Extended Optimal PMU Placement Problem for Voltage Stability Assessment

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Abstract: - This paper proposes extended formulations for the optimal Phasor Measurement Unit (PMU) placement problem in power systems with respect to voltage stability assessment for the cases of Zero Injection Buses (ZIBs), critical buses, and PMU redundancy. Modifications of the Binary Integer Programming (BIP) method to solve the proposed extended PMU placement problem are developed. The extension of the connectivity matrix and observability constraints are introduced on the basis of the analyses of the power system network, Modal Participation Factors (MPFs) from the Eigenvalues of the power flow Jacobian, and the introduction of redundancy for *N-1* PMU outage for the observability of the critical buses. The performances of the modified method and algorithms were validated using the IEEE 14-bus, 30-bus, and 57-bus test networks. Furthermore, a comparison is made with previous results in the literature obtained using the BIP method. The simulation results show that the proposed method and algorithms provide a better framework for the strategic placement of PMUs in practical networks for monitoring the power system status and its margin to voltage collapse. In addition, the modified method and algorithms were shown to give fewer PMUs for redundancy against *N-1* PMU outage when compared to the existing literature.

Key-Words: - Critical bus, modal participation factor, optimal PMU placement, phasor measurement unit, synchrophasors, voltage stability assessment.

1 Introduction

Many Wide Area Monitoring, Protection, and Control (WAMPAC) schemes in power system networks are beginning to integrate time-stamped synchrophasor measurements from Phasor Measurement Units (PMUs). Consequently, there is the need to optimally locate these PMUs in order to reduce the cost of PMU integration and the amount of PMU data that needs to be processed or analysed.

Optimal PMU Placement (OPP) has received considerable attention from researchers in recent years. However, a survey of the literature shows very few publications dedicated to the OPP problem with particular focus on voltage instability. Rather, most publications have focused on OPP for state estimation applications. Voltage instability is peculiar, so are the methods used in its analysis. Thus, it is necessary to take into cognizance the peculiarity of voltage instability during the planning and placement of PMUs, especially in segments of the power system network or critical buses prone to voltage instability. This is because through continuous analyses of measurements from the critical buses in the network, system operators would be able to detect the onset of voltage instability and consequently prevent voltage collapse and blackouts through timely remedial actions at the right locations.

A power system is observable if the available measurements in the system are sufficient in the determination of the voltage magnitude and phase angle at each bus of the system. Power system observability has been examined by [1] under two broad categories: (i) Numerical observability; and (ii) topological observability. A system is numerically observable if the measurement matrix (H) is of full rank and connects all the nodes together. Matrix H is a $m \times N$ matrix, where m is the number of voltage and current phasors, and N is the number of buses.

The required computation for numerical observability can be obtained by using Gaussian elimination or triangular factorization of the Jacobian matrix, gain matrix, or heuristic matrix.

Another method is the computation of the null space of the gain or Jacobian matrices [2]. Topological observability relates to the availability of one or more measurement tree(s) of full rank connecting all the nodes together with either direct measurements from PMUs or calculated measurements. The calculated measurements can be obtained by applying Kirchhoff's and Ohm's laws.

Synchrophasor measurements are measurements obtained from PMUs, and are capable of providing very accurate time-stamped phasors synchronized to an accurate time source such as the Global Positioning System (GPS). Synchrophasor measurements have been proposed for electric power system wide area monitoring, protection and control, parameter estimation, hybrid state estimation, etc. [3,4]. The requirements for optimal placement of PMUs for state estimation applications are different from that of voltage stability applications. Voltage stability applications require the monitoring of the system's status, proximity to instability, and the critical/voltage weak areas of the system. This relates directly to the network topology. Therefore, algorithms for the optimal placement of PMUs for voltage stability assessment should consider the topological observability of the system with special consideration given to voltage weak areas/critical buses.

Several works on Optimal Placement of PMUs (OPP) using mathematical programming, heuristic, and meta-heuristic methods have been covered in the literature. Methods for full network observability based on graph theory and tree search algorithm [5], Tabu Search (TS) algorithm [6], modified binary Particle Swarm Optimization (PSO) [7], probabilistic approach method [8], simulated annealing [9], etc. have been proposed. Methods based on binary integer programming have also been used [10-16].

Although, integer programming may suffer from the problem of being trapped in local minima, the widely used simulated annealing, particle swarm optimization, Tabu search, and genetic algorithm based methods suffer from computational problems as they are iterative in nature, require longer convergence time, and their convergence depend upon the initial guess [14]. Thus, binary integer programming was used in this paper since this would save computational time and resources, and it is suitable for large-scale problems.

Aside [9,16], none of the publications mentioned above considered voltage weak areas/critical buses in the formulation of their OPP problem. Most researchers focused on OPP for state estimation applications. The method by [9] considered OPP for voltage stability margin determination and was implemented using simulated annealing method. Similarly, a modal participation factor-based method combined with binary integer programming was proposed by the authors in [16].

The contributions of this paper include:

- The determination of the minimum number of PMUs and the optimal location to site the PMUs using the critical bus information computed from the eigenvalues and eigenvectors of the Jacobian matrix of the system's power flow.
- The consideration of the effect of zero injection and critical buses on OPP using new rules in the determination of the connectivity matrix as proposed in Sections 3 and 4. Extended formulations of the OPP problem are proposed for the case studies involving the consideration of the impact of the critical buses on voltage collapse and the impact of zero injection buses on the PMU placement sites.
- An extended formulation of the OPP problem for measurement redundancy at the critical buses for a single *N-1* PMU outage is proposed.
- The impact of increased loading on the bus participation factors and the critical buses were investigated.
- A detailed comparison with previous results in the literature is presented.

The observability rules for full topological observability used in this paper include the following [4,11]:

Rule 1: A bus with PMU is regarded as directly observable.

Rule 2: The bus voltage at a remote end of a transmission line can be calculated if the bus voltage and current phasors of the local end are known.

Rule 3: If the voltage phasors at both ends of a line are known, current phasor of the line can be calculated by applying Kirchhoff's Current Law (KCL).

Rule 4: KCL can be applied to a Zero Injection Bus (ZIB) to calculate the unknown current phasors for a branch if the current phasors for all the other adjacent branches are available, since the net injection equals zero.

