel capacities needs an extra PMU as compared to that with fixed channel capacities. However, more number of channels are required if PMUs with fixed channel capacities are used. Therefore, both the cases are compared using TIC. Comparison reveals that, except for IEEE 14 bus system, the TIC of the proposed model with PMUs of varying channel capacity is lesser than that with fixed channel capacities. The savings in the TIC increases with the system size. It is highest for NRPG 246 bus system where the TIC with varying channel capacities is almost 10.4% less than that of with fixed channels.

The effectiveness of the redundant observability formulation in the proposed model is examined by comparing it with that of the cost minimization model under similar operating conditions. PMUs belonging to set A and B are only used in these models. The comparison is done on the basis of SORI.

Table 5 shows the comparison of SORIs obtained with the proposed model against the cost minimization model under normal operating conditions as well as during contingencies. The line taken out for each test system during single line outage is shown in Table 1. It is observed that for all the test systems, both the models require same number of PMUs for complete system observability but the measurement redundancy of the proposed model is higher than the cost minimization model irrespective of the set of PMU used. This increase is more prominent in larger systems like NRPG 246 bus system where there is 10.36% increase in measurement redundancy compared to the cost minimization model when PMUs of set A is used for optimal placement during normal operating conditions. It is also noted that the proposed model gives better measurement redundancy with PMUs of set B than set A due to the presence of additional channels in set B.

In order to further validate the proposed model, it has been compared with existing models in Table 6. In this table, the proposed model uses PMUs belonging to set A and B whereas similar models in [25],[27],[31],[37] and [38] uses PMUs having a fixed channel capacity of six. It is observed that these models have considered only cost minimization as their objective function. So for an effective comparison, the measurement redundancy maximization is removed from the objective function of the proposed model and the performance comparison is done in terms of optimal PMUs required for complete observability and TIC. It is observed that the number of PMUs required is almost same for all these models but the TIC of the PMUs using the proposed model is comparatively lesser than the other models due to the usage of PMUs with varying channel capacities. For instance, the TIC of the IEEE 57 bus system using the proposed model is 8.6% lesser than that of the model in [31]. So it can be inferred that the proposed model is more economical than the models in [25],[27],[31],[37] and [38].

5. Conclusion

In this paper, a redundant observability model for optimal placement of PMUs having varying channel capacity is proposed. The proposed model guarantees complete system observability for normal operating conditions as well as contingencies like single line outage and PMU outages. The effect of usage of PMUs with fixed and varying channel capacities in the proposed model is studied by testing it on various IEEE test systems as well as a NRPG 246 bus system and the results obtained are compared using CUF and TIC. It was found that, the usage of PMUs with varying channel capacities reduces the total number of channels needed for full system observability, thereby improving the CUF and reducing TIC. Moreover, it is observed that the proposed model provides better measurement redundancy than the cost minimization model using the same number of PMUs for complete system observability.

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