Rule 5: The voltage phasor of a ZIB can be calculated by using node equations if the voltage phasors of the incident buses to the ZIB are available.

The rest of this paper is structured as follows: Section 2 presents the basic formulation of the OPP problem and the theory of Binary Integer Programming. Extended rules and formulation for the OPP problem considering ZIBs is described in Section 3. Section 4 gives the modal participation factor algorithm and the formulation of the OPP problem considering critical buses. The OPP problem formulation for the consideration of both ZIBs and critical buses are presented in Section 5, while Section 6 is on the OPP formulation for the case of *N-1* PMU outage. Section 7 describes the test systems used, and the various case studies considered. Section 8 presents and discusses the results obtained. Section 9 summarizes the contribution of this paper.

2 OPP Problem Formulation

2.1 Basic OPP formulation

The Optimal PMU Placement (OPP) problem considered in this paper is aimed at the determination of the minimum number of PMUs and the optimal locations required to achieve complete topological observability for the purpose of monitoring the system's stability status and the margin to voltage collapse.

The objective function of this OPP problem is the minimization of the number of PMUs required for complete system observability. Thus, it is required that each bus is observed by a minimum of one PMU. The basic formulation of the OPP problem using Binary Integer Programming (BIP) approach for any *N*-bus power system can be mathematically described as [11]:

$$S = \min_{x} \sum_{k=1}^{N} x_k \tag{1}$$

subject to:

$$Ax \geq b \tag{2}$$

$$x = [x_1\ x_2 \dots\ x_N]^T \tag{3}$$

where $x_k = \{0,1\}$, $k = \overline{1,N}$, $A \in \Re^{NXN}$ is the connectivity matrix of the considered system obtained from the binary transformation of the bus admittance matrix, *N* is the number of buses in the network, x_k is the PMU placement variable that equals 1 if a PMU is sited at bus *k*, and 0 if otherwise.

The connectivity matrix is defined as $a_{ij} = 1$ if the node *i* and *j* are linked or if $i = j$. $a_{ij} = 0$ if otherwise. $x \in \Re^N$ is the vector of the possible location of the PMUs.

The vector $b \in \Re^N$ is given as:

$$b = [1 1 \dots 1]^T \tag{4}$$

Example:

The objective function for the OPP problem for the IEEE 14-bus system [17] shown in Fig. 1 can be formulated as:

$$\min \quad \{x_1 + x_2 + \dots + x_{14}\}$$

subject to the following observability constraints for buses 1-14 respectively:

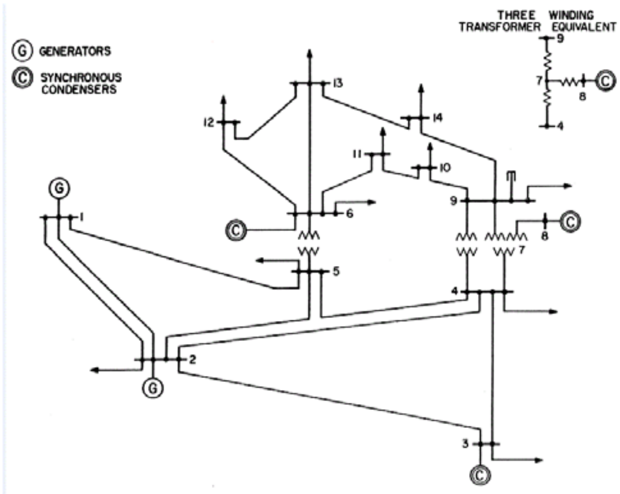
$$f_1 = x_1 + x_2 + x_5 \geq 1$$

$$f_2 = x_1 + x_2 + x_3 + x_4 + x_5 \geq 1$$

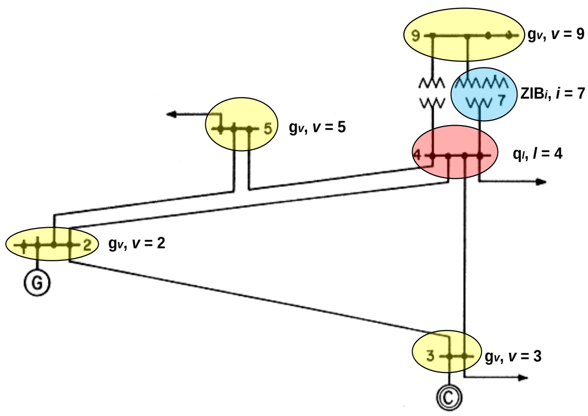
$$\vdots$$

$$f_{14} = x_9 + x_{13} + x_{14} \geq 1$$

$$b = [1\ 1\ \dots\ 1]^T$$



(a)



(b)

Fig. 1. IEEE 14-bus system (a) full network [17]; and (b) segment showing ZIB 7, incident bus 4, and buses connected to bus 4 (buses 2, 3, 5, 9)

The first observability constraint for Bus-1 (f_1) implies that at least one PMU must be located at either of buses 1, 2, or 5 in order to achieve complete observability. The second observability constraint for Bus-2 (f_2) implies that at least one PMU must be located at either of buses 1, 2, 3, 4, or 5. The connectivity matrix A is then obtained from the observability constraints given above.

The obtained optimal solution for this example is:

$x = [0\ 1\ 0\ 0\ 0\ 1\ 1\ 0\ 1\ 0\ 0\ 0\ 0\ 0]^T$

Thus, PMUs would need to be installed at buses 2, 6, 7, 9 for complete topological observability.

2.2 Binary integer programming

The Binary Integer Programming (BIP) method used in this paper for the solution of the modified OPP problems is based on the branch-and-bound algorithm. The branch-and-bound algorithm divides the feasible solution region into sub-problems by searching for the optimal binary integer solution, updating the best binary integer feasible points found, and verifying that the best integer feasible solution has been obtained [18,19].

The branch-and-bound algorithm is implemented in two parts. The branch part of the algorithm involves the creation of a search binary tree and the addition of constraints repeatedly. This branching step chooses a non-integer variable x_j. The constraint $x_j = 0$ is added to form a left branch, while the constraint $x_j = 1$ is added to form the right branch. The added constraints form the nodes which are subsets of a set. The sets are candidate solutions of the search space. For each node, a Linear Programming (LP) relaxation problem is solved using the constraints at that node. The decision to branch or proceed to another node depends on the outcome of the LP relaxation problem.

The bound part of the algorithm uses its bounding step to update the lower and upper bounds on the objective function within a given subset of the set for pruning unnecessary branches. This implies that if the lower bound of a node is greater than the upper bound of another node, the former is discarded from the search.

Two strategies are implemented for the selection of the branch variable in the search tree in MATLAB. The first strategy is to choose the variable with the minimum integer infeasibility, while the second strategy is to choose the maximum integer infeasibility whose value is closer to a predetermined value [19].

Similarly, two strategies are defined in the selection of the next node to search. These are the depth-first search strategy and the best-node first search strategy [19]. For the depth-first strategy, the algorithm chooses an unexplored child node a level down the tree. Alternatively, the algorithm moves to the node one level up in the tree and chooses a child node one level down from that node. The best-node search strategy chooses the node with the lowest bound on the objective function.

3 Modified OPP Problem Formulation Considering Zero Injection Buses (OPPZB)

3.1 Extended rules considering zero injection buses

Zero Injection Buses (ZIBs) are buses with no power or current injection. They are usually transfer buses and do not have a generator or load connected to them. The number of PMUs required for power system observability can be substantially reduced by considering ZIBs. This is because Kirchhoff's Current Law (KCL) can be used to calculate the current measurements in adjacent branches as given in Rule 4 in Section 1. The ZIBs in a network can generally be obtained from the bus data parameters of the power system.

If a zero injection bus *i* has *k* buses connected to it, the number of buses at that node equals *k + 1*. Applying Kirchhoff's Current Law (KCL), if *k* number of buses has phasors that are observable, the remaining bus with zero injection automatically becomes observable [4,11].

The connectivity matrix of the system with ZIBs would need to be modified. The modified connectivity matrix would include the connectivity matrix Z_b of the ZIB.

The modification of the connectivity matrix is based on the following rules proposed in this paper:

Rule 1: The OPP constraints are formulated to eliminate the placement of PMU at the zero injection buses. This is because the location of a PMU at a zero injection bus does not give any additional information.

Rule 2: A zero injection bus would be represented by a variable of zero in the connectivity matrix A to obtain a modified connectivity matrix A_z. The ZIB would then be replaced by a merged fictitious-bus comprising of the ZIB and its associated incident buses in the connectivity matrix Z_b. A variable of zero is used since the effects of the ZIB are already accounted for by the new constraints as given in Rule 4.

Rule 3: All constraints of the adjacent buses incident to a zero injection bus would be assigned a negative variable of 1 for their coefficient parameters in the connectivity matrix Z_b.

Rule 4: The network topology for buses incident to a zero injection bus would be transformed through bus merging. This transformation is only intended for the formulation of the constraint for a fictitious-bus comprising of the ZIB and the incident buses incident to it.

Rule 5: Vector b would be modified according to Rule 4. For the modified vector $\overline{b}$, the coefficients of the fictitious merged buses corresponding to the ZIB would be set equal to the number of buses incident to the ZIB, and the coefficients for the incident buses would be set equal to zero. The rest of the coefficients have value equal to 1 as in the basic OPP problem.

From the rules given in Section 1, it can be deduced that an unknown bus voltage can be calculated if the terminal voltage at the other end and the branch current phasor are known. Therefore, the constraints for the incident buses to the branch can be merged into a single constraint.

3.2 Modified OPP problem formulation considering ZIBs (OPPZB)

The same formulation of the OPP problem proposed in Section 2 applies, but with the connectivity matrix and the vector b modified according to the above rules as follows:

$$S = \min_{x} \sum_{k=1}^{N} x_k \tag{5}$$

subject to:

$$\overline{A}x \geq \overline{b} \tag{6}$$

$$x = [x_1\ x_2 \ldots\ x_N]^T \tag{7}$$

$$\overline{b} = [\overline{b}_1\ \overline{b}_2\ \ldots\ \overline{b}_N]^T \tag{8}$$

where $x_k = \{0,1\}$, $k = \overline{1, N}$. $\overline{A} \in \Re^{NXN}$ the modified connectivity matrix ($\overline{A} = A_z + Z_b$), and the vector $\overline{b}_k$, $k = \overline{1, N}$ are determined according to Rules 1-5. A flowchart for the implementation of the ZIB rules is given in Fig. 2.

where $Z_b(q_l, g_v, v = \overline{1, P_{ql}}) = -1$, $l = \overline{1, j_i}$,

$Z_b(i,l) = \sum_{l=1}^{j_i} Z_b(q_l, u)$, $i = \overline{1, r}$, $u = \overline{1, N}$,

$\overline{A}(i,:) = A(i,:)$, $\overline{b_i} = \left[b_1 : b_{i-1, j_i}, b_{i+1} \cdots b_N\right]^T$.

q_l is the *lth* incident bus connected to the *ith* Zero Injection Bus (ZIB), g_v is the *vth* bus connected to the *lth* incident bus, P_{ql} is the number of buses connected to the incident bus q_l, j_i is the number of incident buses connected to the *ith* ZIB. u is the *uth* bus in the network, r is the number of ZIBs, N is the number of buses in the network (Fig. 1b).

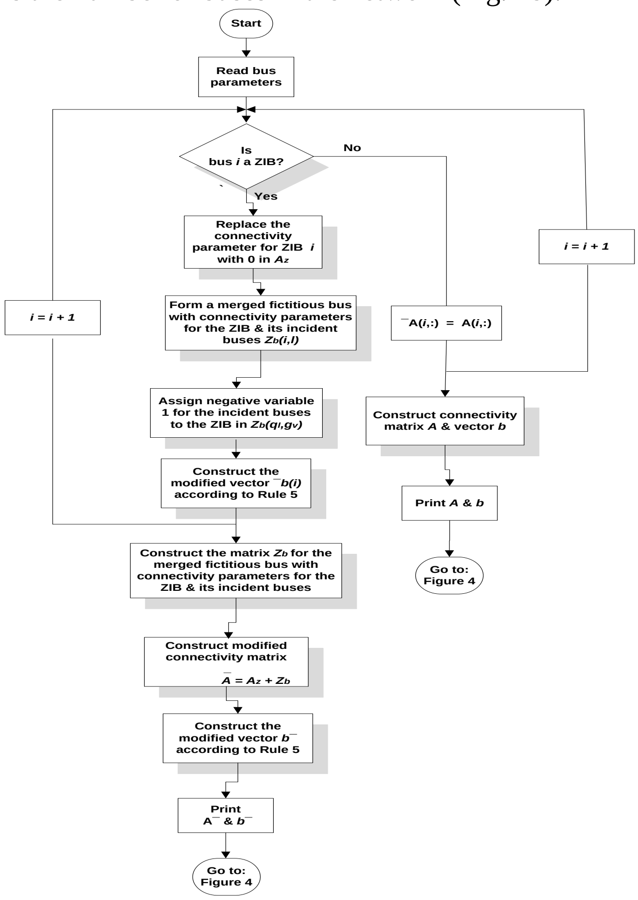


Fig. 2. Flowchart of the proposed ZIB rules

Using the IEEE 14-bus network as an example, bus-7 is identified as a zero injection bus based on the bus parameters of the network.

The constraints corresponding to this can be derived for all the buses incident to bus-7 as follows:

$$f_4 = x_2 + x_3 + x_4 + x_5 + x_7 + x_9 \geq 1$$

$$f_7 = x_4 + x_7 + x_8 + x_9 \geq 1$$

$$f_8 = x_7 + x_8 \geq 1$$

$$f_9 = x_4 + x_7 + x_9 + x_{10} + x_{14} \geq 1$$

The transformation of the topology as a result of the merging of the buses incident to bus 7 is given by the constraint for the fictitious bus f_{Zb7}:

$$f_{Zb7} = f_4 + f_7 + f_8 + f_9$$

$$= x_2 + x_3 + 3x_4 + x_5 + 4x_7 + 2x_8 + 3x_9 + x_{10} + x_{14} \geq 3$$

For the example from Section 2, and on the basis of Rules 1-5, the corresponding connectivity matrices are:

$$A_z = \begin{bmatrix} 1&1&0&0&1&0&0&0&0&0&0&0&0&0 \\ 1&1&1&1&1&0&0&0&0&0&0&0&0&0 \\ 0&1&1&1&0&0&0&0&0&0&0&0&0&0 \\ 0&1&1&1&1&0&1&0&1&0&0&0&0&0 \\ 1&1&0&1&1&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&1&0&0&0&0&1&1&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&1&0&0&0&0&0&0 \\ 0&0&0&1&0&0&1&0&1&1&0&0&0&1 \\ 0&0&0&0&0&0&0&0&1&1&1&0&0&0 \\ 0&0&0&0&0&1&0&0&0&1&1&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&1&1&0 \\ 0&0&0&0&0&1&0&0&0&0&0&1&1&1 \\ 0&0&0&0&0&0&0&0&1&0&0&0&1&1 \end{bmatrix}$$

$$Z_b = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&-1&-1&-1&-1&0&-1&0&-1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&1&3&1&0&4&2&3&1&0&0&0&1 \\ 0&0&0&0&0&0&-1&-1&0&0&0&0&0&0 \\ 0&0&0&-1&0&0&-1&0&-1&-1&0&0&0&-1 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \end{bmatrix}$$

$$\overline{A} = \begin{bmatrix} 1&1&0&0&1&0&0&0&0&0&0&0&0&0 \\ 1&1&1&1&1&0&0&0&0&0&0&0&0&0 \\ 0&1&1&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&1&0&1&1&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&1&0&0&0&0&1&1&1&0 \\ 0&1&1&3&1&0&4&2&3&1&0&0&0&1 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&1&1&0&0&0 \\ 0&0&0&0&0&1&0&0&0&1&1&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&1&1&0 \\ 0&0&0&0&0&1&0&0&0&0&0&1&1&1 \\ 0&0&0&0&0&0&0&0&1&0&0&0&1&1 \end{bmatrix}$$

The modified vector $\overline{b}$ is given by:

$\overline{b} = [1 1\ 1\ 0\ 1\ 1\ 3\ 0\ 0\ 1\ 1\ 1\ 1\ 1]^T$

Solving the optimization problem using the obtained matrix $\overline{A}$ results to an optimal solution of:

$x = [0\ 1\ 0\ 0\ 0\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0]^T$

Thus, PMUs would need to be installed at buses 2, 6, 9 for complete topological observability. No PMU is installed on bus 7 (ZIB) and the total number of PMUs to obtain full observability is reduced.

4 Modified OPP Problem Formulation Considering Critical Buses (OPPCB)

4.1 Modal participation factor algorithm

The derivation of Modal Participation Factor (MPF) in power system is based on the calculation of the Jacobian matrix of the power flow. The eigenvalues of the Jacobian matrix of the power system load flow are associated with the system's mode which gives the relationship between reactive power and the bus voltages.

The modes with the smallest eigenvalues are the most prone to voltage instability. Similarly, the eigenvectors describe the mode shape and indicate the mechanism of voltage instability in terms of the voltage weak areas prone to instability.

The considered system Jacobian can be obtained from the linearized steady-state power flow equations given by [20]:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_{P\theta} & J_{PV} \\ J_{Q\theta} & J_{QV} \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix} \tag{9}$$

$J_{P\theta}$, J_{PV}, $J_{Q\theta}$, J_{QV} are the Jacobian matrices for the real and reactive power sensitivities with respect to voltage angles and voltage magnitudes. ΔP and ΔQ are the increments in real and reactive powers respectively, $\Delta\Theta$ and ΔV are increments of the angle and voltage.

In order to focus the study on the reactive power demand and supply problem of the power system, as well as to minimize computational effort by reducing the dimensions of the Jacobian matrix J, the real power increment is assumed to be zero. This is supported by the fact that voltage collapse primarily occur as a result of reactive power deficit leading to loss of voltage control [20]. Thus, the angle increment is expressed as a function of the voltage increment. This permits the evaluation of the bus voltage increment *(ΔV)* and reactive power increment *(ΔQ)* for different operating conditions.

The reduced Jacobian resulting from this is given by:

$$0 = J_{P\theta}\,\Delta\theta + J_{PV}\,\Delta V \tag{10}$$

From Eqs. (9) and (10) above,

$$\Delta\theta = -J_{P\theta}{}^{-1} J_{PV}\,\Delta V \tag{11}$$

$$\Delta Q = J_{Q\theta}\,\Delta\theta + J_{QV}\,\Delta V \tag{12}$$

Substituting for $\Delta\theta$ in Eq. (12)

$$\Delta Q = J_{Q\theta}(-J_{P\theta}{}^{-1} J_{PV}\,\Delta V) + J_{QV}\,\Delta V \tag{13}$$

The reduced Jacobian is given by:

$$J_R = J_{QV} - J_{Q\theta} J_{P\theta}{}^{-1} J_{PV} \tag{14}$$

$$\Delta V = J_R^{-1} \Delta Q \tag{15}$$

The matrix J_R represents the linearized relationship between the incremental changes in the bus voltage (ΔV) and the bus reactive power injection (ΔQ).
Eigenvalue analysis of J_R results in the following expression:

$$J_R = \Phi \Lambda \Gamma \quad (16)$$

where Φ is the right eigenvector matrix of J_R, Γ is the left eigenvector matrix of J_R, Λ is the diagonal eigenvalue matrix of J_R.
The right eigenvector is said to give the mode shape which defines the relative activity of the state variables when a particular mode is excited. The left eigenvector weighs the contribution of this activity to this mode [20].
Eq. (16) can be re-written as:

$$J_R^{-1} = \Phi \Lambda^{-1} \Gamma \quad (17)$$

From Eqs. (15) and (17),

$$\Delta V = \Phi \Lambda^{-1} \Gamma \Delta Q \quad (18)$$

if $\Gamma\Phi = I$,
After left multiplication of Eq. (18) by Γ and since the geometrical product between the right and left eigenvectors is equal to the identity matrix I [20], Eq. (19) is obtained.

$$\Gamma \Delta V = \Lambda^{-1} \Gamma \Delta Q \quad (19)$$

The modal voltage variations ($v = \Gamma \Delta V$) can be related to the modal reactive power variation ($q = \Gamma \Delta Q$):

$$v = \Lambda^{-1} q \quad (20)$$

The relative modal participation factor of bus k in mode i is defined as [24]:

$$P_{ki} = \Phi_{ki} * \Gamma_{ik}, \quad i = \overline{1, N} \quad k = \overline{1, N} \quad (21)$$

where P_{ki} is the modal participation factor of bus k to mode i, Φ_{ki} is the right eigenvector matrix of J_R of bus k for the ith mode, Γ_{ik} is the left eigenvector matrix of J_R of bus k to the ith mode.

High values of P_{ki} indicate the buses most prone to voltage collapse. Thus, signifying the critical buses which need to be monitored with PMUs. The critical buses identification used in this paper is based on the bus modal participation factors derived from the smallest eigenvalues and their associated eigenvectors of the system's reduced Jacobian matrix J_R close to the point of voltage collapse.
The consideration of zero injection buses in the OPP problem formulation results to the use of fewer PMUs in order to achieve complete observability of the power system. PMU placement at sensitive/critical buses in the power system provides measurements from critical buses in the network.

These measurements can be used to provide the power system status (situational awareness) as the system operating conditions changes. The results from [16] show that the computation time for the OPP solution incorporating the critical buses information was reduced when compared to that without critical buses consideration.
Fig. 3 shows the process involved in the calculation of the modal participation factor.

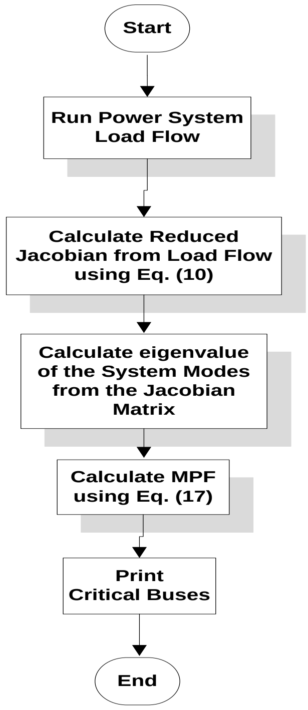


Fig. 3. Flowchart of the modal participation factor algorithm

4.2 Formulation of the modified OPP problem considering critical buses (OPPCB)

An equality constraint incorporating the information of the critical buses as described in Subsection 4.1 is used as an additional constraint in the OPP problem formulation in this subsection. This is done in order to assign a higher priority to the critical buses in the system.
The proposed formulation of the problem is given as:

$$S = \min_{x} \sum_{k=1}^{N} x_k \quad (22)$$

subject to:

$$Ax \geq b \quad (23)$$

$$\tilde{A} x = \tilde{b} \quad (24)$$

$$x = [x_1 \; x_2 \ldots x_N]^T \quad (25)$$

$$\tilde{b} = [\tilde{b}_1 \; \tilde{b}_2 \; \ldots \; \tilde{b}_N]^T \quad (26)$$

where $x_k \in \{0,1\}$, $\tilde{b}_k \in \{0,1\}$, $k = \overline{1, N}$.

The same formulation used in Subsection 2.1 applies here except for $\tilde{A}x = \tilde{b}$ which represents the constraints for the critical buses. The connectivity matrix for the critical buses $\tilde{A} \in \Re^{N \times N}$ is defined as $\tilde{a}_{ij} = 1$ if i is connected to j and bus i is a critical bus. If otherwise, $\tilde{a}_{ij} = 0$. Similarly, $\tilde{b}_k = 1$ if bus k is a critical bus. If otherwise, $\tilde{b}_k = 0$, $\tilde{b} \in \Re^N$.

The identification of the critical buses in this paper is based on the bus modal participation factors derived from the smallest eigenvalues and the associated eigenvectors of the system's reduced Jacobian matrix J_R obtained using Eq. (21). The critical buses are the buses with the highest participation factors.

The equality constraint used in Eq. (24) is a hard constraint. This requires that the constraint is satisfied by ensuring that the critical buses are observable in the OPP solution.

Using the IEEE 14-bus network as an example, the constraints for the critical buses (Buses 9, -10, and -14) are given by:

$$f_9 = x_4 + x_7 + x_9 + x_{10} + x_{14} = 1$$
$$f_{10} = x_9 + x_{10} + x_{11} = 1$$
$$f_{14} = x_9 + x_{13} + x_{14} = 1$$

The vector $\tilde{b}$ for the equality constraint and the matrix $\tilde{A}$ are given as:

$$\tilde{b} = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 1]^T$$

$$\tilde{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

The obtained optimal solution is:

$$x = [0\ 1\ 0\ 0\ 0\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0]^T$$

Thus, PMUs would need to be installed at buses 2, 6, 8, 9 for complete topological observability.

5 Modified OPP Problem Formulation Considering ZIBs and Critical Buses (OPPZCB)

The proposed OPP problem formulation for a case considering the critical buses and the zero injection buses (OPPZCB) is given below:

$$S = \min_{x} \sum_{k=1}^{N} x_k \tag{27}$$

subject to:

$$\bar{A}x \geq \bar{b} \tag{28}$$
$$\tilde{A}x = \tilde{b} \tag{29}$$
$$x = [x_1\ x_2 \ldots\ x_N]^T \tag{30}$$
$$\bar{b} = [\bar{b}_1\ \bar{b}_2\ \ldots\ \bar{b}_N]^T \tag{31}$$
$$\tilde{b} = [\tilde{b}_1\ \tilde{b}_2 \ldots\ \tilde{b}_N]^T \tag{32}$$

where $x_k \in \{0,1\}$, $\tilde{b}_k \in \{0,1\}$, $\bar{b}_k$, $k = \overline{1,N}$ are determined as in Section 3.

The same formulation used in Section 4 applies here except for the introduction of the modified connectivity matrix $\bar{A} \in \Re^{NXN}$ formulated for the case involving zero injection buses. The connectivity matrix for the critical buses $\tilde{A} \in \Re^{N \times N}$ is as given in Section 4.

6 OPP Problem Formulation Considering Single PMU Outage, Critical Buses, and ZIBs

It is desirable that measurements at the critical buses do not cease in the event of a PMU outage, loss of communication network, or loss of time synchronization. This is because during emergency/contingency situations, the most important measurements are the measurements from the critical buses in the system. In reality, the measurements from the critical buses are adequate for providing situational awareness of the power system.

The critical bus equality constraint in the OPP problem given in Sections 4 and 5 can be modified to ensure that priority is given to the critical buses during PMU placement such that each critical bus have at least one PMU observing it in the event of a *N-1* PMU outage. This is done by changing the vector $\tilde{b}_k \in \{0,1\}$ in Sections 4 and 5 to $\tilde{b}_k \in \{0,2\}$. This implies that each critical bus is monitored by at least two PMUs during normal operating conditions, and by at least one PMU during a *N-1* PMU outage. The proposed OPP problem formulation for the OPPZCB case (Section 5) for a single PMU outage with respect to voltage stability assessment is given below:

$$S = \min_{x} \sum_{k=1}^{N} x_k \tag{33}$$

subject to:

$$\bar{A}x \geq \bar{b} \tag{34}$$

$$\tilde{A}x = \tilde{b} \tag{35}$$

$$x = [x_1\, x_2 \ldots x_N]^T \tag{36}$$

$$\bar{b} = [\bar{b}_1\, \bar{b}_2\, \ldots\, \bar{b}_N]^T \tag{37}$$

$$\tilde{b} = [\tilde{b}_1\, \tilde{b}_2 \ldots\, \tilde{b}_N]^T \tag{38}$$

where $x_k \in \{0,1\}$, $\tilde{b}_k \in \{0,2\}$, $\bar{b}_k$, $k = \overline{1,N}$ are determined as in Section 3. The same formulation applies for the case of OPPCB (Section 4) except for the modification of the equality constraint $\tilde{b}_k$.

7 Test Systems and Case Studies

7.1 Test system

The proposed algorithm is tested on the IEEE-14 bus, IEEE-30 bus, and IEEE 57-bus benchmark test systems. The implementation of the algorithms was carried out using MATLAB Version 7.12.0.635 (R2011a) and MATLAB Global Optimization Toolbox.

7.2 Case studies

Case studies were simulated for the OPP problem on the test systems considered above.

These case studies include the following:

a. Minimizing the number of PMUs for complete topological observability with and without zero injection buses.

b. Minimizing the number of PMUs for complete topological observability with and without zero injection buses, and with the critical buses as an additional constraint to the optimization problem. The calculations are performed according to the algorithms in Fig. 4.

c. Minimizing the number of PMUs for complete topological observability with and without zero injection buses, and with the critical buses for *N-1* PMU outage.

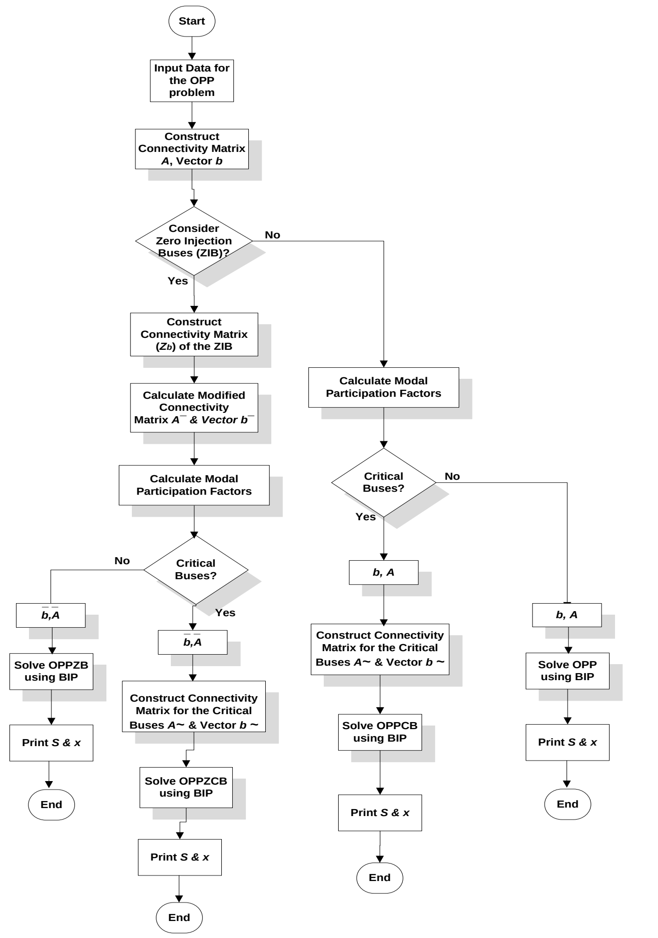


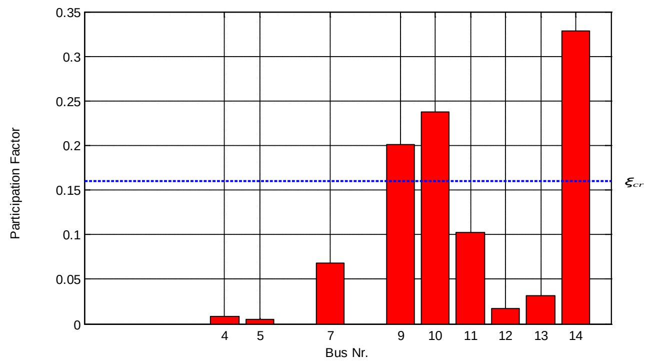
Fig. 4. Flowchart of the proposed algorithm

8 Results and Discussion

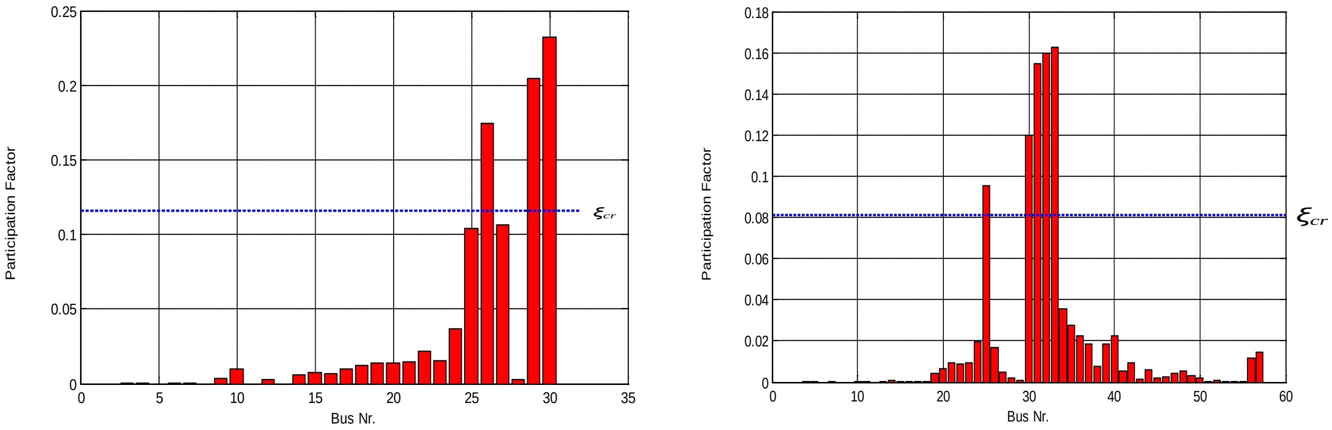
8.1 Results

Table 1 gives the participation factors obtained for various incremental loading of the load buses in the IEEE 14-bus system. From Table 1, the highest participation factor for load level 1 (steady-state condition) is 0.3287 (bus 14).

All buses with participation factors greater than or equal to the given threshold ξ_{cr} are identified as the critical buses, where ξ_{cr} = 50 % of the highest participation factor obtained for the test system.



(a)



(b) (c)

Fig. 5. Bus modal participation factor for (a) IEEE 14-bus test system; (b) IEEE 30-bus test system; and IEEE 57-bus test system

Table 1
Participation factor for IEEE-14 bus test system under load increase scenario

	Load Level	Bus 4	5	7	9	10	11	12	13	14
1.	Participation Factor (λ = 0.0)	0.0088	0.0046	0.068	**0.2020**	**0.2380**	0.1030	0.0169	0.0311	**0.3287**
2.	Participation Factor (λ = 0.1)	0.0093	0.0049	0.0680	**0.2038**	**0.2379**	0.1004	0.0159	0.0301	**0.3288**
3.	Participation Factor (λ = 0.2)	0.0098	0.0054	0.0696	**0.2059**	**0.2380**	0.0977	0.0150	0.0293	**0.3296**
4.	Participation Factor (λ = 0.3)	0.0107	0.0062	0.0703	**0.2082**	**0.2371**	0.0946	0.0140	0.0284	**0.3304**
5.	Participation Factor (λ = 0.4)	0.0122	0.0076	0.0720	**0.2116**	**0.2362**	0.9050	0.0128	0.0272	**0.3302**
6.	Participation Factor (λ = 0.5)	0.0156	0.0108	0.0744	**0.2168**	**0.2343**	0.0844	0.0112	0.0254	**0.3271**

Table 2
OPP Results for the various problem formulation in Sections 3, 4, and 5

Test System	Zero Injection Buses	OPPZB Placement Using BIP (Section 3)	Critical Buses	OPPCB Placement Using BIP (Section 4)	OPPZCB Placement Using BIP (Section 5)
IEEE 14-bus	7	2, 6, 9	9, 10, 14	2, 6, 8, 9	2, 10, 13
IEEE 30 -bus	6, 9, 11, 25, 28	1, 5, 10, 12, 19, 23, 27	26, 29, 30	2, 4, 6, 10, 11, 12, 15, 18, 25, 27	1, 2, 12, 16, 19, 21, 26, 27
IEEE 57-bus	4, 7, 11, 21, 22, 24, 26, 34, 36, 37, 39, 40, 45, 46, 48	1, 6, 10, 15, 20, 25, 29, 32, 41, 49, 54	25, 30, 31, 32, 33	1, 6, 15, 17, 19, 22, 25, 27, 32, 36, 38, 41, 46, 51, 52, 55, 57	1, 6, 10,13, 19, 25, 29, 32, 41, 49, 54

Table 3
Comparison of OPP results with results in the literature with respect to the number of PMUs

Test System	OPPZB Placement Using BIP (Section 3)	OPPZCB Placement Using BIP (Section 5)	[10]	[12]	[13]
IEEE 14-bus	3	3	3	3	3
IEEE 30 -bus	7	8	7	7	N/A
IEEE 57-bus	11	11	12	11	14

Table 4
Comparison of OPP results with results in the literature for the case of a single PMU outage

Test System	OPPCB Placement (Critical Buses Only)	OPPZCB Placement (Critical Buses & ZIB)	Nr. of PMUs for OPPCB Placement (Critical Buses Only)	Nr. Of PMUs for OPPZCB Placement (Critical Buses & ZIB)	[10]	[12]	[13]
IEEE 14-bus	2, 7, 9, 11, 13	2, 9, 10, 13	5	4	7	7	7
IEEE 30 -bus	2, 4, 6, 10, 11, 12, 15, 20, 25, 26, 27, 29	N/A	12	N/A	17	15	N/A
IEEE 57-bus	1, 2, 6, 13, 19, 22, 25, 27, 30, 32, 33, 36, 39, 41, 44, 47, 51, 52, 55	1, 6, 10,15, 20, 25, 29, 30, 32, 33, 41, 49, 54	19	13	26	26	29

Rows 2-6 of Table 1 give the result obtained for the IEEE 14-bus test system for real power and reactive power load increments with a constant power factor (p.f), for up to 50 % ($\lambda = 0.5$), where λ is the loading factor for the load increments.

Figs. 5a-5c give the plots of the participation factors calculated according to Eq. (21) for the test systems used.

Only buses 9, 10, and 14 have participation factors above the given threshold. Therefore, the critical buses for the IEEE 14-bus test system are buses-9, 10, and 14. The impact of increased system loading on the bus participation factors obtained for load level 1 (steady-state conditions) was carried out.

Table 2 presents the result for the optimal PMU locations for the OPP problem formulations for the OPPZB case as given in Section 3, OPPCB case (Section 4), and the OPPZCB case (Section 5). Table 3 presents the comparison of the number of PMUs obtained for the OPPZB and OPPZCB problem formulations with the results available in some previous publications.

Table 4 gives the results for the PMU placement for the case when a single PMU outage is considered for the OPPCB and OPPZCB problem formulations, and their comparison with the results available in some previous publications where the critical buses were not used.

8.2 Discussion

From the results obtained as given in Table 1, it was observed that the increase in system loading in all the load buses did not have any effect on the initial critical buses obtained as demonstrated with the IEEE 14-bus system.

Comparison of the results in Table 2 shows that the number of PMUs required for complete observability is reduced when zero injection buses were considered in the problem formulation. The number of PMUs required for complete system observability for the IEEE 14-bus test system when zero injection buses are considered reduces to 3. Similarly, for the IEEE 30-bus test system, the number of PMUs reduces from 10 to 7.

For the IEEE 57-bus test system, the number of PMUs required reduces to 11 from the previous 17 for the base case where zero injection buses and the critical buses information were not considered.

The optimal locations obtained for the base case without ZIBs and critical buses are:

- IEEE 14-bus test system: {2, 6, 7, 9}
- IEEE 30-bus test system: {4, 7, 9, 10, 12, 18, 24, 25, 27, 28}
- IEEE 57-bus test system: {1, 4, 6, 13, 19, 22, 25, 27, 29, 32, 36, 39, 41, 45, 47, 51, 54}

From the results obtained for the three test systems, it was observed that the application of the proposed rules in Section 3 resulted to the elimination of the placement of PMUs at the Zero Injection Buses (ZIBs) for OPP problems involving the consideration of the ZIBs. This is in accordance with Rule 1 proposed in Section 3, and is due to the fact that the placement of PMUs at the zero injection buses does not provide any additional information. Thus, the placement of a PMU at the ZIBs is an inefficient way of utilising the limited PMUs available. With the correct placement of PMUs at the critical load buses or non-ZIBs, other system information from the power system can easily be streamed by the PMUs as analogue measurements (real and reactive powers) and digital bits (circuit breaker/disconnector status information).

From the foregoing, the effect of combining ZIBs and the critical buses in the OPP problem formulation and solution include: i) the reduction in the computation time of the OPP solution as shown by the authors in [16]; ii) improved computational efficiency since the number of iterations required is considerably reduced [16]; iii) allows for emphasis on the power system's critical buses especially during phased (multi-stage) PMU placement; iv) reduction in the number of PMUs required for complete topological observability; and v) The critical buses are given priority in the formulation of the OPP problem. This ensures that the critical buses are covered by a minimum of two PMUs during steady-state conditions, and by at least one PMU during a single *N-1* PMU outage. Thus, the critical buses are observable for *N-1* PMU outage due to the loss of measurements from Voltage Transformers (VTs)/Current Transformers (CTs), PMU failure, loss of communication channel, or loss of time synchronization.

Consequently, redundant measurements relating to a particular critical bus can be used for bad data detection and measurement verification since two sources of measurements are simultaneously obtainable from that critical bus. Also, comparisons done with the existing methods in the literature for a single PMU outage showed that fewer additional PMUs are required as shown in Table 4. For example, a 43 % reduction in the total number of PMUs for a single PMU outage was recorded for the IEEE 14-bus test system when compared with [10], [12-13].

9 Conclusion

This paper extends the findings from the authors' paper [16] and investigates the proposal of new formulations and modification of the methods of solution of the OPP problem for voltage stability assessment in power systems. It incorporates the impact of the critical/voltage weak buses and the buses with zero injection on the full observability of the system.

Five OPP problems were formulated and solved using the Binary Integer Programming approach. This includes:

- Basic formulation from available methods existing in the literature; and
- New modified formulations with the consideration of Zero Injection Buses (ZIBs), incorporating the critical buses obtained from modal participation factors, and PMU outage in the structure of the OPP problem.

The application of the proposed algorithm for ZIBs in the study networks showed that PMU placement on a ZIB was automatically eliminated. This is an advantage since ZIBs do not provide any additional information compared to that obtainable from the load buses. Thus, additional information available at the load buses can be streamed by the PMUs as analogue measurements or binary information.

The proposed algorithm identifies the critical buses prone to voltage instability and uses this as one of the constraints for the optimization problem. This is aimed at increasing the accuracy of synchrophasor-based voltage stability assessment schemes to prevent voltage instability/voltage collapse through the identification of the weak areas of the system during the PMU placement stage. A problem formulation for a single PMU outage shows that redundancy can easily be provided at the critical buses in the power system with fewer PMUs when voltage stability assessment is considered, rather than with most of the existing methods where state estimation is the objective. Simulation results show that the proposed OPP problem formulations and their solutions are simple, effective, and can be used for the optimal placement of PMUs in practical systems, such that continuous situational awareness of the critical buses is provided even when there is an outage of the main PMU covering a critical bus. Also, faster convergence rate with less iteration results to less computational time. Therefore, the proposed method is suitable for large power system networks.

Future research work would consider the implementation of the proposed method for wide area voltage stability assessment/monitoring.

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